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ANALYZING A MACROPRUDENTIAL INSTRUMENT DURING THE COVID-19 PANDEMIC USING BORDER COLLISION BIFURCATION

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RESUMEN: Bank Indonesia, el banco central de Indonesia, ha realizado ajustes en un instrumento de política macroprudencial llamado índice de intermediación macroprudencial (IIM) para impulsar el crecimiento de los préstamos en el contexto de la recuperación económica nacional debido a la pandemia de COVID-19. En este artículo, se desarrolla un modelo dinámico de préstamo bancario con comportamiento procíclico, y se equipa con el instrumento predecesor del IIM denominado requerimiento de reserva basado en la relación préstamo-depósito (RR-RPD). Examinamos los efectos de los parámetros RR-RPD en la dinámica del préstamo utilizando el análisis de bifurcación de colisión de fronteras para determinar los valores umbral de los parámetros RR-RPD para que se pueda mantener la estabilidad del equilibrio del préstamo. Este modelo se aplica a los datos mensuales de los bancos comerciales de Indonesia antes y durante la pandemia de COVID-19 para evaluar la región de estabilidad de los parámetros del instrumento.

Palabras Clave: Política macroprudencial, Dinámica de préstamos, Bifurcación de colisión fronteriza, COVID-19.

ABSTRACT: Bank Indonesia, the central bank of Indonesia, has made adjustment settings in a macroprudential policy instrument called macroprudential intermediation ratio (MIR) to boost loan growth in the context of national economic recovery due to the COVID-19 pandemic. In this paper, a dynamic model of bank loan with procyclicality behavior is developed, and it is equipped with the predecessor of the MIR instrument called loan-to-deposit ratio based reserve requirement (LDR-RR). We examine the effects of LDR-RR parameters on the dynamics of loan using the border collision bifurcation analysis to determine the threshold values of the LDR-RR parameters so that the stability of loan equilibrium can be maintained. This model is applied to monthly data of Indonesian commercial banks before and during the COVID-19 pandemic to assess the stability region of the instrument parameters.

Keywords: Macroprudential policy, Loan dynamics, Border collision bifurcation, COVID-19.

1. Introduction

In March 2021, in order to boost loan distribution to the businesses sector in the framework of national economic recovery due to the COVID-19 pandemic while maintaining financial system stability, Bank Indonesia, the central bank of Indonesia, updated its macroprudential policy instrument called macroprudential intermediation ratio (MIR) by adding the export money order from the bank balance sheet component and eliminating the upper disincentive parameter in the calculation of MIR (Bank Indonesia Press Conference, 2021; Governors Board of Bank Indonesia Regulation, 2021). Since it was first established, there are several changes on the setting of MIR instrument carried out by Bank Indonesia in order to promote economic growth and maintain financial system stability. The MIR instrument is aimed to manage banking intermediation function without forgetting the precautionary principle by controlling the bank's intermediation ratio, to fit the capacity and target of economic growth (Bank Indonesia, 2021).

The MIR instrument is constructed from the predecessor macroprudential instruments called loan-to-deposit ratio based reserve requirement (LDR-RR) and loan-to-financing ratio based reserve requirement (LFR-RR). Since it was first implemented in March 2011 until now, this macroprudential instrument has undergone several regulatory changes. In December 2013, the upper bound of LDR target was lowered to bound loan growth; in August 2015, the LDR base was changed to LFR in order to boost loan growth; in August 2016, the lower bound of LFR target was increased to boost loan growth; in July 2018, the LFR base was changed to financing-to-funding ratio (FFR) to expand the type of banking intermediation (Wijayanti et al., 2020). In November 2019, the lower and upper bounds of MIR target were increased, and the lower and upper disincentive values were determined based on the level of non-performing loan ratio and capital adequacy ratio (CAR) of banks that must meet certain criteria. (Bank Indonesia Regulation, 2019). Recently, in March 2021 there was a MIR arrangement as previously described responding the COVID-19 pandemic.

Changes in the setting of dynamic instrument parameters are carried out to control the growth of banking loan. Loan is the component of a bank's balance sheet that generates the most profit among other types of assets. Even so, loan is an asset that has the highest risk profile. When a bank channels too much loan, the availability of liquid assets will be at a low level, so this will interfere the bank's ability to meet its short-term obligations. Conversely, too low loan channeled by banks will interfere the bank's intermediation function, namely capital allocation and financial intermediaries in the economy (Kahou & Lehar, 2017).

The general purpose of this research is to examine the effect of changes in LDR-RR parameter settings on the dynamics of banking loan in Indonesia using a dynamic model of banking loan based on gradient adjustment process. The reason for using the LDR-RR instrument rather than the MIR in this paper is because the formula is simpler, which only involves loan and deposit variables while the MIR formula involves more bank balance sheet variables such as issued securities, bonds, and export money order. Even so, the essence that became the foundation remains the same, changes in the setting of LDR-RR and MIR parameters that have been done are aimed at controlling loan growth. We analyze the LDR-RR parameters using border collision bifurcation theory to determine the boundary regions of the LDR-RR parameters values that maintain the stability of loan equilibrium. Based on the Indonesian banking data before and during the COVID-19 pandemic, the model’s parameters are estimated using spiral optimization method. As a specific purpose of the research, we apply the analytical and numerical results to analyze the MIR's March 2021 policy in responding the impact of the COVID-19 pandemic on Indonesian banking loan.

