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Content and Problem Analysis in Turkish and Singaporean Mathematics Textbooks: The Case of Multiplying Fractions

Suphi Önder Bütüner[1]

1) Yozgat Bozok University, Turkey

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Suphi Önder Bütüner
Yozgat Bozok University

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Abstract

In this study, Turkish and Singaporean textbooks were compared in terms of teaching content for multiplying fractions, a subject that most students have difficulty in understanding. The study analyzed the 6th-grade mathematics textbook published by the Turkish Ministry of National Education and its Singaporean counterpart. While the Singaporean textbook covered all meanings of multiplying fractions, the Turkish textbook did not include the operator meaning of multiplying fractions. Compared to the Turkish textbook, the Singaporean textbook included more solution strategies. The number line model was not used in the textbooks of either country, and only one representation format was used to model a fraction multiplication problem. The Singaporean textbook included more fraction multiplication problems than did the Turkish textbook. Many problems in both textbooks were of a one-step fashion and required numerical answers.

Keywords: Turkish and Singaporean textbooks, teaching content, problem analysis, multiplying fractions

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Análisis de Contenido y Problemas en los Libros de Texto de Matemáticas de Turquía y Singapur: el Caso de la Multiplicación de Fracciones

Suphi Önder Bütüner
Yozgat Bozok University

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Resumen

En este estudio se comparó la enseñanza de la multiplicación de fracciones en libros de texto turcos y singapurenses, tema en el que la mayoría de los estudiantes tiene dificultades de comprensión. El estudio analizó libros de texto de matemáticas de sexto publicados por el Ministerio de Educación Nacional de Turquía y su homólogo de Singapur. Mientras que los libros de texto de Singapur cubrían todos los significados de multiplicar fracciones, los libros de texto turcos no incluían el significado de operador. En comparación con los libros de texto turcos, los libros de texto de Singapur incluían más estrategias de solución. El modelo de recta numérica no se usó en los libros de texto de ninguno de los países, y solo se usó un formato de representación para modelar un problema de multiplicación de fracciones. Los libros de texto de Singapur incluían más problemas de multiplicación de fracciones que los libros de texto turcos. Muchos problemas en ambos libros de texto eran de un solo paso y requerían respuestas numéricas.

Palabras clave: Libros de texto turcos y singapurenses, contenido didáctico, análisis de problemas, multiplicación de fracciones

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To better understand their progress in science and mathematics education, countries participate in international assessments such as the TIMSS, PISA, and PIRLS. One of these, the TIMSS (Trends in International Mathematics and Science Study**)** is held every four years at the level of the 4th and 8th grades of primary education and provides comparative data regarding the achievement levels of countries in the science and mathematics subjects. Far East countries shared top ranks on the 2015 TIMMS, as was the case on the 1999, 2007, and 2011 TIMMS. Turkey displayed a mediocre performance in numbers and other learning areas on the TIMSS. There may be many reasons for the difference between the mathematics performance of Far Eastern countries and that of Turkey. One reason could be differences in the content of the textbooks. In this study, selected Singaporean and Turkish mathematics textbooks were compared by focusing on their presentations of the multiplication of fractions. Conducting this research is important for the following four reasons: (1) the importance of textbook quality on student learning, (2) limited number of studies on the comparison of between Turkey and East Asian countries in terms of their textbooks, (3) difficulties encountered in multiplying fractions, and (4) helping improve the content of multiplying fractions in textbooks.

As is known, Singapore has maintained a very strong performance in recent international student assessment tests whereas Turkey scored much lower scores than Singapore. It is true that student achievement is affected by a variety of factors; nevertheless, such a big difference between the two countries might be resulting from their mathematics curricula (Reys, Reys, & Chavez, 2004). Textbooks are generally believed to be a bridge between the intended curriculum and the implemented curriculum, and the TIMSS data illustrate that teachers use textbooks as the primary source in lessons (Beaton, Mullis, Martin, Gonzales, Kelly, & Smith, 1996). Likewise, Kilpatrick, Swafford and Findell (2001) argued that textbooks primarily determine what teachers teach in classes. Similarly, the content of mathematics classes and the way this content is delivered are also influenced by textbooks (Alajmi, 2012; Hirsch, Lappan, Reys & Reys, 2005; Li, 2000). Thus, textbooks can substantially affect students' attitudes towards learning as well as the problem-solving strategies they use in math classes (Cai & Ni, 2011; Fan, Zhu & Miao, 2013). Additionally, since different textbooks provide different learning opportunities for students, textbook comparison studies can help explain the differences between students' achievement levels (Mesa, 2004; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002; Zhu & Fan, 2006).

