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Are 9th grade students ready to engage in the theoretical discursive process in geometry?

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Are 9th Grade Students Ready to Engage in the Theoretical Discursive Process in Geometry?

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Abstract

This study was conducted to examine whether newly enrolled 9th grade students were ready to directly engage in the theoretical discursive process from the perspective of Duval's Cognitive Model. The sample of the study was comprised of 51 newly enrolled 9th grade students between the ages of 14 and 15, who had not received any prior geometry instruction. These 51 students were posed two open-ended questions that would enable them to make a transition between perceptual and discursive apprehension. According to the findings obtained from the study, many of the students could not display the necessary behaviors for theoretical discursive process. Students were mostly unsuccessful in converting discursive information into perceptual information, in writing discursive information based on perceptual information, and making inferences based on discursive information. These findings indicate that recent graduates of secondary school are not ready enough to directly engage in theoretical discursive process and, thus, they could experience difficulties in such high order skills as providing proof requiring the theoretical discursive process.

Keywords: Duval's cognitive model, theoretical discursive process, geometrical figure apprehension.

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¿Están Listos los Estudiantes de Noveno Grado para Participar en el Proceso de Discurso Teórico en Geometría?

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Resumen

Este estudio se realizó para examinar si los estudiantes de noveno grado recién matriculados estaban listos para participar directamente en el proceso discursivo teórico desde la perspectiva del modelo cognitivo de Duval. La muestra del estudio estuvo compuesta por 51 estudiantes de noveno grado recién matriculados entre las edades de 14 y 15 años, que no habían recibido ninguna instrucción previa en geometría. Muchos de los estudiantes no pudieron mostrar los comportamientos necesarios para el proceso discursivo teórico. Los estudiantes en su mayoría no lograron convertir información discursiva en información perceptiva, escribir información discursiva basada en información perceptiva y hacer inferencias basadas en información discursiva. Estos hallazgos indican que los recién graduados de la escuela secundaria no están lo suficientemente preparados para participar directamente en el proceso discursivo teórico y, por lo tanto, podrían experimentar dificultades en habilidades de orden tan alto como proporcionar pruebas que requieren el proceso discursivo teórico.

Palabras clave: Modelo cognitivo de Duval, proceso discursivo teórico, aprehensión de figuras geométricas.

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There is no doubt that using figures to solve problems in geometry is highly beneficial because they provide an integrative presentation of all the constituent relations of a geometrical situation (Duval, 1995). Identifying geometrical properties based on figure constitutes the most important step in a problem-solving process. However, that students draw a conclusion in relation to a figure under the influence of its appearance is one of the common situations that mathematics teachers often encounter. For this reason, one of the points that needs to be focused on in mathematics education is how such explanations encountered in learning environments as "because it looked like a right angle" or "because it worked in another problem case," made under the influence of the appearance of the figure, can be transformed to statements that are based on definitions, axioms and theorems (Jones, 2000). According to Duval (1998), who names such kinds of explanations as the theoretical discursive process, defines explanations based on definitions, axioms and theorems as those made through deduction. Thus, providing proof or logical deduction for geometrical properties is essentially a process of constructing theoretical discursive process.

According to the approaches (Van Hiele model and Duval's cognitive model) that seek to explain geometrical reasoning students need certain behaviors to engage in the theoretical discursive process (Jones, 1998). For example, according to the Van Hiele model, in order for students to make proof, they should know the properties of geometric figures and be able to recognize the logical relationships among these properties (Fuys, Geddes and Tischler, 1988; Güven, 2006; Mason, 1998). But it has limitations such as the emphasis on sequential and hierarchical levels of geometry understanding, Duval's cognitive model is more attractive because it is concerned with understanding the cognitive processes (Ramatlapama and Berger, 2018). According to Duval (1998, 1995), who bases geometric reasoning on cognitive processes, students can engage in the theoretical discursive process only if they look geometrical figure mathematically. The mathematical way of looking at figures in geometry requires that students can establish accurate interactions between their perceptual and discursive apprehensions.

Generally, in school mathematics, particularly high school geometry lessons are regarded as a transitional phase in making logical deductions and providing proof (National Council of Teachers of Mathematics Standards (NCTM, 2000; Sriraman, 2004). Thus, in the educational programs in Turkey, grade 9 (high school) geometry is regarded as a transition to the level of deduction with the assumption that secondary school graduates know the properties of geometric features and the logical relationships among these features. Moreover, as a natural outcome of the education students receive, they can look geometric figure mathematically and can establish accurate interactions between their perceptual and discursive apprehensions when they look at a figure. By means of the present study, to what extent this expectation is realistic and the response to the following research question were investigated: "Does the education provided to secondary school students enable them mathematical way of looking at figures?" To this end, by making use of Duval's cognitive model, the current study aimed to explain whether or not newly enrolled high school students (year 9) and those who had not yet received any instruction in geometry were ready for theoretical discursive process.

While reviewing the related literature, it is possible to encounter numerous studies on students' geometrical figure apprehension (Llinares and Clement, 2014; Michael, Gagatsis, Avgerinos, Kuzniak, 2011; Michael, 2013; Torregrosa and Quesada, 2008). While the participants of some of these studies were teacher candidates (Llinares and Clement, 2014; Torregrosa and Quesada, 2008), in other studies, the participants were comprised of high school students (Michael, Gagatsis, Avgerinos, Kuzniak, 2011; Michael, 2013). In studies on high school students, the structure of geometrical figure apprehension of different grade level was examined. In these studies, it was found that students' figure apprehension generally developed as students proceeded from one grade level to another, that students experienced difficulties in questions related mostly to sequential and discursive apprehension, and that the mistakes that students made in their responses to questions were predominantly related to dominance of the perceptual apprehension on the looking at the figure, when compared to the other processes (Michael, Gagatsis, Avgerinos, Kuzniak, 2011; Michael, 2013). These studies entail important findings related to high school students' use of figure apprehensions. However, while solving problems and making logical deductions, students also need to establish relationships among these processes (Duval, 1995, 1998). Hence, in addition to these studies, those examining how students establish relationships among perceptual and discursive apprehension are also needed. Because between perceptual and discursive apprehension is essential for engaging theoretical discursive process.