2. Literature studies

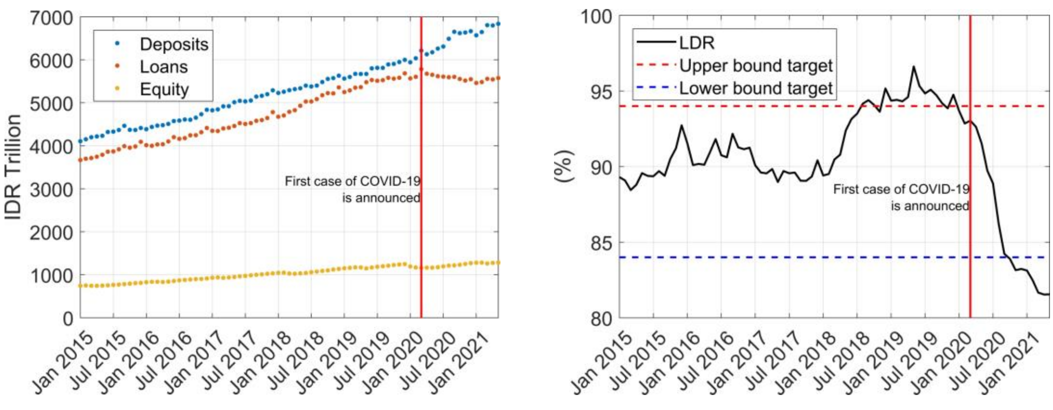
Efforts to assess the effect of changes in the value of LDR-RR parameters can be made using the banking industry model. The banking industry model views bank as an industrial company that wants to maximize its objective function, such as profit, while still meeting some existing constraints. In Mathematics, this reliable optimization can be solved by using the Lagrange multiplier method analytically to obtain optimal portfolio selection from bank balance sheet and interest rate variables. The most famous model is the Monti-Klein model (Klein, 1971; Monti, 1972). The following researchers used the Monti-Klein model to examine the effects of LDR-RR instruments on optimal banking portfolios. Gunadi and Harun (2011) examined the effect of the lower disincentive parameter and the lower bound of LDR target on the bank's optimal portfolio and sensitivity. Meanwhile, Satria et al. (2016) examined the effect of the lower and upper disincentive parameters as well as the lower and upper bounds of LDR target against the bank's optimal portfolio.

Mathematically, research to build a model of banking dynamics has been done for quite a long time. In (Sumarti et al, 2013; Sumarti, 2014; Sumarti et al., 2018), some models were built by investigating the dynamics of loan and deposit volume based on the Monti-Klein profit function, Lotka-Volterra model, and logistic model with harvesting, respectively. The model of Sumarti et al. (2018) is then developed by Ansori (2021) becoming a system of differential equations that includes LDR-RR policy. Recently, Ansori et al. (2021b) used an algorithm for simulating banking network to assess the optimal values of the LDR-RR parameters that can optimize the banking system stability.

The model studied in this paper uses a gradient adjustment process, which is a dynamic model where the determination of the quantity of a company's products in the next period is based on gradient information in the current period (Bischi et al., 2010). This gradient information is a partial derivative of an objective function, for example profit, to the quantity of a product. In some literatures, such gradient information is also commonly referred to as bounded rational expectation (Elsadany, 2010; Elsadany, 2017). The bank, which is also a company, has a product in the form of loan. Therefore, the bank's behavior in determining the amount of loan can be modeled by the gradient adjustment process. The use of gradient adjustment process to model banking loan dynamics can be found in (Fanti, 2014; Brianzoni & Campisi, 2021). Fanti (2014) examined the effect of CAR policy on the dynamics of loan competition between two banks, while Brianzoni and Campisi (2021) reviewed Fanti model involving competition between large and small banks.

3. Data and model

3.1. Data of Indonesian commercial banks



(a) (b)

Figure 1. (a) Deposits, loans, and equity data and (b) loan-to-deposit ratio (LDR) of commercial banks in Indonesia in the period January 2015 - May 2021

Data used in this study is monthly data of deposits, loans, and equity of commercial banks in Indonesia in the period January 2015 to May 2021, as show in Figure 1. The data is taken from the financial services authority of Indonesia (Otoritas Jasa Keuangan, 2021). The first case of COVID-19 in Indonesia is announced in March 2020. From Figure 1a, we can observe that in the period before and after the case was announced, the commercial banks' deposits and equity tend to increase every time, on the other hand, the loans tend to increase in the period before the case is announced but decline afterwards. This shows that the loans of commercial bank in Indonesia are affected by the COVID-19 pandemic. In Figure 1b, the loan-to-deposit ratio (LDR) of commercial banks during the pandemic period decreases dramatically. This phenomenon made Bank Indonesia to set new arrangements on the MIR instrument in March 2021, in order to boost banking loan growth.

3.2. Mathematical model

Suppose at the time t, the funding side of a bank's balance sheet consists of deposits D_t and equity E_t. The financing side consists of loan L_t and liquid assets A_t. Liquid assets contains reserve requirement (RR) G_t.

The identity of balance sheet requires the funding total must be equal to the financing total. Liquid assets act as the balancing variable:

$$A_t = D_t + E_t - L_t \tag{1}$$

By explicitly bringing up RR into equation (1), non-RR liquid assets ($A_t - G_t$) is written as:

$$A_t - G_t = D_t + E_t - L_t - G_t \tag{2}$$

The equity must meet the capital adequacy ratio (CAR) policy. CAR is the ratio between capital (equity) and risk-weighted assets (RWA). In this paper, it is assumed that loan has a risk profile of 100% and liquid assets have a risk profile of 0%, so the bank's CAR becomes:

$$\frac{E_t}{\mathrm{RWA}} = \frac{E_t}{1\times L_t + 0\times A_t} = \frac{E_t}{L_t} \geq \kappa_0 \tag{3}$$

where κ_0 is the minimum CAR determine by the central bank that must be met by the bank, where $0 < \kappa_0 < 1$.