In line with the above, it has been observed that textbook comparison studies have become widespread in recent years. In the relevant studies, especially, textbooks of countries such as China, Korea, Japan, Taiwan, Singapore, and Finland, which top international assessments such as the TIMSS and PISA, were used. In most of the studies, the textbooks used were analyzed in terms of their content or problem type (Hong & Choi, 2014). In this study, selected Singaporean and Turkish mathematics textbooks were compared. Textbooks from these two countries were selected because these counties represent different levels of performance on the TIMSS. While 8th-grade Singaporean students performed well, 8th-grade Turkish students performed below the TIMSS scale average. Mathematics textbooks used in Turkey were compared with the textbooks of the countries ranked in the top five on international assessments (only Singaporean textbooks) in terms of design characteristics (Erbaş, Alacacı, & Bulut, 2012) and problem type (Özer & Sezer, 2014). In Toprak and Özmantar's study, Turkey and Singapore 5th grade mathematics textbooks were compared in terms of learning opportunities offered to students through worked-examples and questions posed (potential cognitive demand, reasoning and proof) (Toprak & Özmantar, 2020). In Bütüner's study, Turkish and Singaporean textbooks were compared with respect to the instructional content of the unit on division in fractions (Bütüner, 2020).

In this study, selected Singaporean and Turkish mathematics textbooks were compared regarding how the topic of multiplying fractions was presented. This topic was chosen for several reasons. First, previous studies indicated that students have more difficulty in multiplying and dividing fractions compared to adding and subtracting fractions (Huang, Liu, & Lin, 2009). Second, the multiplication of fractions has an important role in teaching the division of fractions because the inverse multiplying algorithm is used when dividing fractions (Kar, Güler, Şen, & Özdemir, 2018).

The present study aimed at identifying the strengths and weaknesses of the textbooks and the learning opportunities offered by these textbooks according to the content of multiplying fractions. In addition, in line with the findings of this study, various suggestions have been offered to curriculum designers working within the Ministry of Education in Turkey and its counterpart in Singapore. It is believed that the results of this study will provide curriculum designers with remarkable insights on minimizing apparent weaknesses in the textbooks. Therefore, they can substantially improve the content of multiplying fractions. In addition, the results of this study will provide them with an information about the performances of Turkish and Singaporean students in the future international assessments, particularly regarding multiplying fractions. The next section presents a literature review of textbook comparison studies on multiplying fractions and the importance of multiplying fractions as part of the curriculum.

Theoretical Framework

The extant literature comprises studies that compared textbooks in terms of their multiplying fractions content (Kar et al., 2018; Son, 2005; Son & Senk, 2010; Watanabe, Lo, & Son, 2017). For instance, Son and Senk (2010) compared Korean and American mathematics textbooks; Kar et al. (2018) compared Turkish and American mathematics textbooks; and Watanabe et al. (2017) compared Japanese, Korean, and Taiwanese mathematics textbooks in terms of their multiplying fractions content. Son and Senk (2010) compared Korean and American mathematics textbooks in terms of their content on multiplying and dividing fractions and the subject-relevant problems the textbooks covered. They found that Korean and American textbooks were aimed at presenting the meaning underlying the algorithm of multiplying fractions at the beginning of the topic. Son and Senk stated that the topic of multiplying fractions was presented in Korean textbooks through the meaning of repetitive summing of multiplication; but in American textbooks, the topic is taught through paper-folding activities and the area model based on the idea of “finding part of a part. While a synchronous conceptual and procedural understanding of the subject in the presentation of multiplying fractions was the basis in Korean textbooks, American textbooks prioritized the conceptual understanding of the subject, i.e., the procedural understanding of the subject being presented in a symbolic form, like $\frac{1}{4}\ of\ \frac{1}{3} = \frac{1}{3} x \frac{1}{4}$, was delayed. In Korean textbooks, all meanings of fraction multiplication (i.e., repeated addition, operator, and “taking a part of a part”) are in focus, and all of the models, including area, length, and set, are used within the subject. Apart from these, Korean textbooks included more solution strategies in multiplying fractions compared to American textbooks and presented the simplification operation. When the textbooks were compared in terms of problem types, most of the problems in the textbooks of both countries measured procedural understanding. Most of the problems in both countries' textbooks included one-step problems that required numerical answers and were presented in a symbolic form. However, Korean textbooks were richer than American textbooks in terms of multi-step problems. Similarly, Watanabe et al. (2017) compared Japanese, Korean, and Taiwanese mathematics textbooks in terms of multiplying fractions. In Japanese textbooks, unlike in Korean and Taiwanese textbooks, the multiplication of fractions was presented after teaching the division of a proper fraction by a positive whole number. The teaching of multiplying fractions begins in the 4th grade in Taiwan and in the 5th grade in Japan and Korea. In the textbooks of all three countries, operations are performed on unit fractions so that students can better understand multiplying fractions. In Korean and Taiwanese textbooks, the operator meaning of fractions is referred to at an earlier stage when compared to Japanese textbooks. Additionally, problems regarding the operator meaning of fractions were encountered in the 3rd grade in Korean, 5th grade in Taiwanese, and 6th grade in Japanese textbooks. However, in Korean textbooks, the problems in the 3rd grade that were suitable for the operator meanings of fractions (e.g., "How many apples do three-quarters of eight apples make?") were considered as multiplication problems. However, these problems were included in the Korean textbook for students to allow the comprehension of the concept of a fraction. Taiwanese textbooks included $\frac{a}{b} xc$ type problems in the 5th grade that were suitable for the use of the number line model (e.g., "How many kilometers would be covered when three-quarters of a 120 km-road is traveled?"). Japanese textbooks also taught fractions by combining the area and number line models to provide students with the opportunity to apply proportional reasoning in multiplying fractions (see Figures 1-2).