Theoretical Framework

Duval's Cognitive Model

Duval (1995) has sought to explain the types of processes involved when looking at a geometric figure. Duval stated that these processes were made up of four geometrical figure apprehension processes: perceptual apprehension, discursive apprehension, sequential apprehension and operative apprehension (Duval, 1995). According to Duval, each of these carries out different functions, which enable the comprehension of mathematical relationships in geometric figures, and solving problems very often requires an interaction among these four processes. However, for an accurate establishment of this interaction, these apprehensions should be developed separately (Duval, 1995).

Perceptual apprehension is the process which includes knowledge acquired when one looks at a figure for the first time and is related to the structure (external appearance) of the figure. It includes such processes as providing information about the name and size of the figure and becoming aware of the fundamental geometric elements (point, line segment, triangle, circle…) that make up the figure. Moreover, identifying the subfigure also takes part in the perceptual apprehension process. This apprehension is static and does not enable one to recognize the relationships among the subfigures (Duval, 1995). It is impossible to identify the mathematical properties of a geometric figure merely through perceptual apprehension. For this to happen, some preliminary information about the figure should be given. Based on the preliminary information provided, establishing a relationship between a figure and mathematical principles (definition, theorem, axiom, etc.) to draw a conclusion is named as discursive apprehension (Duval, 1995, 1999; Michael, 2013).

In learning environments, the transformation of students' discourses into theoretical discursive processes can be attained with the replacement of explanations derived from the appearance of the figure with conclusions drawn based on definitions, axioms and theorems. According to Duval (1998), such a transformation is only possible by looking at a figure mathematically. To be able to look at a figure mathematically, accurate interactions should be established between perceptual and discursive apprehensions. In such an interaction, the perceptual information presented on a figure (the information presented on the figure: point, line segment, angle, etc.) should be accurately converted to discursive information (sentences or symbols showing the mathematical relationships on a figure, such as the lengths of the two line segments are equal or line segment AB is the angle bisector, etc.) or the given discursive information should be converted accurately to perceptual information. However, the result should be obtained based only on discursive information (Duval, 1998). In this process, while perceptual apprehension enables one to recognize the perceptual information on the figure (line segment, angle, point, etc.), discursive apprehension enables one to gain discursive information based on perceptual data, which in turn leads to the construction of new information. Thus, discursive apprehension serves two different functions: The first function is to establish a link between mathematical principles and the geometric figure; that is, expressing visual data utilizing mathematical principles, while the second function is to enable the construction of new information by utilizing mathematical principles (Llinares and Clemente, 2014). Evidently, to be able to look at a figure mathematically, it is essential to display certain behaviors (e.g., converting perceptual information into discursive information).

Based on Duval's explanations regarding perceptual and discursive perception and the transitions between them, the behaviors arising from the interaction between discursive and perceptual apprehensions can be presented as below (Table 1).

As can be seen in Table 1, to be able to engage in the theoretical discursive process in geometry, the perceptual information presented on the figure should be accurately converted to discursive information and vice versa by establishing correct interactions between perceptual and discursive apprehensions. In fact, to prevent the influence of the appearance of the figure, conclusions should be drawn based merely on discursive information. As these behaviors show what must be done to engage in the theoretical discursive process, it is possible to consider these behaviors as criteria for cognitive readiness in engaging in the theoretical process within Duval's Cognitive Model.

Table 1

Interaction Between Discursive and Perceptual Apprehension (The Mathematical Way of Looking at a Figure) and Sample Student Behaviors

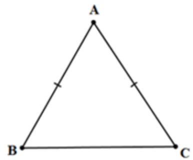
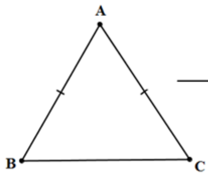
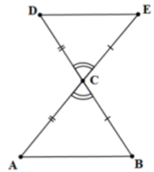
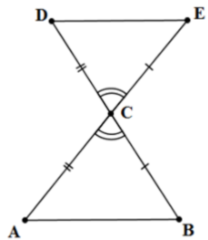
Student Behavior	Sample Student Behavior
1st Behavior Converts the given discursive information to perceptual information. *(Transition from discursive to perceptual apprehension)*	$\lvert \mathbf{AB} \rvert = \lvert \mathbf{AC} \rvert \longrightarrow$ 
2nd Behavior Converts the given perceptual information to discursive information. *(Transition from perceptual to discursive apprehension)*	 $\longrightarrow \lvert \mathbf{AB} \rvert = \lvert \mathbf{AC} \rvert$

Table 1 (continue)

Interaction Between Discursive and Perceptual Apprehension (The Mathematical Way of Looking at a Figure) and Sample Student Behaviors

Student Behavior	Sample Student Behavior
3rd Behavior Draws a conclusion based on the discursive information obtained from the figure. *(Acquiring new information using discursive apprehension)*	 **if** $m\left(D\hat{C}E\right)=m\left(A\hat{C}B\right)$ $\lvert AC\rvert=\lvert DC\rvert$ $\lvert EC\rvert=\lvert BC\rvert$ **Hypothesis** **then triangle DEC and triangle ABC are congruent triangles** **Conclusion**
4th Behavior Does not become influenced by the appearance of a figure while drawing a conclusion. *(Drawing a conclusion based on perceptual apprehension)*	If  then [DE] // [AB] *(A probable conclusion that can be drawn under the influence of the appearance of the figure*.)