In Indonesia, RR is the sum of primary RR G_t^p, secondary RR G_t^s, and LDR-RR G_t^{LDR}:

$$G_t = G_t^p + G_t^s + G_t^{LDR} \tag{4}$$

where $G_t^p = \rho_p D_t$ ($0 < \rho_p < 1$), $G_t^s = \rho_s D_t$ ($0 < \rho_s < 1$), and G_t^{LDR} is formulated as:

$$G_t^{LDR} = \begin{cases} 0 & \text{if } \lambda_{lb} \leq L_t/D_t \leq \lambda_{ub} \\ \gamma_{lb}(\lambda_{lb} - L_t/D_t)D_t & \text{if } L_t/D_t < \lambda_{lb} \\ \gamma_{ub}(L_t/D_t - \lambda_{ub})D_t & \text{if } L_t/D_t > \lambda_{ub} \text{ and } E_t/L_t < \kappa_1 \\ 0 & \text{if } L_t/D_t > \lambda_{ub} \text{ and } E_t/L_t \geq \kappa_1 \end{cases} \tag{5}$$

where λ_{lb} is the lower bound of LDR target, λ_{ub} is the upper bound of LDR target, γ_{lb} is the lower disincentive parameter, γ_{ub} is the upper disincentive parameter, and κ_1 is the incentive CAR. Those parameters yield the following conditions:

$$0 < \lambda_{lb} < \lambda_{ub}\ ,\ 0 < \gamma_{lb} < 1,\ 0 < \gamma_{ub} < 1, \text{ and } \kappa_0 \leq \kappa_1 \tag{6}$$

Based on the data in Figure 1, the average CAR of commercial banks in Indonesia is 23.21%. This number exceeds the value of incentive CAR of LDR-RR which is 14%. In fact, in (Bank Indonesia Regulation, 2010) the CAR exceeds 19%, see in Figure 2. In (Governors Board of Bank Indonesia Regulation, 2021), this 19% value is used to distinguish the amount of the lower disincentive parameter imposed on banks that have MIR value less than the lower bound target. If the bank's CAR is more than the value of CAR incentive but less than 19% then the bottom disincentive parameter is charged at 0.1, while if the bank's CAR ratio is more than 19% then the bottom disincentive parameter is charged at 0.15. Because the average CAR of commercial banks in Indonesia exceeds the value of CAR incentive, in this model the bank is assumed to always have a CAR that is not less than the value of CAR incentive, that is $E_t/L_t \geq \kappa_1$. To simplify calculations, suppose the bank's CAR is always constant:

$$\frac{E_t}{L_t} = \kappa \tag{7}$$

where $\kappa \geq \kappa_1$. The constant CAR assumption can be viewed as the average of banking CAR.

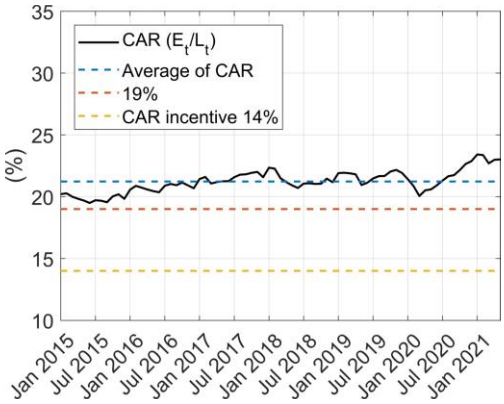


Figure 2. The CAR of commercial banks in Indonesia in simplified calculation (equity:loans).

Equation (7) causes the last two lines of equation (5) can be omitted, so that the LDR-RR calculation becomes:

$$G_t^{LDR} = \begin{cases} 0 & \text{if } \lambda_{lb} \leq L_t/D_t \\ \gamma_{lb}(\lambda_{lb} - L_t/D_t)D_t & \text{if } L_t/D_t < \lambda_{lb} \end{cases} \tag{8}$$

The bank's profit π_t is calculated as follows:

$$\pi_t = r_t^L L_t + r_A(A_t - G_t) - r_t^D D_t - r_E E_t - C_t \tag{9}$$

where r_t^L, r_A, r_t^D, r_E, and C_t respectively denote loan interest rate, rate of return of liquid assets, deposit interest rate, equity costs, and operating expenses.

Inspired by Fanti (2014) and Brianzoni and Campisi (2021), loan is assumed to follow the following gradient adjustment process:

$$L_{t+1} = L_t + \alpha_L L_t \frac{\partial \pi_t}{\partial L_t} \tag{10}$$

where α_L is called the speed of adjustment parameter, $\alpha_L > 0$.

Model (10) can be viewed as procyclical behavior of the bank in distributing loan. When the economy in the current period is good, in this case $\partial \pi_t / \partial L_t > 0$, the bank will increase the distribution of loan in the next period. Vice versa, when the economy in the current period is not good, in this case $\partial \pi_t / \partial L_t < 0$, the bank will reduce the distribution of loan in the next period. Because the LDR-RR instrument is a countercyclical instrument, that is against procyclicality, the model is very suitable for analyzing the impact of LDR-RR instrument on banking loan dynamics.