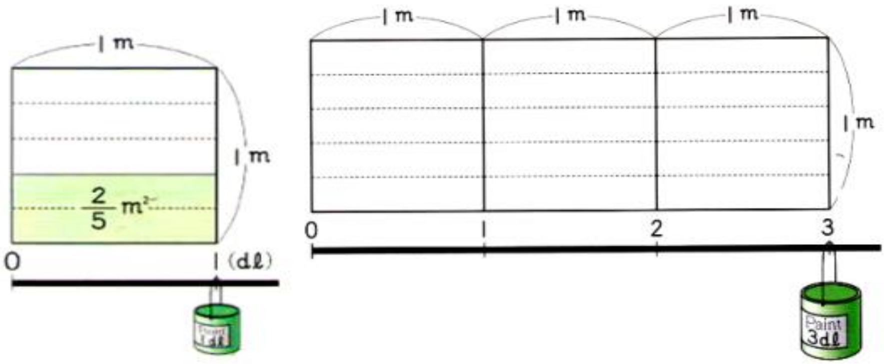


Figure 1. Multiplying fractions in Japanese mathematics textbooks (On this fence, 1 dl of green paint is enough for an area of 25 m2. How many m2 of this fence can be painted by 3 dl of paint?) (Gakkho Tosho, 1995, p.4).

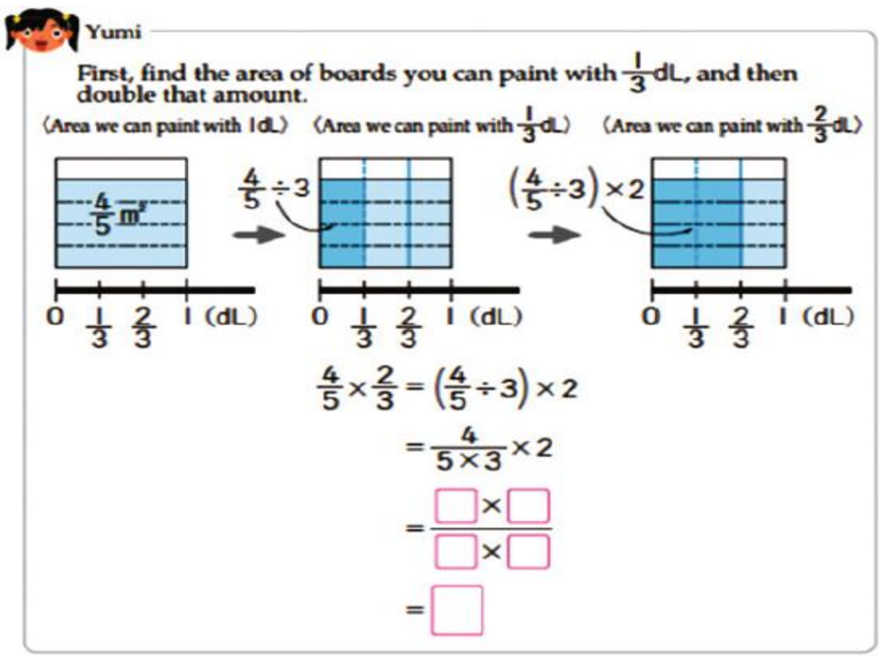


Figure 2. Multiplying fractions in Japanese mathematics textbooks.

Kar et al. (2018) compared Turkish and American mathematics textbooks in terms of their multiplying fractions content and the subject-relevant problems the textbooks covered. They found that Turkish and American textbooks, at the beginning, aimed to present the meaning underlying the algorithm of multiplying fractions. They stated that the multiplication of fractions in Turkish textbooks was based on the meaning of repetitive addition of multiplication; in American textbooks, the multiplication of fractions was presented by considering the idea of “taking part of a part” and using paper-folding activities and field models. While a synchronous conceptual and procedural understanding of the subject in the presentation of multiplying fractions was the basis in Turkish textbooks, American textbooks prioritized the conceptual understanding of the subject, whereas the procedural understanding of the subject was delayed. The number of problems on multiplying fractions in American textbooks was threefold higher than in Turkish textbooks. In American textbooks, there was a higher number of solution strategies for multiplying fractions compared to Turkish textbooks. Most of the problems in the textbooks included one-step problems that required numerical answers and measured procedural knowledge.

Why the Operation of Multiplying Fractions?