Method

This study aimed to reveal whether newly enrolled high school students who had not yet received any instruction in geometry had cognitive readiness to engage in theoretical discursive process. To this end, requirements for theoretical discursive process were identified based on Duval's Cognitive Model (see Table 1) and two open-ended questions were prepared to measure these requirements. These questions were administered to the students during one of the convenient class hours. Subsequently, to observe the students' mathematical behaviors and draw conclusions from these observations regarding their cognitive processes (Goldin, 1997), approximately 10-to-15-minute clinical interviews were held on a voluntary basis with three self-expressive students selected from each category of results obtained from the analyses of the answers to the open-ended questions (see Table 3). With the permission of the students, their responses were recorded. In these interviews, the students were asked to explain and justify their answers. To this end, the students were posed the question, "Could you please explain and justify your response?" In this way, the justifications underlying students' written responses were tried to be revealed.

Sample

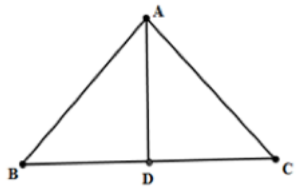
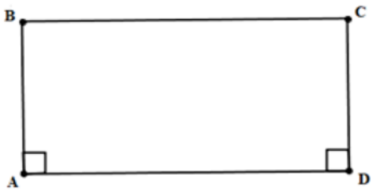
The study was carried out in a state high school (Anatolian High School), which admits its students through a centralized exam in Turkey and is at a moderate level of success in its region with reference to student scores. Excluding vocational schools, Anatolian High Schools are the most preferred type of high school among those aiming to provide academic education to students. One such school was chosen to implement the study in order to address the common student level. In the selected high school, there were 75 students in year 9. These students were introduced formal proof for the first time in the geometry course. The fact that students had not previously attended a geometry course with instruction on formal proof in their high school was paid attention to since the study aimed towards the readiness of students' theoretical discursive process. For this reason, 24 students were not included in the study as their teacher had started teaching the geometry units in their class. Thus, the sample of the study was comprised of 51 fourteen-to-fifteenyear-old 9th grade students who had just enrolled in high school and had not previously received a geometry course in their high school. These students had learned triangles, polygons, geometric objects and transformation geometry within the scope of sub-learning topics of geometry in secondary school. At the end of secondary school, students are expected to explain the features of geometric figures, logical relationships among them and as a natural outcome of education accurately establish an interaction between their perceptual and discursive apprehensions when they look at a figure. When they become high school students, they are expected to systematically prove the geometric relationships by theoretical discursive process as of year 9.

Data Collection Instrument

For theoretical discursive process, three fundamental behaviors need to be realized: converting discursive information into perceptual information, arriving at discursive information based on the perceptual information, and arriving at logical conclusions based only on the discursive information (Torregrosa and Quesada, 2008; Llinares and Clemente, 2014). Thus, whether or not students were ready to engage in the theoretical discursive process were tried to be revealed based on whether or not they displayed these behaviors. Accordingly, two open-ended questions that could reveal each behavior were prepared together with two experts holding a doctoral degree in the field of math education. Subsequently, a pilot study was conducted at a high school to determine, by consulting expert opinion, whether or not the questions could reveal the behaviors expected of the students, and the questions were revised to take their final shape. The questions that were prepared and the behaviors that were aimed to be measured are as follows (see Table 2).

In the first question, a geometric figure with discursive information was presented, and the students were asked to convert the discursive information to perceptual information by using appropriate symbols. In the second question, the students were given a figure with perceptual information (such as perpendicularity) displaying certain mathematical relationships and were asked to write the mathematical properties of the given geometric figure. The figure given in the second question was designed to display with mathematical properties such as “parallelism” and “perpendicularity”. For example, while at first sight the figure seemed like a rectangle with all its angles being 90

Table 2
Measurement Instruments

The Questions Prepared to Measure students' competencies in the theoretical discursive process	The Behaviors To Be Measured
Try to show the given information on the geometric figure by using symbols • $\lvert AB\rvert=\lvert AC\rvert$ • $\lvert AC\rvert=\lvert BC\rvert$ • $[AD]\perp[BC]$ • $m(\hat{C})=m(\hat{B})$ • $[AD]$, is the angle bisector of angle BAC. • $[AD]$ is a median. 	• Converting discursive information to perceptual information accurately.
Look at the geometric figures below and write their mathematical properties using appropriate symbols and representations in the space provided on the right 	• Converting the given perceptual information to discursive information accurately. • Arriving at a conclusion based on the discursive information obtained through the figure. • Not being influenced by the appearance of the figure while arriving at a conclusion.

degrees, it is, in fact, in the most general sense, a trapezoid, and it cannot be claimed that all its angles are 90 degrees.

Data Analysis

The qualitative data obtained from the responses given to the open-ended questions and the interview constitute the data of the study. The data obtained were analyzed in two phases. Initially the written responses to the open-ended questions and subsequently the qualitative data obtained in the interviews were analyzed. The written responses given to the open-ended questions were analyzed by two researchers. Three previously formed categories were used in data analysis (see Table 3). These categories were used to group students according to the behaviors of accurately converting discursive information into perceptual information, arriving at accurate discursive information based on the perceptual information, and arriving at logical conclusions based only on the discursive information. The first group of the categorization was comprised of students who had perfectly displayed the behavior measured via the two open-ended questions (see Table 2). The second group was made up of students who had displayed some deficiencies in the behaviors measured. Finally, the third group consisted of students who did not display the behaviors at all. For example, while the written responses to the first question was analyzed, the students who could convert all the given discursive information to perceptual information completely were assigned to group 1, while those students who had some deficiencies (such as not being able to convert some discursive information into perceptual information), but could convert some of the discursive information into perceptual information were assigned to group 2 and finally those students who could not accurately convert any of the discursive information into perceptual information were assigned to group 3. In order to validate the categorization and to clarify the number of students in each category, the data were analyzed by two researchers separately and independent of each other. Subsequently, the obtained data were compared and the responses of the students assigned to different groups were re-examined. At the end of the examination, the researchers arrived at a common conclusion and a complete agreement was established. In this way, the number of students in each category, which both researchers agreed upon, were revealed. The student numbers that were determined arranged into tables, which are presented in the findings section. The categories identified and their explanations are as follows:

Table 3
The Categories Used to Group Students and Their Explanations

1st CATEGORY: Displays the behavior accurately	2nd CATEGORY: Responding accurately despite some deficiencies in displaying the behavior	3rd CATEGORY: Unable to display the behavior
This category includes the students who answered all the questions accurately (by writing the expected responses).	This category includes the students who provided accurate answers but could not write some of the expected answers.	This category includes the students who provided erroneous answers or did not respond to the questions at all.