Partial derivative of π_t against L_t is:

$$\frac{\partial \pi_t}{\partial L_t} = \begin{cases} r_t^L + L_t \frac{\partial r_t^L}{\partial L_t} - \left[r_A + (r_E - r_A)\kappa + \frac{\partial C_t}{\partial L_t} \right] & \text{if } L_t/D_t \geq \lambda_{lb} \\ r_t^L + L_t \frac{\partial r_t^L}{\partial L_t} - \left[r_A + (r_E - r_A)\kappa + \frac{\partial C_t}{\partial L_t} \right] + r_A \gamma_{lb} & \text{if } L_t/D_t < \lambda_{lb} \end{cases} \tag{11}$$

Loan interest rate is assumed to follow the inverse of loan demand with $\partial r_t^L / \partial L_t < 0$ (Freixas & Rochet, 2008). Without loss of generality, we assume:

$$r_t^L = a_L - b_L L_t \tag{12}$$

where $a_L > 0$ and $b_L > 0$. Furthermore, it is assumed that the rate of return on liquid assets and equity costs are both constant, where $0 < r_A < 1$ and $0 < r_E < 1$. Also, the marginal operating expenses against loan $\partial C_t/\partial L_t$ is assumed to be constant $\partial C_t/\partial L_t = c_L$, where $0 < c_L < 1$.

Thus, the loan model (10) is changed to:

$$L_{t+1} = \begin{cases} L_t + \alpha_L L_t(a_L - \Lambda - 2b_L L_t) & \text{if } L_t/D_t \geq \lambda_{lb} \\ L_t + \alpha_L L_t(a_L - \Lambda + r_A\gamma_{lb} - 2b_L L_t) & \text{if } L_t/D_t < \lambda_{lb} \end{cases} \quad (13)$$

where $\Lambda = r_A + (r_E - r_A)\kappa - c_L$.

Equation (13) means that the determination of loan in the next period depends on the position of the bank's LDR against the lower bound of LDR target. When the bank's LDR is smaller than the lower bound of LDR target, the bank will be penalized by adding more reserves in Bank Indonesia. This penalty makes banks suffer from higher equity costs (Satria et al., 2016). Therefore, in order to avoid or reduce the amount of the penalty, in the next period the bank will increase the distribution of loan as much as $\alpha_L r_A \gamma_{lb} L_t$ more than if it does not get penalty before.

Meanwhile, the dynamics of deposit are assumed to follow the discrete logistic model below:

$$D_{t+1} = D_t + \alpha_D D_t\left(1 - \frac{D_t}{K_D}\right) \quad (14)$$

where $\alpha_D > 0$ is the growth rate of deposit and $K_D > 0$ is the carrying capacity. The assumption of this logistic model is in line with those in (Sumarti et al., 2018; Ansori et al., 2019b; Ansori et al., 2021a).

4. Results

4.1. Border collision bifurcation analysis

The deposit model (14) has an equilibrium $D^* = 0$ or $D^* = K_D$. But $D^* = 0$ is not desirable because it makes the LDR hasving denominator of zero, therefore the equilibrium of deposit chosen here is $D^* = K_D$. From the loan model (13), we can get loan equilibrium points L^* as follows:

$$L^*_{(0)} = 0 \quad (15)$$

$$L^*_{(1)} = \frac{a_L - \Lambda}{2b_L} \text{ if } L^* \geq \lambda_{lb} K_D \quad (16)$$

$$L^*_{(2)} = \frac{a_L - \Lambda + r_A\gamma_{lb}}{2b_L} \text{ if } L^* < \lambda_{lb} K_D \quad (17)$$

To make sure $L^*_{(1)}$ and $L^*_{(2)}$ positive, it must be fulfilled that:

$$a_L > \Lambda \quad (18)$$

The focus of this paper is to examine the values of the lower disincentive parameter γ_{lb} and the lower bound of LDR target λ_{lb}. Note that in equation (17), when γ_{lb} goes up, $L^*_{(2)}$ also goes up. As a result, $L^*_{(2)}$ will move closer to the border $\lambda_{lb} K_D$. When the equilibrium $L^*_{(2)}$ intersects or collides with the border $\lambda_{lb} K_D$, in other words $L^*_{(2)} = \lambda_{lb} K_D$, consequently the equilibrium point will lose its stability through a bifurcation called border collision bifurcation. Border collision bifurcation is a bifurcation that occurs in a system due to equilibrium points colliding with borders when parameters vary (Wong, 2011).

Using the definition of border collision bifurcation, parameters γ_{lb} and λ_{lb} are analyzed to study their effect on the stability of loan equilibrium. The result of the analysis is stated in the following theorem.

Theorem 1.

(a) *[The case of lower disincentive parameter] The loan equilibrium* $L^*_{(2)}$ *can lose its stability through border collision bifurcation when* $\gamma_{lb} = \gamma_{lb}^{BC}$, *where* $\gamma_{lb}^{BC} = \frac{2b_L\lambda_{lb}K_D-(a_L-\Lambda)}{r_A}$. *The equilibrium* $L^*_{(2)}$ *is stable when* $\gamma_{lb} < \gamma_{lb}^{BC}$.