Fractions play a significant role in learning mathematics because they constitute the basis of the topics of percentage, ratio, and decimal numbers. Past studies have shown that students have difficulty in multiplying fractions and are prone to various misconceptions. Many students know how to multiply two fractions but cannot explain what 3/4 x 1/2 means. Teaching and learning the rules for calculating fractions might be more straightforward for students. However, only a rule-based approach to problems causes students to lose their ability to reason (Aksu, 1997). For example, in Newton, Willard, and Teufel’s (2014) study, a number of students found the result of $\frac{2}{9}x\frac{7}{9}$ as $\frac{14}{9}$. While multiplying two mixed numbers, some students started multiplying the whole numbers within themselves ($3\frac{5}{7}x4\frac{3}{8}=12\frac{15}{56}$). Another mistake made by students in the study was multiplication by equalizing the denominator. While performing the $\frac{2}{3}x\frac{1}{5}$ operation, some students calculated the result of $\frac{10}{15}x\frac{3}{15}=\frac{30}{15}=2$ by equalizing the denominators of fractions. Considering these mistakes, the students who were extremely bound by the rules thought that the result of the multiplication of two proper fractions could be a fraction greater than 1. From time to time, students have doubts about the accuracy of the result even though they have found the correct answer by multiplying fractions because they think that multiplication with natural numbers will always yield high results, and they consequently transfer this fact to multiplying fractions (Hansen, 2017). Therefore, first, students should be taught what multiplying fractions means, and they should be able to explain why the multiplication algorithm is used as well as what the underlying meaning is. The presentation of multiplying fractions in textbooks should be based on communicating multiple meanings of multiplying fractions, using multiple representation forms and calculation strategies, and promoting the synchronous development of conceptual-procedural understanding (Son, 2012). It has been recommended in past studies that multiplying fractions should start with the meaning of repetitive addition (Wu, 2001) because students use the meaning of repetitive addition in multiplication operations with natural numbers. For instance, in the $3x4$ operation, 3 represents the number of groups, and 4 represents the number of elements in each group. When this idea is transferred to multiplying fractions, the presentation of the problem "If there is $\frac{2}{3}lt$ of water in each of the 6 glasses, how much water is there in the jug when all the water in these glasses is poured into the jug?" in symbolic form becomes $6x\frac{2}{3}$ (6 is the number of glasses, $\frac{2}{3}lt$ is the amount of water in each glass). The solution to the problem will be the sum of 6 pieces of $\frac{2}{3}\ lt$ of water based on the meaning of the repetitive sum of multiplication. Ali ate $\frac{2}{3}$ of the 24 chocolate bars in a package. The symbolic representation of the problem of "What is the number of the chocolate bars Ali ate?" is $\frac{2}{3}x24$. Although multiplication has a commutative property, it is important that students learn that the meaning of each condition is differently represented in symbolic forms (Van de Walle, Karp, & Bay-Williams, 2013). After being taught to take part of a part of a positive whole number, students can transition into taking part of a part of a fraction. When multiplying fractions, the use of different representation forms is imperative. Ainsworth (2006) highlighted the importance of using different forms of representation, emphasizing that the use of two forms of representation is better. Similarly, Gagatsis and Shiakalli

(2004) underlined that a single form of representation reveals only certain aspects of the concept, and that multiple forms of representation have the potential to complement each other.

Method

The Textbooks Analyzed in the Study

In this study, mathematics textbooks in Singapore and Turkey were analyzed for their content on the subject of multiplying fractions. The textbooks included in the analysis were those used during the 2019-2020 academic year. In the Singaporean education system, Primary Education consists of a four-year foundation stage between grades 1 and 4 and a two-year orientation stage covering grades 5 and 6. The purpose of primary education is to provide students with a good education in the English language, in their mother tongue, and in mathematics. In Singapore, currently, four mathematics textbooks are used in primary education: *My Pals Are Here! New Syllabus Primary Mathematics*, *Shaping Maths*, and *Targeting Mathematics*. In Turkey, multiplying fractions is taught in the 6th grade. In the 6th grade, there are two textbooks published by the Ministry of National Education. This study analyzed the 6th-grade Turkish mathematics textbook written by Çağlayan, Dağıstan, and Korkmaz (2018) and a Singaporean mathematics textbook, *Targeting Mathematics 5A*. The mathematics textbooks of both countries were approved by their Ministries of Education and subsequently used in schools.

Content Analysis

In a number of studies found in the previous literature, it can be observed that textbooks have been compared using content and problem analysis (Alajmi, 2012; Fan & Zhu, 2000; Kar et al., 2018; Li, Chen, & An, 2009; Özer & Sezer, 2014; Son & Hu, 2016; Son & Senk, 2010; Zhu & Fan, 2006). Son and Senk (2010) examined how and when the meanings of multiplying fractions are presented in American and Korean textbooks, as well as what kinds of solution strategies are used. Similarly, Kar et al. (2018) examined when and how the meanings of multiplying fractions were presented in American and Turkish textbooks, and what kinds of solution strategies were used. Li, Chen, and An

(2009) compared Chinese, Japanese, and American textbooks at macro and micro levels with respect to the content on division by fractions. When comparing the textbooks at the macro level, the authors focused on the class levels in which the multiplying fractions operation was presented, the organization of the content (e.g., a separate section after multiplying fractions), the order of the content, and the number of pages allocated for the content. Further, they compared how the meaning of multiplying fractions was presented in the textbooks, how the multiplication algorithm was created, and what kinds of solution strategies were covered. Additionally, the problems related to multiplying fractions present in the textbooks were classified.