After the students' written responses given to the open-ended questions were categorized by the researchers, the qualitative data obtained from the clinical interviews were analyzed. After the recorded interviews were transcribed, the students' oral responses given during the interviews and their written responses to the open-ended questions were compared. Both the consistent and conflicting aspects of the written and oral responses were tried to be identified during the comparisons. When an inconsistency was observed between a student's written and oral response to the extent of impacting his/her category, the students' category identified for that particular question was changed based on the data obtained from the interview. Thus, as a result of the data analysis, one student who was placed in "Group 3- No response" based on his/her written responses to the open-ended questions was moved into "Group 3- Erroneous responses produced under the influence of the appearance of the figure" according to the findings obtained from the interview (see Table 6).

Findings

Converting the Given Discursive Information to Perceptual Information

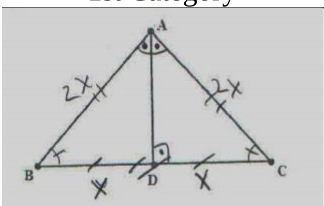
The first of the open-ended questions was asked to determine whether the students could convert the given discursive information to perceptual information accurately. The distribution of the number of students according to the categories identified based on the data obtained from the analysis is as follows (Table 4).

Table 4
The Distribution of the Number of Students Based on Their Responses to the First Question

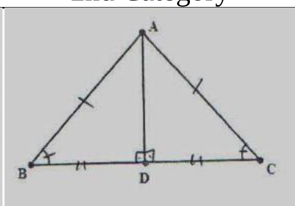
Behavior	1st Category	2nd Category	3rd Category
Can convert the given discursive information to perceptual information accurately.	2	32	17

As can be seen in Table 4, only two of the students were able to completely convert the discursive information given about the figure to perceptual information. On the other hand, even though 32 students in the 2nd category could convert some of the discursive information to perceptual information accurately, they could not convert some of the information at all. Finally, the 17 students in the 3rd category converted the discursive information to perceptual information erroneously. When the number of students in each group are taken into consideration, it can be claimed that the majority of the students displayed the behavior insufficiently or erroneously. Some of the student responses for each category are presented below (see Figure 1).

1st Category



2nd Category



3rd Category

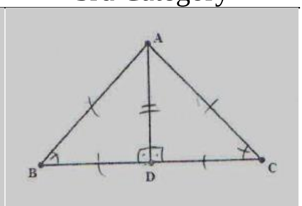


Figure 1. Some student responses from each of the three categories

As can be observed from the responses, the student in the 1st Category converted all the given discursive information to perceptual information accurately. While the student was doing so, s/he established different perceptual information for different angles and sides. However, when the response of the student in the 3rd category was examined, it was observed that the student established the same perceptual information for the different discursive information (e.g. [AD] is a median and $|AB| = |AC|$) given and this caused the student to make mistakes in converting discursive information to perceptual information.

It is obvious that students who make such mistakes will obtain erroneous results when they use the perceptual information, they form on a geometric figure to determine the figure's mathematical properties. Thus, in the interviews held with the students, when students were given the perceptual information, they had created themselves (see Figure 2) and asked to explain the mathematical properties of the figure, based on their perceptual information, they arrived at wrong results. One excerpt from an interview with a student is as follows:

> Researcher: Please write the mathematical properties of the figure by considering the symbols on the given figure. (The researcher shows Figure 2.)
> Student: Here side AC and side DC are shown to be equal...
> Researcher: What else can be said?
> Student: Side BD, side DC and side AB are also of equal length. And there, in D, there are two equal angles. (The student points to angles ADB and ADC.)

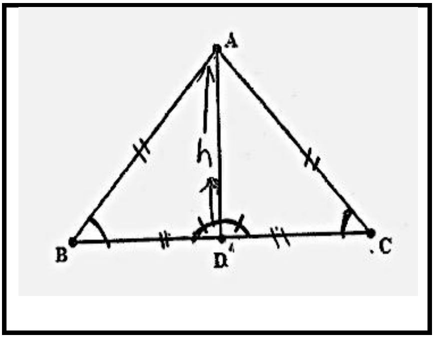


Figure 2. The perceptual information previously formed by the student

As can be observed in the interview excerpt, the student arrives at erroneous results by giving the same perceptual information for different verbal information. Thus, this shows that students are unsuccessful even in the most fundamental behavior they need to display in solving geometric problems.

When the response of the student in the second category is examined, it is observed that s/he converted some of the given discursive information to perceptual information accurately. However, s/he could not convert some of the discursive information (that AD line segment is an angle bisector, and BC and AC side lengths are equal) to perceptual information at all. In the interviews held with the students, when students were asked why they could not show some of the information on the figure, they stated that they did not read all the information on the figure and that previously this had not caused any problems. An excerpt of an interview held with one of these students is as follows:

> Researcher: You haven't shown on the figure that the side is a median.
> Student: I didn't read that; I didn't see it.
> Researcher: Why didn't you read it?
> Student: I don't read all the information when solving problems. It doesn't cause any problems. The information is given on the figure anyway.