(b) *[The case of lower bound of LDR target] The loan equilibrium point* $L^*_{(1)}$ *can lose its stability through border collision bifurcation when* $\lambda_{lb} = \lambda_{lb}^{BC_1}$, *where* $\lambda_{lb}^{BC_1} = \frac{a_L-\Lambda}{2b_LK_D}$. *The equilibrium* $L^*_{(1)}$ *is stable when* $\lambda_{lb} < \lambda_{lb}^{BC_1}$. *Meanwhile, the loan equilibrium* $L^*_{(2)}$ *can lose its stability through border collision bifurcation when* $\lambda_{lb} = \lambda_{lb}^{BC_2}$, *where* $\lambda_{lb}^{BC_2} = \frac{a_L-\Lambda+r_A\gamma_{lb}}{2b_LK_D}$. *The equilibrium* $L^*_{(2)}$ *is stable when* $\lambda_{lb} > \lambda_{lb}^{BC_2}$.

Proof

(a) The border collision bifurcation occurs when $L^*_{(2)} = \lambda_{lb}K_D$ or in other words $\frac{a_L-\Lambda+r_A\gamma_{lb}}{2b_L} = \lambda_{lb}K_D$. Through simple calculations with focus on finding value γ_{lb}, we can get $\gamma_{lb} = \frac{2b_L\lambda_{lb}K_D-(a_L-\Lambda)}{r_A}$. The equilibrium $L^*_{(2)}$ is stable in the sense of border collision bifurcation if $L^*_{(2)} < \lambda_{lb}K_D$ which is $\gamma_{lb} < \frac{2b_L\lambda_{lb}K_D-(a_L-\Lambda)}{r_A}$, in other words, $L^*_{(2)}$ will never collide with the border $\lambda_{lb}K_D$. (

(b) In a similar way, but the focus changes to find the value of λ_{lb}, for the case $L^*_{(1)} = \lambda_{lb}K_D$, we get $\lambda_{lb} = \frac{a_L-\Lambda}{2b_LK_D}$, and for the case $L^*_{(2)} = \lambda_{lb}K_D$, we get $\lambda_{lb} = \frac{a_L-\Lambda+r_A\gamma_{lb}}{2b_LK_D}$. The equilibrium $L^*_{(1)}$ is stable when $L^*_{(1)} > \lambda_{lb}K_D$ or $\lambda_{lb} < \frac{a_L-\Lambda}{}$ and the equilibrium $L^*_{(2)}$ is stable when $L^*_{(2)} < \lambda_{lb}K_D$ or $\lambda_{lb} > \frac{a_L-\Lambda+r_A\gamma_{lb}}{2b_LK_D}$. □.

From Theorem 1, the loan equilibrium L^* is stable when the LDR-RR's parameters yield the following conditions: $\lambda_{lb} < \lambda_{lb}^{BC_1} = \frac{a_L-\Lambda}{2b_LK_D}$ or ($\gamma_{lb} < \gamma_{lb}^{BC} = \frac{2b_L\lambda_{lb}K_D-(a_L-\Lambda)}{r_A}$ and $\lambda_{lb} > \lambda_{lb}^{BC_2} = \frac{a_L-\Lambda+r_A\gamma_{lb}}{2b_LK_D}$). If these three are combined, an illustration of the stability region of loan equilibrium can be seen in Figure 3.

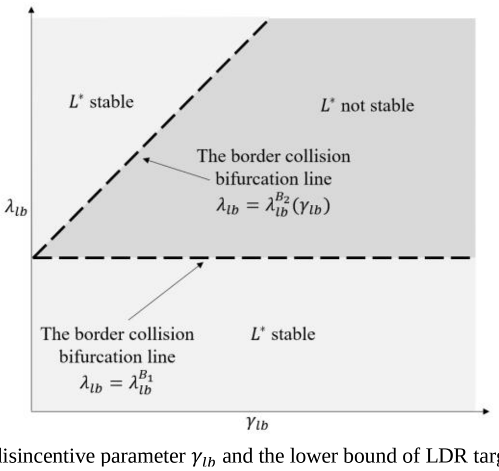


Figure 3. The region of the lower disincentive parameter γ_{lb} and the lower bound of LDR target λ_{lb} for the stability of loan equilibrium L^*. The light gray region is a stable region, and the dark gray region is an unstable region.

Because $\lambda_{lb}^{BC_1} < \lambda_{lb}^{BC_2}$, the stability region $\lambda_{lb} < \lambda_{lb}^{BC_1}$ or $\lambda_{lb} > \lambda_{lb}^{BC_2}$ means that the requirement for stable loan is the value of the lower bound of LDR target must be set small enough or large enough. This makes banks control their LDR in quite low or quite high level. Of course, it is not expected that banks have a small LDR because it can cause liquidity excess problem as experienced by the Indonesian banking sector in the post-financial crisis 1997 (Bathaluddin et al., 2012). Therefore, the stability region $\lambda_{lb} > \lambda_{lb}^{BC_2}$ is preferred and the stability region $\lambda_{lb} < \lambda_{lb}^{BC_1}$ will not be the highlight of this paper.

An analysis of the influence of other parameters of the model on the stability of loan equilibrium is carried out by observing the sign of partial derivative of the border collision bifurcation values γ_{lb}^{BC} and $\lambda_{lb}^{BC_2}$ with respect to the other parameters. Thus, the following theorem is obtained.