In line with the studies in the previous literature, mathematics textbooks used in Singapore and Turkey were analyzed through content and problem analyses in terms of their content for multiplying fractions. The present study focused on the following research questions: 1. How is multiplication of fractions presented in the lessons of *Targeting Mathematics 5A* and 6th-grade Turkish mathematics textbooks? (a) To what grade level(s) do the textbooks focusing on multiplication in fractions belong? (b) What is the number of pages and percentage allocated to the unit on multiplication in fractions in the textbooks? (c) What is the order of topics in the textbooks regarding multiplication in fractions? (d) In what way and when is the subject of "multiplication of fractions" introduced in the textbooks? (e) What meanings (repeated addition, operator, multiplication as taking a part of a part) are developed? (f) Are conceptual understanding and procedural fluency developed simultaneously in each curriculum? (g) What are the representational models (e.g., area, set, length) provided in the textbooks? (h) What are the computational strategies included in the textbooks? 2. How are problems in multiplication of fractions presented in the textbooks? What expectations are evident in these problems?

Problem Analysis

Content and problem analyses are not entirely independent because there is a close relationship between the teaching style and testing and assessment practices (Black & William, 1998). For instance, a textbook that is rich in conceptual knowledge, mathematical reasoning, representations, and problem-solving categories indicates that importance is given to students' conceptual learning. On the contrary, a textbook that is rich in single-step problems considered as a procedural knowledge category and that requires only numerical answers serves to improve students' procedural fluency (Son & Senk, 2010). In this study, considering previous studies, the problems were categorized by the four characteristics presented in Table 2. The number of steps was classified into two categories as problems that require a "single computational step (O)" or "multiple computational steps (M)." The answer type was divided into three categories as "Numerical answer only (NA)," "Numerical expression required (such as +, -, x; NE)," and "Explanation or solution required (ES)." Problem type was classified into two categories in terms of context as "problems relevant and/or irrelevant to daily life" and "problems presented in symbolic, verbal, and multiple forms" (Kar et al., 2018; Son, 2012; Son & Senk 2010).

Cognitive expectation comprises conceptual knowledge, procedural knowledge, mathematical reasoning, representation, and problem-solving categories. If a student must know the meaning of a rule to solve a mathematical problem, then it is categorized as conceptual knowledge. For instance, the way to determine whether students possess a conceptual understanding of the multiplying fractions operation is to directly ask them what multiplying fractions means (Kar et al., 2018). If the problem requires the student to find a solution using only procedures and algorithms, then the problem is categorized as procedural knowledge. Procedural knowledge requires knowing when and how to appropriately use procedures (Kilpatrick, Swafford, & Findell, 2001). If a student is expected to evaluate, explain, and justify the solution strategy, then the problem is categorized as mathematical reasoning. If a student needs to use visual representations such as figures, graphs, and tables, or if s/he needs to interpret the visual representations to solve the problem, the problem is categorized as representation. The problem-solving category includes verbal, daily life problems that students are asked to solve (Son, 2012; Son & Senk, 2010). The problems requiring students to estimate the answers in the studies conducted in recent years were included in the mathematical reasoning category (Son & Hu, 2016; Kar et al., 2018). Kar et al. (2018) added the problem-posing category to the cognitive expectation characteristic in problem analysis. In this study, cognitive expectation comprises "conceptual knowledge," "procedural knowledge," "mathematical reasoning," "representation," "problem-solving," and "problem-posing" categories. The framework used to analyze the problems is presented in Table 1.

Table 1

The theoretical framework of problem analysis

Characteristics	Categories (Codes)
Number of Steps	Single computational step (S) Multiple computational steps (M)
Response Type	Numerical Answer only (NA) Numerical Expression required (NE)
	Explanation or solution required (ES)
Context	Daily Life (DL) - Irrelevant to Daily Life (IDL)
	In Symbolic Form (ISF)
	In Verbal Form (IVeF)
	In Visual Form (IViF)
	In Multiple Forms (e.g., Visual+Verbal; IMF)
Cognitive Expectation	Conceptual Knowledge (CK) Procedural Knowledge (PK)
	Mathematical Reasoning (MR) Representation (R)
	Problem-Solving (PS) Problem Posing (PR)

Examples of problems falling under each aforementioned category are presented in Table 2. Each category under which a problem falls is explained in Table 2, along with their reasons. The problems were categorized by taking earlier studies into account (Kar et al., 2018; Son & Senk, 2010; Yang et al., 2010).

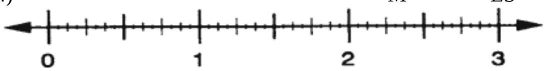
Table 2.