As can be understood from the interview excerpt, the student has said that s/he does not read some of the given information due to the problem-solving habits he gained in geometry and thus could not show some of the information on the figure. Furthermore, s/he states that this behavior has not previously caused any problems for him/her.

Converting the Given Perceptual Information to Discursive Information

The second of the open-ended questions was asked to measure two behaviors. The first of these aimed to determine whether students could convert the given perceptual information to discursive information. When the responses to the open-ended questions were examined, it was found that students converted perceptual information to discursive information either accurately or did not convert them at all. That is, students were classified only according to the 1st and 3rd categories. As there were no students who had not written incomplete discursive information, no student was placed in the 2nd category. The distribution of student numbers across the categories defined is presented in Table 5.

Table 5

The Distribution of the Number of Students Based on Their Responses to the Second Question

Behavior	1st Category	2nd Category	3rd Category
Converts the given perceptual information to discursive information accurately	23	0 (zero)	28

As can be observed in Table 5, twenty-three students converted the given perceptual information to discursive information accurately by using the necessary symbols and representations to show the perpendicularity of the angles on the figure. On the other hand, 28 students could not convert the given perceptual information to discursive information. Consequently, this behavior, which enables one to acquire the necessary discursive information (hypotheses) to arrive at a conclusion without being influenced by the appearance of the figure, was not displayed by most of the students (3rd behavior, see Table 1). In addition, as the perceptual information given on figures constitutes the data of a geometrical problem, it can also be asserted that most of the students do not determine the given data in a problem. Some student responses in relation to categories are shown in Figure 3.

1st Category

3rd Category

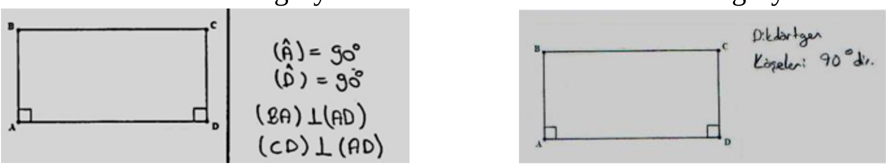


Figure 3. The responses of some of the students in relation to the specified categories

As can be observed in Figure 3, the student in the 1st category converted the given information of perpendicularity to discursive information accurately. However, the student in the 3rd category has directly written results (such as it is a rectangle and its angles are 90 degrees) related to the mathematical properties of the figure without writing the discursive information. In the interviews held with some students in the 3rd category, students stated that they did not see the need to convert perceptual information to discursive information and that it was meaningless to write the relationships they saw when they looked at the figure. An excerpt from an interview held with one of the students is presented below:

> Researcher: Why didn't you state that angles A and D are 90 degrees?
> Student: It's already stated on the figure. Is there a need to rewrite it?
> Researcher: What do you think?
> Student: I think it's not necessary. It's clearly seen in the figure, it's pointless to write it…

As can be seen in the interview script, the student finds it meaningless to convert the perceptual information presented on the figure to discursive information. For this reason, these students directly wrote explanations about the mathematical properties of the figure.

Arriving at a Result Based on the Discursive Information Obtained from the Figure

The second of the open-ended questions was asked to measure two behaviors. The second behavior is to write correct mathematical properties of a figure based on the perceptual information obtained from the figure. When the answers were examined, it was found that only two of the students had written accurate mathematical properties about the figure, while the others provided erroneous answers or did not answer at all, thereby not providing any explanation about the mathematical properties of the figure. Consequently, in terms of this behavior, these students were grouped only in the 1st and 3rd categories. Since those who provided erroneous responses or no response at all had not displayed the behavior, they were placed in the 3rd category and it is for this reason that the 3rd category was divided into two subcategories, namely "3rd category- Erroneous responses produced under the influence of the appearance of the figure" and "3rd category- No responses." The distribution of the number of students according to categories is shown in Table 6.

Table 6

The Distribution of Student Numbers Based on Their Responses to the Second Question

Behavior	1st Category	3rd Category	
• Arrives at a conclusion based on the discursive information obtained from the figure. • Is not influenced by the appearance of the figure while making deductions.	Correct responses	Erroneous responses produced under the influence of the appearance of the figure	No responses
	2	38	11

As can be seen in the table, most of the students are comprised of students who gave wrong answers because they were influenced by the appearance of the figure. All these students arrived at the conclusion that the figure was a rectangle and thus wrote the properties of a rectangle. Unlike these students, the students who wrote the mathematical properties correctly were those students who based their response on discursive information without being influenced by the appearance of the figure. On other hand, those students who could not arrive at any conclusion only converted perceptual information to discursive information. Some student responses in relation to categories are presented in Figure 4.

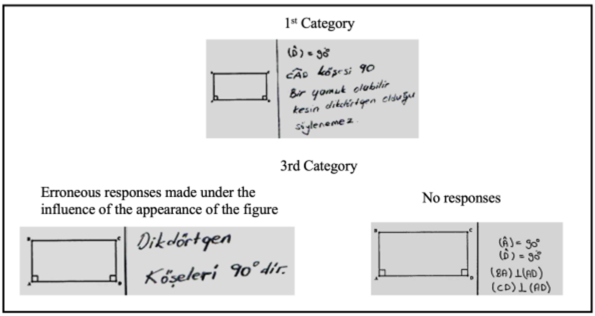


Figure 4. Some student responses

As can be observed in Figure 4, the student in the 1st category has stated that angle A and angle D are 90 degrees, so it can definitely not be a rectangle but could be a trapezoid. This indicates that the student determined the mathematical properties of the figure based on preliminary information (discursive information). In other words, the student first acquired certain discursive information (e.g. that angles A and D are 90 degrees) and then arrived at a conclusion based on this information.