Theorem 2. *The stability of loan equilibrium can be maintained when at least one of the following occurs:*

(i) *The carrying capacity of deposits K_D is increased.*
(ii) *The loan interest rate is lowered (a_L is lowered or b_L is increased).*
(iii) *The cost of equity r_E is increased.*
(iv) *The bank's CAR κ is increased if, $r_E > r_A$.*
(v) *The bank's CAR κ is lowered if $r_E < r_A$.*
(vi) *Marginal cost of loan c_L is increased.*
(vii) *The lower disincentive γ_{lb} is lowered.*
(viii) *The lower bound of LDR target λ_{lb} is increased.*

Proof Note that $\frac{\partial \gamma_{lb}^{BC}}{\partial K_D} = \frac{2b_L\lambda_{lb}}{r_A} > 0$, $\frac{\partial \gamma_{lb}^{BC}}{\partial a_L} = -\frac{1}{r_A} < 0$, $\frac{\partial \gamma_{lb}^{BC}}{\partial b_L} = \frac{2\lambda_{lb}K_D}{r_A} > 0$, $\frac{\partial \gamma_{lb}^{BC}}{\partial r_E} = \frac{\kappa}{r_A} > 0$, $\frac{\partial \gamma_{lb}^{BC}}{\partial r_A} = -\frac{2b_L\lambda_{lb}K_D-(a_L-[r_E\kappa+c_L])}{r_A^2} < 0$, $\frac{\partial \gamma_{lb}^{BC}}{\partial \kappa} = \frac{r_E-r_A}{r_A}\begin{cases} > 0 & \text{if } r_E > r_A \\ \leq 0 & \text{if } r_E \leq r_A \end{cases}$, $\frac{\partial \gamma_{lb}^{BC}}{\partial c_L} = \frac{1}{r_A} > 0$, $\frac{\partial \gamma_{lb}^{BC}}{\partial \lambda_{lb}} = \frac{2b_LK_D}{r_A} > 0$, $\frac{\partial \lambda_{lb}^{BC_2}}{\partial K_D} = -\frac{a_L-\Lambda+r_A\gamma_{lb}}{2b_LK_D^2} < 0$, $\frac{\partial \lambda_{lb}^{BC_2}}{\partial a_L} = \frac{1}{2b_LK_D} > 0$, $\frac{\partial \lambda_{lb}^{BC_2}}{\partial b_L} = -\frac{a_L-\Lambda+r_A\gamma_{lb}}{2b_L^2K_D} < 0$, $\frac{\partial \lambda_{lb}^{BC_2}}{\partial r_E} = -\frac{\kappa}{2b_LK_D} < 0$, $\frac{\partial \lambda_{lb}^{BC_2}}{\partial r_A} = -\frac{1-(\kappa+\gamma_{lb})}{2b_LK_D} < 0$, $\frac{\partial \lambda_{lb}^{BC_2}}{\partial \kappa} = -\frac{r_E-r_A}{r_A}\begin{cases} < 0 & \text{if } r_E > r_A \\ \geq 0 & \text{if } r_E \leq r_A \end{cases}$, $\frac{\partial \lambda_{lb}^{BC_2}}{\partial c_L} = -\frac{1}{2b_LK_D} < 0$, $\frac{\partial \lambda_{lb}^{BC_2}}{\partial \gamma_{lb}} = \frac{r_A}{2b_LK_D} > 0$.

Since the loan equilibrium is stable when $\gamma_{lb} < \gamma_{lb}^{BC}$ and $\lambda_{lb} > \lambda_{lb}^{BC_2}$, the requirement in maintaining the stability of loan equilibrium is a change in the value of those other parameters that can raise γ_{lb}^{BC} and reduce $\lambda_{lb}^{BC_2}$. By looking at the signs of the partial derivative calculations above, we can conclude as mentioned in the theorem. □.

4.2. The model's parameters estimation

In this subsection, the loan model (13) and deposit model (14) are applied to Indonesian commercial banks data in Figure 1a. By estimating the model's parameters, the graph of these models are expected to be fitted with the actual data graph. The expectation is the model can describe the real state of Indonesian banking, so that the macroprudential analysis of Indonesian banking can be conducted. Mathematically, the estimation of parameter value is done using the least square method, which minimizes the residual error between the model and the data. In this paper, the residual error used is the mean absolute percentage error (MAPE), which is defined as follows:

$$\text{MAPE} = \frac{1}{N}\sum_{k=1}^{N}\left|\frac{\text{data}_k-\text{model}_k}{\text{data}_k}\right| \quad (19)$$

where N is the number of data.

In this paper, the minimization problem of residual error (19) is solved using a metaheuristic method called spiral optimization, that is method inspired by spiral phenomena in nature such as snail shell spirals, whirlpool spirals, and galactic spirals. This method was first introduced by Tamura and Yasuda (2011). The technique of this method is to rotate a set of points with a certain angle and scale to the global optimal point in each iteration. Spiral optimization (SpO) algorithm for a minimization problem is provided by Algorithm 1. The recent applications of SpO in various fields involving optimization issues can be found in (Sidarto & Kania, 2015; Ansori et al., 2019a; Josaphat et al., 2021).

The parameters of loan model (13) and deposit model (14) are estimated using Algorithm 1. The vector x consists of parameters α_L, a_L, b_L, r_E, r_A, c_L, α_D, dan K_D. These parameters except K_D are estimated in interval [0, 1], meanwhile parameter K_D is estimated in interval [1, 100] × maximum data of deposit. The function F is replaced by MAPE in (19). The model values in the MAPE calculation are obtained by substituting the values of parameters into the models and then doing iterations. The algorithm is run 20 times with the settings $m = 5000$, $\theta = \pi/4$, $r = 0.95$, $k_{max} = 500$. The choose of m and k_{max} values are subjective, but the reason of choosing the values of θ and r as written above is to produce the rotation scheme not too fast or too slow so that the search region can be explored effectively (Ansori et al., 2021a).