Problem examples and codes (Kar et al., 2018, p. 208; Son & Senk, 2010, p. 125)

Samples	Coding			
	Number of Steps	Answer Type	Context	Cognitive Expectation
1) a) $\frac{3}{4} x \frac{2}{5} =?$	S	NA	IDL, ISF	PK

Table 2. (Continue)

Problem examples and codes (Kar et al., 2018, p. 208; Son & Senk, 2010, p. 125)

Samples	Coding			
	Number of Steps	Answer Type	Context	Cognitive Expectation
2) 3/8 of the audience in a cinema were adults, and the rest were children. If 2/3 of the children were girls, what was the proportion of girls in the audience?	S	NA	DL, IVeF	PS
3) Please explain what $\frac{1}{4} x \frac{2}{5}$ means and solve the operation.	M	ES	IDL, ISF	CK
4)  Please solve the question “what is 1/2 of 3/8?” using the number line above and explain your solution.	M	ES	IDL, IMF	S, MR

The first problem presented in Table 2 is a one-step problem requiring only a numerical answer, irrelevant to daily life, and presented in a symbolic form. It is sufficient to know the multiplication algorithm to be able to solve the problem; thus, in terms of cognitive expectation, it falls under the procedural knowledge category. The second problem is a single-step problem requiring a numerical answer, relevant to daily life, and in verbal form. It falls under the problem-solving category in terms of cognitive expectation. In the third problem, because the students are asked what multiplying fractions means, the problem falls under the conceptual knowledge category and is a multi-step problem requiring explanation; the problem is in symbolic form and irrelevant to daily life. The fourth problem has multiple steps and requires explanation; it is asked in multiple forms and is irrelevant to daily life. Therefore, the problem is categorized under the representation and mathematical reasoning categories because it requires explanation and the use of visual representations. The data analysis was conducted by three different academicians. First of all, in these textbooks, the number of pages containing the topic of multiplication in fractions was determined, after which the ratio of that number to the number of pages in each textbook was calculated.

Afterward, three different researchers identified and noted how this topic was introduced, as well as what kinds of models (length, area, cluster models) and strategies in solving problems were used in these textbooks. They then compared these notes and reached a consensus on a model of the steps that the textbooks followed in teaching the division of fractions. Thus, the similarities and differences in the teaching of the division of fractions in the Turkish and Singaporean textbooks were determined.

Findings

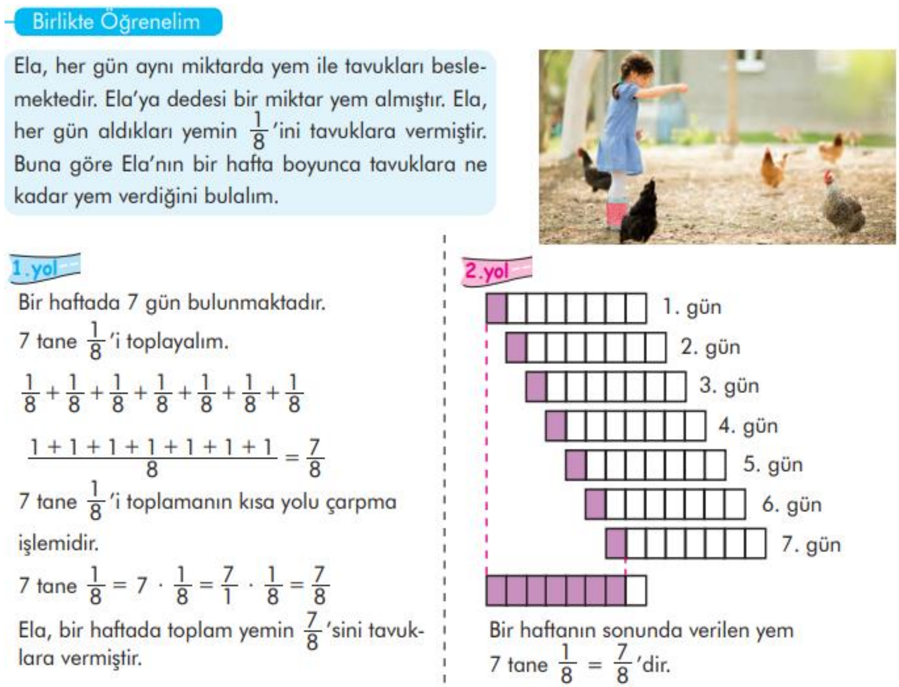
How is Multiplication of Fractions Presented in the Textbooks?

Grade level, number of pages and percentage, order of topics.

The topic of multiplying fractions is covered in the 6th-grade mathematics textbook in Turkey, whereas it is covered in the 5th-grade mathematics textbook in Singapore. In the Turkish textbook, five (2.08 %) pages were allocated to multiplying fractions, whereas 18 (5.42 %) pages were allocated in the Singaporean textbook. In the Turkish textbook, the subjects of addition, subtraction, multiplication, and division with fractions are located in one unit and are sequentially listed. In addition, in Singapore, addition and subtraction with fractions are taught in 4th grade, and multiplication with fractions is taught in 5th grade. In the Singaporean textbook, the subject of multiplying fractions was presented in subheadings. These subheadings of multiplying fractions were "the multiplication of a proper fraction by a positive whole number," "the multiplication of an improper fraction by a positive whole number," "the multiplication of a proper fraction by a proper fraction," "the multiplication of a proper fraction by an improper fraction," "the multiplication of an improper fraction by an improper fraction," and "the multiplication of a mixed number by a positive whole number." In the Turkish textbook, subheadings were not used in presenting the content of multiplying fractions. The textbook included the multiplication of a positive whole number by a proper fraction, the multiplication of a positive whole number by a mixed number, and the multiplication of a proper fraction by a proper fraction.