As the discursive information s/he acquired was insufficient to consider the figure as a rectangle, the student arrived at the conclusion that the figure must be a trapezoid. Similar findings were also obtained during the interviews held with the students. When this student was asked why s/he believed the figure was not a rectangle, s/he stated that it would be insufficient to consider the figure as a rectangle when only angles A and D are stated to be 90 degrees. An excerpt from the interview held with the student is as follows:

> Researcher: What can you say about the mathematical properties of the given figure?
> Student: Angles A and D are 90 degrees. But it doesn't have to be a rectangle.
> Researcher: Why?
> Student: Because the other angles (refers to angles B and C) don't have to be 90 degrees. They can change.
> Researcher: What else can be said?
> Student: What else… I think it can be a trapezoid, I mean if we turn it like this (meaning rotating the figure to make side DC the base), it will look more like a trapezoid.
> Researcher: Why did you think it was a trapezoid?
> Student: As these angles are 90 degrees (points to angles A and D), these sides become parallel (sides AB and DC); that's why it becomes a trapezoid.

As can be observed in the interview excerpt, the student has used the given discursive information (that angles A and D are 90 degrees) without being influenced by the appearance of the figure and arrived at a conclusion in relation to the mathematical properties of the figure. That is, s/he has determined the hypotheses and arrived at a conclusion based on this information. However, most of the students wrote the mathematical properties of the figure under the influence of the appearance of the figure. As can be understood from Figure 4, the student (in the 3rd category- wrong answers) draws the conclusion that the figure is a rectangle and states that all the angles are 90 degrees. As can be seen, the student arrives at a conclusion under the influence of the appearance of the figure and then writes mathematical properties suitable to this conclusion. Similar findings were also yielded by the interviews. When the student was asked during these interviews how s/he had arrived at that conclusion, it was understood that s/he was influenced by the appearance of the figure and expressed mathematical properties that were in agreement to this conclusion.

An excerpt from an interview with one of the students is presented below:

> Researcher: What can you say about the mathematical properties of the given figure?
>
> Student: The figure is a rectangle. All its angles are 90 degrees...
>
> Researcher: Why did you arrive at the conclusion that it is a rectangle?
>
> Student: Because all its angles are 90 degrees...
>
> Researcher: How did you understand that all its angles are 90 degrees?
>
> Student: Because it is a regular quadrilateral. (Here the student refers to the appearance of the figure.)

As can be seen in the interview excerpt, the student arrives at a conclusion by being influenced by the appearance of the figure and s/he determines the mathematical properties of the figure based on this conclusion. In other words, s/he states that the figure is a rectangle without referring to any mathematical reason and then states the properties of a rectangle. This shows that the student initially states a conclusion and then writes hypotheses to explain the conclusion.

In the 3rd category, there are not only students who wrote erroneous mathematical properties but also students who converted perceptual information to discursive information but did not write any mathematical properties based on this information (3rd category-no response). However, it was understood during the interviews that although a student had converted perceptual information to discursive information in his/her written response but had not written a mathematical property based on this information, s/he had actually arrived at a conclusion by being influenced by the appearance of the figure and when asked questions about the mathematical properties of the figure, s/he drew a conclusion based on the appearance of the figure. For this reason, this student was placed into "3rd category- Erroneous responses produced under the influence of the appearance of the figure."

Following is an excerpt from an interview with one of the students:

> Researcher: What can you say about the mathematical properties of the given figure?
>
> Student: Angles A and D are 90 degrees. Side BA is perpendicular to side AD, and side CD is perpendicular to side AD.
>
> Researcher: What else can be said?
>
> Student: Nothing else is given.
>
> Researcher: Then let me reword the question... What can be said about angles B and C?
>
> Student: Angles B and C (the student thinks for a while) ... 90 degrees.
>
> Researcher: What can be said about the given figure?
>
> Student: Well it's a rectangle… That is what's given.
>
> Researcher: Why is it a rectangle?
>
> Student: (referring to the appearance of the figure) Because its sides are perpendicular.
>
> Researcher: Why didn't you write this information?
>
> Student: I understood the question as "write the mathematical properties presented on the figure."
>
> Researcher: How do you think the question should have been worded?
>
> Student: I think it should have been "Write the properties that are not presented on the figure."

As can be observed, even though the student found the perceptual information on the figure sufficient and did not write any other information, when asked questions about the figure, s/he gave responses based on the appearance of the figure. When the student was asked why s/he had not written the mathematical properties she expressed during the interview, s/he said that s/he had misunderstood the question and thought that s/he had to write only the mathematical properties presented on the figure. Moreover, the student, who thought that converting perceptual information to discursive information meant writing the mathematical properties presented on the figure, believed that the question should be reworded as "Write the mathematical properties not presented on the figure." This indicates that the student regards the conversion of perceptual information into discursive information not as an essential hypothesis formation process to draw a conclusion, but a process to determine the mathematical properties of the given figure.

Discussion and Conclusion

By means of the current study, whether students at the very beginning of their high school education possess cognitive readiness to pass on to the theoretical discursive process has been investigated. To this end, some of the behaviors needed for the theoretical discursive process were identified by utilizing Duval's Cognitive Model, and these were used to explain students' cognitive readiness. Based on the findings obtained in the study, it can be asserted that most of the students could not display the behaviors that were essential for the theoretical discursive process. The students were generally unsuccessful in converting discursive information to perceptual information, writing discursive information based on perceptual information and making deductions based on discursive information. This indicates that 9th grade students in Turkey contrary to assumption do not have readiness in directly engaging in the theoretical discursive process and, thus, will be unsuccessful in higher order skills such as providing proof, which necessitates the theoretical discursive process. In fact, in numerous studies conducted on high school students, students' proof-writing ability was found to be very low (Healy and Hoyles, 1998; McCrone and Martin, 2004; Senk, 1985).