Algorithm 1 (spiral optimization)

Initiation: m is the number of search points ($m \geq 2$), θ is the rotation angle ($0 < \theta < 2\pi$), r is the rotation scale ($0 < r < 1$), k_{max} is the maximum iteration ($k_{max} > 1$), I is the search region ($I \subset \mathbb{R}^n$)

Process:

(i) $k = 0$.

(ii) Generate randomly initial points $x_i(0) \in \mathbb{R}^n$, $i = 1,2,\dots,m$ in the search region I.

(iii) Set $x^* = x^g$, where $F(x^g) = \min_{i=1,2,\dots,m} F(x_i(0))$.

(iv) Update x_i in a way $x_i(k+1) = S_n(r,\theta)x_i(k) - (S_n(r,\theta) - I_n)x^*$, $i = 1,2,\dots,m$, where $S_n(r,\theta) = r\prod_{i=1}^{n-1}\prod_{j=1}^{i} R_{n-i,n+1-j}^{(n)}$. I_n is an $n \times n$ identity matrix, $R_{i,j}^{(n)}$ is a rotation matrix whose entry is exactly the same as in the identity matrix except on entry-ii, entry-ij, entry-ji, and entry-jj which are respectively replaced by $\cos\theta$, $\sin\theta$, $\sin\theta$, and $\cos\theta$.

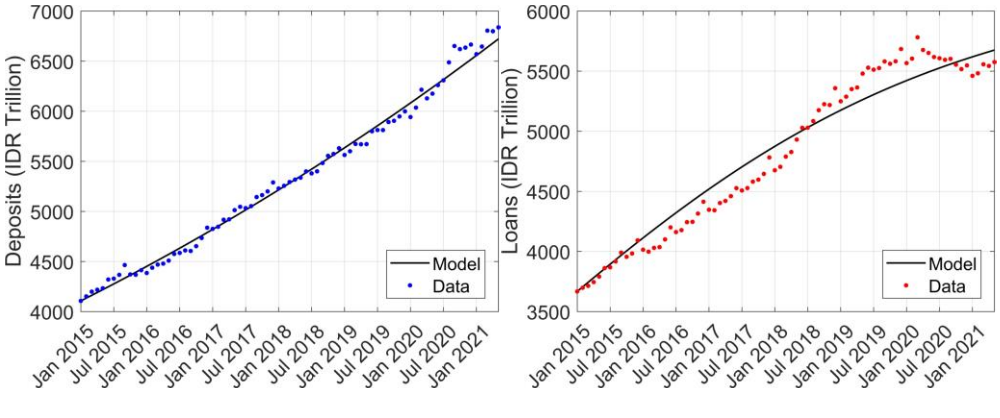
(v) Update x^* in a way $x^* = x^g$, where $F(x^g) = \min_{i=1,2,\dots,m} F(x_i(k+1))$.

(vi) If $k = k_{max}$ the algorithm is complete, but if $k < k_{max}$ the iteration must be updated with setting $k = k + 1$ and redo the process (iv).

Output: x^* is the minimum point.

Table 1. Estimated value of the parameters of model (13) and (14) using the data in Figure 1

Parameter	Value
α_D	0.0077
K_D	34184.82
MAPE$_D$	1.01%
α_L	0.7544
a_L	0.9115
b_L	2.67e-6
r_E	0.4656
r_A	0.7266
κ	0.2123
c_L	0.2069
MAPE$_L$	2.56%



(a) (b)

Figure 4. Graphs comparison between the model with the data of (a) deposits and (b) loans

The output of algorithm is the best-estimated parameters of the models. The best result of the estimation is presented in Table 1. In the table we can see that the MAPE values on both models are quite small, which is less than 3%, so it can be said that these models can fit the real data really well. The graphs of model versus data are presented in Figure 4.

4.3. Policy analysis

An analysis is conducted to observe whether the macroprudential policy that have been implemented by Bank Indonesia in responding the COVID-19 pandemic can ensure the loan equilibrium in the stable condition. The parameters' value in Table 1 obtained from the fitting of Indonesian banking data are substituted into the stability regions $\gamma_{lb} < \frac{2b_L\lambda_{lb}K_D-(a_L-\Lambda)}{r_A}$ or $\lambda_{lb} > \frac{a_L-\Lambda+r_A\gamma_{lb}}{2b_LK_D}$ resulting:

$$\gamma_{lb} < 0.2512\lambda_{lb} - 0.0460 \text{ or } \lambda_{lb} > 0.1831 + 3.9851\gamma_{lb} \quad (20)$$

Referring to the recent MIR policy in response to the impact of the COVID-19 pandemic, the lower disincentive parameter γ_{lb} is set at 0.15 if the bank's CAR ratio exceeds 19%. Meanwhile, the lower bound of MIR target λ_{lb} is set at 84%. By observing these values, we can see that they are in the stable region, as shown in Figure 5 by the red dot. Thus, the recent MIR policy is in accordance with theoretical studies aimed at ensuring the stability of loan equilibrium.