Introducing fraction multiplication, meanings for fraction multiplication, representational models, computational strategies, developing conceptual understanding, and procedural fluency.

In the Turkish textbook, the teaching of multiplying fractions began on the basis of the relationship between multiplication and repeated addition (Figure 3); then, the multiplication algorithm $(ax\frac{b}{c} = \frac{a}{1}x\frac{b}{c} = \frac{axb}{1xc})$ was presented by highlighting the need to write 1 in the denominator of the positive whole number when a positive whole number and a fraction were multiplied.


Birlikte Öğrenelim

Ela, her gün aynı miktarda yem ile tavukları beslemektedir. Ela'ya dedesi bir miktar yem almıştır. Ela, her gün aldıkları yemin $\frac{1}{8}$'ini tavuklara vermiştir. Buna göre Ela'nın bir hafta boyunca tavuklara ne kadar yem verdiğini bulalım.

1.yol

Bir haftada 7 gün bulunmaktadır.

7 tane $\frac{1}{8}$'i toplayalım.

$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$

$\frac{1+1+1+1+1+1+1}{8}=\frac{7}{8}$

7 tane $\frac{1}{8}$'i toplamanın kısa yolu çarpma işlemidir.

7 tane $\frac{1}{8} = 7 \cdot \frac{1}{8} = \frac{7}{1} \cdot \frac{1}{8} = \frac{7}{8}$

Ela, bir haftada toplam yemin $\frac{7}{8}$'sini tavuklara vermiştir.

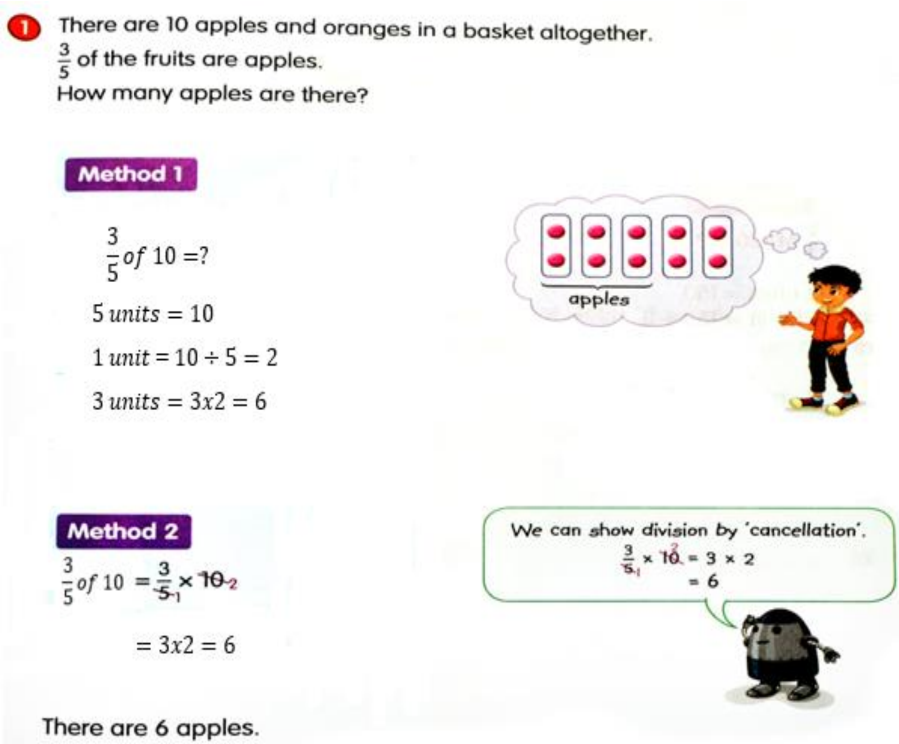
2.yol

1. gün
2. gün
3. gün
4. gün
5. gün
6. gün
7. gün

Bir haftanın sonunda verilen yem 7 tane $\frac{1}{8} = \frac{7}{8}$'dir.

Figure 3. The symbolic and visual representations of the meaning of repetitive addition in multiplying fractions in the Turkish textbook when a positive whole number and a fraction are multiplied.[1]

When the problem and solution presented in Figure 3 are examined, it can be seen that the knowledge of multiplication by natural numbers is transferred to the multiplication by fractions. As understood from multiplication by natural numbers, in the operation of 3x4, 3 represents the number of groups, and 4 represents the number of objects in each group. The problem in the Turkish textbook that states "Ela has given $\frac{1}{8}$ of the feedstuff to chickens every day for a week. So, how much feedstuff has Ela fed the chickens in a week?" is solved by expressing the problem in symbolic form as $7x\frac{1}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$. In the second alternative, the solution recorded in symbolic form is presented in visual form, meaning 7 $\frac{1}{8}$s make $\frac{7}{8}$In the Singaporean textbook, the teaching of multiplying fractions began by presenting the "operator" meaning of multiplying fractions. In the Singaporean textbook, there was no emphasis that 1 should be written in the denominator of a positive whole number when it is multiplied by a fraction ($\frac{a}{b}xc=\frac{axc}{b}$), and examples were presented showing that the operation could be carried out through simplification. The "set model" was used in the visual solution of the problem in Figure 4 that states, "There are 10 fruits (apples + oranges) in a basket. $\frac{3}{5}$ of these fruits are apples. How many apples are there?" First, 10 apples were divided into five groups, each of which had an equal number of objects. After determining that there were two apples in each group, the total number of apples in three groups, $(3x2)$, was calculated. The second solution presented with the help of visual representations immediately below the first solution was expressed in symbolic form. The required simplifications were made after noting $\frac{3}{5}$ of 10 is $\frac{3}{5}x10$, and the result of 6 was obtained based on $3x2$.