In the theoretical discursive process, it is important for students to convert discursive information to perceptual information and vice versa. However, most of the students in this study were unsuccessful in displaying this basic level behavior of converting the given discursive information to perceptual information. According to Duval (1998), the most important reason underlying this is that in learning environments importance is attached to increase in knowledge, while cognitive and perceptual processes are neglected. Thus, the teaching of concepts should not be the sole focus in primary and secondary school education. Behaviors essential for theoretical discursive process should also be given place in learning environments because mathematics is not a branch of science consisting of concepts and mathematical results found by some people, but a way of thinking (Cuoco and Goldenberg, 1996).

Another behavior that is essential for engagement in theoretical discursive process is to be able to make deductions based on the discursive information derived from the figure. However, the findings obtained indicate that most students are influenced by the appearance of figures when making deductions.

Similar findings were revealed in numerous other studies (Michael, 2013; Ubuz, 1999). While Ubuz (1999) attributed it to the fact that students were not at the necessary Van Hiele level, Duval (1995, 1998) attributed it to the dominance of perceptual apprehension and its influence on the discursive processes. According to Duval (1995, 1998), drawing conclusions in relation to figures should begin with discursive apprehension. If discursive apprehension does not dominate the reasoning process, perceptual apprehension becomes dominant and impacts students' reasoning processes. Even though he utilizes different concepts in his explanations, Fischbein (1993), like Duval (1998), believes that when discursive information does not dominate the reasoning process, the appearance of figures will affect the conclusions (Fischbein, 1993; Fischbein and Nachlieli,1998). On the other hand, Harel and Sowder (1998) attribute students' behavior of making deductions based on the figures' appearance to their being within the Perceptual Proof Scheme. According to Harel and Sowder (1998), students within this scheme apprehend figures as static and cannot take into consideration the different conditions of figures.

When both the written responses and the interview oral responses of students who were influenced by the appearance of the figures were examined, it was found that students initially made claims by being influenced by the appearance of the figure and then put forward hypotheses to explain their conclusions. When the students' reasoning processes were examined, it can be claimed that most students could not reason deductively and, hence, arrived at conclusions under the influence of the appearance of the figures. In deductive reasoning, the process begins with hypotheses, and conclusions are essentially derived from these hypotheses (Özlem, 1994). Thus, similar results were reported in a study by Healy and Hoyles (1998), which examined students' proof-writing ability and their opinions about the role of proof. In this study, it was found that most students could not reason deductively while they arrived at a conclusion. Furthermore, in studies where students' being influenced by the appearance of figures is attributed to their not being at the necessary Van Hiele level (Ubuz, 1999) actually implicitly emphasize students' weaknesses in the deductive reasoning process. The rationale is that a person who proceeds to the upper levels in Van Hiele should not be influenced by the appearance of figures when drawing conclusions, which is naturally a reference to the deductive reasoning process characteristic of the upper levels.

When the other reasoning processes (inductive and abduction) other than deductive reasoning are considered, the only type of reasoning that starts with a conclusion is the abduction reasoning process. The course of an abduction process that seeks to reach possible explanations that would validate a conclusion to be resulting from observation is *conclusion-rule-hypothesis* and the hypotheses arrived at are not absolute explanations (Meyer, 2010). Thus, it can be stated that the abduction thinking process was dominant in most of the students who participated in the study. All these results should not lead to the general conclusion that the abduction process always leads students to make wrong conclusions. However, it can be concluded that when it is considered that an abduction process begins with an observation, and when students do not have the necessary instruments for them to make sufficient observations, using the abduction reasoning process can lead to wrong conclusions.

The most important instrument for students to make a sufficient observation is a dynamic geometry software because by means of the software, geometric figures can easily be made on the computer screen. Such properties as angle, side, perimeter and area can be measured, and the geometric figures made with certain associations can be moved around on the screen. Consequently, all the measured properties of figures also change dynamically (Güven and Karataş, 2009). For this reason, it can be said that learning environments in which dynamic geometry software is used are more conducive to the abduction reasoning process.

Based on the results of the present study, teachers are recommended to determine how students interactively use their perceptual and discursive apprehension before starting the geometry course and integrate into their lesson activities that would accurately develop the relationship between these two processes. Activities that specifically enable students to understand that the appearance of figures is deceptive should be integrated into the lessons. Moreover, the approach to discover the properties or identify the common properties of geometric figures as of primary education should, over time, leave their place to a process directed by definitions, axioms or theorems.

References

Coşkun, F. (2009). *Ortaöğretim öğrencilerinin Van Hiele geometri anlama seviyeleri ile ispat yazma becerilerinin ilişkisi* [Correlation between secondary school pupils Van Hiele levels of understanding of geometry and their proof writing skill] (Unpublished master's thesis). Karadeniz Teknik University, Trabzon, Turkey.

Cuoco, A., Goldenberg, E. P. & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *The Journal of Mathematical Behavior,15*(4), 375-402.

Duval, R. (1995). Geometrical pictures: Kinds of representation and specific processings. In R. Sutherland & J. Mason (Ed.), *Exploiting mental imagery with computers in mathematics education* (pp. 142–157). Berlin, Heidelberg: Springer Berlin Heidelberg. Retrieved from http://dx.doi.org/10.1007/978-3-642-57771-0_10

Duval, R. (1998). Geometry from a cognitive point of view. In C. Mammana & V. Villani (Ed.), *Perspectives on the teaching of geometry for the 21st century* (pp. 37–52). Dordrecht, Netherland: Kluwer.

Duval, R. (1999). *Representation, vision and visualization: Cognitive functions in mathematical thinking. Basic issues for learning*. Retrieved from ERIC database (ED466379)

Fischbein, E. (1993). The theory of figural concepts. *Educational studies in mathematics*, *24*(2), 139-162.