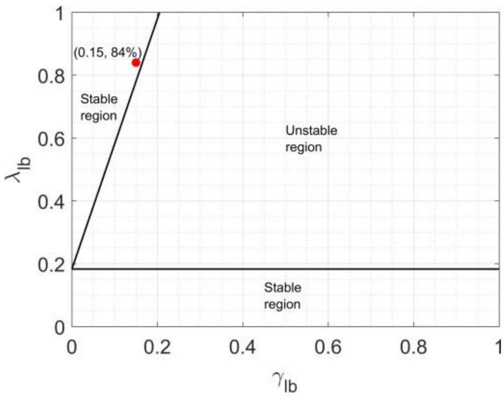


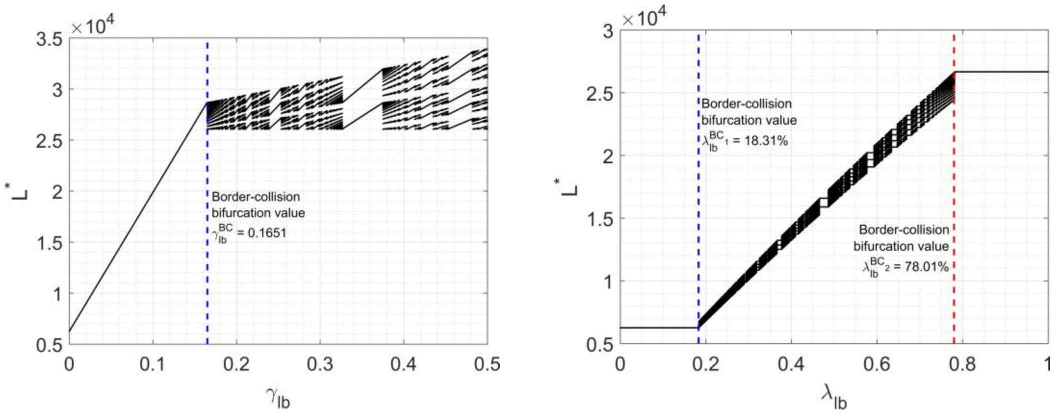
Figure 5. The region of the lower disincentive parameter γ_{lb} and the lower bound target λ_{lb} for the loan stability of Indonesian commercial banks

Examples of uses of the stability region (20) are as follows. If Bank Indonesia first sets the value of the lower disincentive parameter γ_{lb} by 0.15, then the value of the lower bound target parameter λ_{lb} must be set more than $0.1831 + 3.9851(0.15) = 78.01\%$. Conversely, if Bank Indonesia first sets the lower bound target λ_{lb} by 84%, then the lower disincentive parameter γ_{lb} should be set less than $0.2512(84\%) - 0.0460 = 0.1651$. These results express more discretion of policy determinants in stating the target values of the lower disincentive parameter γ_{lb} and the lower bound target λ_{lb}.

What happen to the loan equilibrium if the lower disincentive parameter or the lower bound of the target is set outside the bound value in the equation (20)? What happens at the point of unstable loan equilibrium? To know this, a bifurcation diagram is required to present the behaviour of the equilibrium point to its bifurcation parameters, which is in this case the lower disincentive parameter γ_{lb} and the lower bound target λ_{lb}. The bifurcation diagrams of lower disincentive parameter and lower bound are presented in Figure 6.

In Figure 6a, the rising straight line from the left says that the value of loan equilibrium increases as the disincentive parameter increases before reaching the border collision bifurcation value γ_{lb}^{BC} which is characterized by a blue dashed line. When the lower disincentive parameter crosses the border collision bifurcation line, the loan equilibrium loses its stability causing its value rises erratically in the form of periodic orbits.

From figure 6b, the horizontal line on the left shows that the value of loan equilibrium is quite small for a quite small lower bound target parameter. When the lower bound target is bigger than the border collision bifurcation value $\lambda_{lb}^{BC_1}$ or less than than the border collision bifurcation value $\lambda_{lb}^{BC_2}$, which is depicted by a blue dashed line and a red dashed line respectively, the loan equilibrium loses its stability generating periodic orbits. Generally, Bank Indonesia does not want banks' LDR level to be at a low level, so the lower bound of target which is quite small must be ignored. Such loan instability will potentially disrupt the economic growth, where if there is a sudden event that occurs at an unstable loan equilibrium, the loan value has the potential to become very high or very low and uncontrollable. This needs to be avoided. The way to avoid this is to set the value of the lower disincentive parameter and the lower bound target meets the stability conditions.



(a) (b)

Figure 6. Bifurcation diagrams of parameter (a) the lower disincentive γ_{lb} and (b) the lower bound target λ_{lb}.

5. Conclusion and recommendation

This paper examines the effects of two LDR-RR parameters, namely the lower disincentive parameter and the lower bound of LDR target, on the loan dynamics of Indonesian commercial banks using border collision bifurcation analysis. The results showed that in ensuring the stability of loan equilibrium, there is a smallest upper bound (supremum) for the lower disincentive parameter and the largest lower bound (infimum) for the lower bound target parameter. If the values of these parameters are set outside the stability region, it will result unstable loan equilibrium.

Using monthly data of Indonesian commercial banks in the period before and during the pandemic (January 2015 - May 2021), the model's parameters are estimated using spiral optimization method. Numerical results show that the lower disincentive parameter value 0.15 and the lower bound target 84% contained in MIR policy in March 2021 are in a stable region.

The method used in this paper can be a recommendation for Bank Indonesia as LDR-RR or MIR regulator to check whether the parameters of the instrument will make loan stable or even unstable. The steps are as follows: 1) Using more recent data, the parameters of deposit model and loan model are estimated. 2) The estimated value of the parameters is then substituted into the value of the border collision bifurcation, so that the region of loan stability is obtained. 3) By using the stability region, check whether the value of the lower disincentive parameter and the lower bound target are within the stability region. 4) If the parameters are outside the stability region, then Bank Indonesia needs to reset the parameters value in such a way that the value is within the stability region.

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