Fischbein, E. (1999). Intuitions and schemata in mathematical reasoning. *Educational Studies in Mathematics*, *38*, 11–50

Fischbein, E. & Nachlieli, T. (1998). Concepts and figures in geometrical reasoning. *International Journal of Science Education*, *20*(10), 1193-1211.

Fischbein, E., & Kedem, I. (1982). Proof and certitude in the development of mathematics thinking. In A. Vermandel (Ed.), *Proceedings of the sixth international conference for the psychology of mathematics education* (pp. 128-131). Antwerp, Belgium: Universitaire Installing Antwerpen

Fuys, D., Geddes, D., & Tischler, R. (1988). The Van Hiele Model of Thinking in Geometry among Adolescents. *Journal for Research in Mathematics Education*. Monograph, 3, I-196. doi:10.2307/749957

Goldin, G., A. (1997). Chapter 4: Observing Mathematical Problem Solving through Task-Based Interviews. *Journal for Research in Mathematics*

Education. Monograph, 9, 40-177. doi:1. Retrieved from http://www.jstor.org/stable/749946 doi:1

Güven, B. (2006). *Öğretmen adaylarının küresel geometri anlama düzeylerinin karakterize edilmesi. [Characterizing student mathematics teachers' levels of understanding of spherical geometry.]* Unpublished doctoral thesis, Karadeniz Technical University, Trabzon, Turkey

Güven, B., Karataş, İ. (2009). The effect of dynamic geometry software (cabri) on pre-service elementary mathematics teachers' achievement about locus problems. Ankara University, *Journal of Faculty of Educational Sciences*, Ankara, Turkey, *42*(1), 1-31.

Harel, G. & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *Research in collegiate mathematics education III*, *7*, 234-282.

Healy, L. & Hoyles, C. (1998). *Justifying and proving in school mathematics: Technical report on the nationwide survey*. London, United Kingdom: Institute of Education, University of London.

Jones, K. (1998). Theoretical frameworks for the learning of geometrical reasoning. In *Proceedings of the british society for research into learning mathematics* (Vol. 18, pp. 29–34). Retrieved from http://www.bsrlm.org.uk/IPs/ip18-12/BSRLM-IP-18-12-5.pdf

Jones, K. (2000). Providing a foundation for deductive reasoning: Students' interpretations when using dynamic geometry software and their evolving mathematical explanations. *Educational studies in mathematics*, *44*(1), 55-85.

Llinares, S. & Clemente, F. (2014) Characteristics of Pre- Service Primary School Teachers' Configural Reasoning, *Mathematical Thinking and Learning, 16*(3), 234-250, DOI: 10.1080/10986065.2014.921133

Mason, M. M. (1998): The van Hiele levels of geometric understanding. In*: The Professional Handbook for Teachers Geometry*: Boston: McDougal-Littell/Houghton-Mifflin, p. 4–8.

McCrone, S. M., & Martin, T. S. (2004). Assessing high school students' understanding of geometric proof. *Canadian Journal of Math, Science & Technology Education*, *4*(2), 223-242.

Meyer, M. (2010). Abduction—A logical view for investigating and initiating processes of discovering mathematical coherences. *Educational Studies in Mathematics*, *74*(2), 185-205.

Michael, P. (2013). Geometrical figure apprehension: cognitive processes and structure (Unpublished doctoral dissertation). The University of Cyprus, Cyprus.

Michael, P., Gagatsis, A., Avgerinos, E., & Kuzniak, A. (2011). Middle and High school students' operative apprehension of geometrical figures. *Acta Didactica Universitatis Comenianae–Mathematics, 11*, 45-55.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Özlem, D. (1994). *Mantık* [Logic]. İstanbul, Turkey: Ara Yayıncılık.

Ramatlapana K. and Berger M. (2018). Prospective mathematics teachers' perceptual and discursive apprehensions when making geometric connections. *African Journal of Research in Mathematics, Science and Technology Education*, Retrieved from https://doi.org/10.1080/18117295.2018.1466495

Senk, S. (1989). Van Hiele levels and achievement in writing geometry Proofs. *Journal for Research* in *Mathematics Education, 20*(3), 309-321.

Senk, S. L. (1985). How well do students write geometry proofs? *Mathematics Teacher, 78,* 448-456.

Sriraman, B. (2004). Gifted ninth graders' notions of proof: Investigating parallels in approaches of mathematically gifted students and professional mathematicians. *Journal for the Education of the Gifted, 27*(4), 267-292.

Stylianides, G. J. & Stylianides, A. J. (2008). Proof in school mathematics: Insights from psychological research into students' ability for deductive reasoning. *Mathematical Thinking and Learning*, *10*(2), 103-133.

Torregrosa, G. & Quesada, H. (2008). The coordination of cognitive processes in solving geometric problems requiring proof. *In Proceedings of the Joint Meeting of PME* (Vol. 32, pp. 321-328).

Türkiye Cumhuriyeti Milli Eğitim Bakanlığı [MoNE]. (2013a). Ortaöğretim Matematik Dersi (9, 10, 11 ve 12. Sınıflar) Öğretim Programı [Republic of Turkey, Ministry of National Education secondary school mathematics curriculum (9, 10, 11 and 12. grades], Ankara, Turkey.

Türkiye Cumhuriyeti Milli Eğitim Bakanlığı [MoNE]. (2013b). Ortaokul Matematik Dersi (5, 6, 7 ve 8. Sınıflar) Öğretim Programı [Republic of Turkey, Ministry of National Education primary school mathematics curriculum (5, 6, 7 and 8. grades], Ankara, Turkey.

Ubuz, B. (1999). 10. ve 11. sınıf öğrencilerinin temel geometri konularındaki hataları ve kavram yanılgıları [10th and 11th grade students' basic geometry errors and misconceptions on the subject]. Hacettepe Üniversitesi Eğitim Fakültesi Dergisi,*17*(17).

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