1 There are 10 apples and oranges in a basket altogether.
$\frac{3}{5}$ of the fruits are apples.
How many apples are there?

Method 1

$\frac{3}{5}\, of\ 10 = ?$

$5\ units = 10$

$1\ unit = 10 \div 5 = 2$

$3\ units = 3x2 = 6$

Method 2

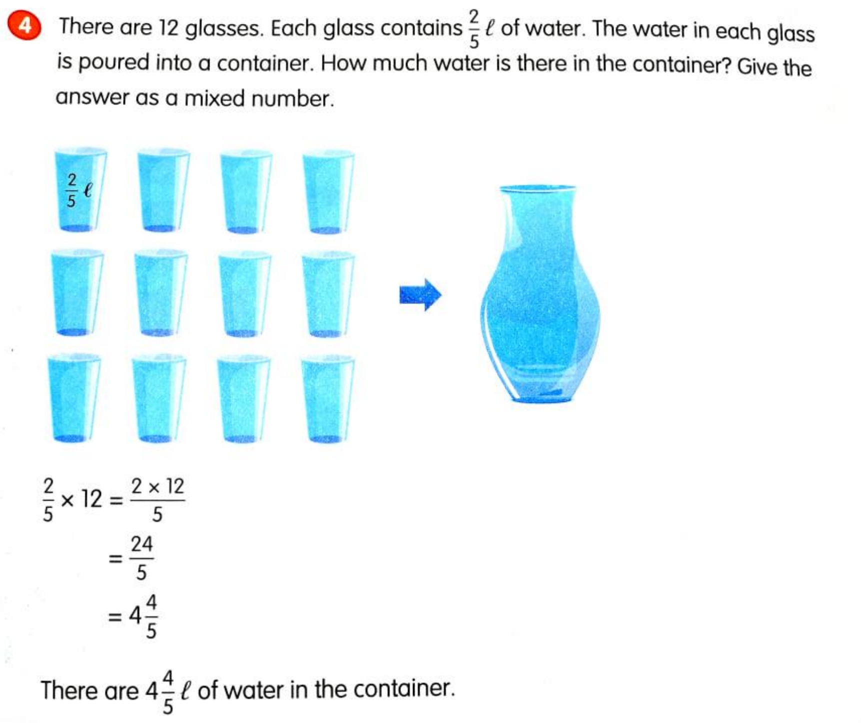
$\frac{3}{5}\, of\ 10 = \frac{3}{\cancel{5}_1} \times \cancel{10}_2$

$= 3x2 = 6$

There are 6 apples.

Figure 4. The operator meaning of multiplication and its solution through simplification in the multiplication of a fraction and a positive whole number in the Singaporean textbook.

In the following section of the Singaporean textbook, multiplication of fractions was performed following the repetitive addition meaning in multiplication. However, it was found that the appropriate symbolic form, which corresponds to the visual representations used in solving the problem, was not used in the textbook. The visual representation used in solving the problem in the textbook that states "There is $\frac{2}{5}\,lt$ water in 12 identical glasses. The water in each glass is poured into a jug. What is the amount of water in the jug?" points out the meaning of repetitive addition in multiplication. Therefore, it emerges as a deficiency that the symbolic equivalent of the visual representation in the textbook was presented as $\frac{2}{5}x12$ ($\frac{2}{5}$ of 12), even though the symbolic equivalent of the visual representation was $12x\frac{2}{5}$, because $\frac{3}{5}$ of 10 was expressed as $\frac{3}{5}x10$ in the introduction of the subject in the textbook; however, the presence of the commutative property of multiplication was not modeled through conceptual problems using various representation forms. The solution in the Singaporean textbook that was prepared considering the repetitive addition meaning in multiplication is presented in Figure 5.



4 There are 12 glasses. Each glass contains $\frac{2}{5}\ell$ of water. The water in each glass is poured into a container. How much water is there in the container? Give the answer as a mixed number.

$$\frac{2}{5} \times 12 = \frac{2 \times 12}{5}$$
$$= \frac{24}{5}$$
$$= 4\frac{4}{5}$$

There are $4\frac{4}{5}\ell$ of water in the container.

Figure 5. The inconsistency between the visual and symbolic representations in the Singaporean textbook.

In the Singaporean textbook, there are problems wherein the meaning of repetitive addition in multiplication is modeled through visual representations and presented in an appropriate symbolic form. Figure 6 demonstrates a visual representation of a problem expressed in the symbolic form in accordance with repetitive addition in multiplication. 4 $\frac{7}{3}$s were added in the problem, and the result of the operation of $4x\frac{7}{3}$ was calculated.

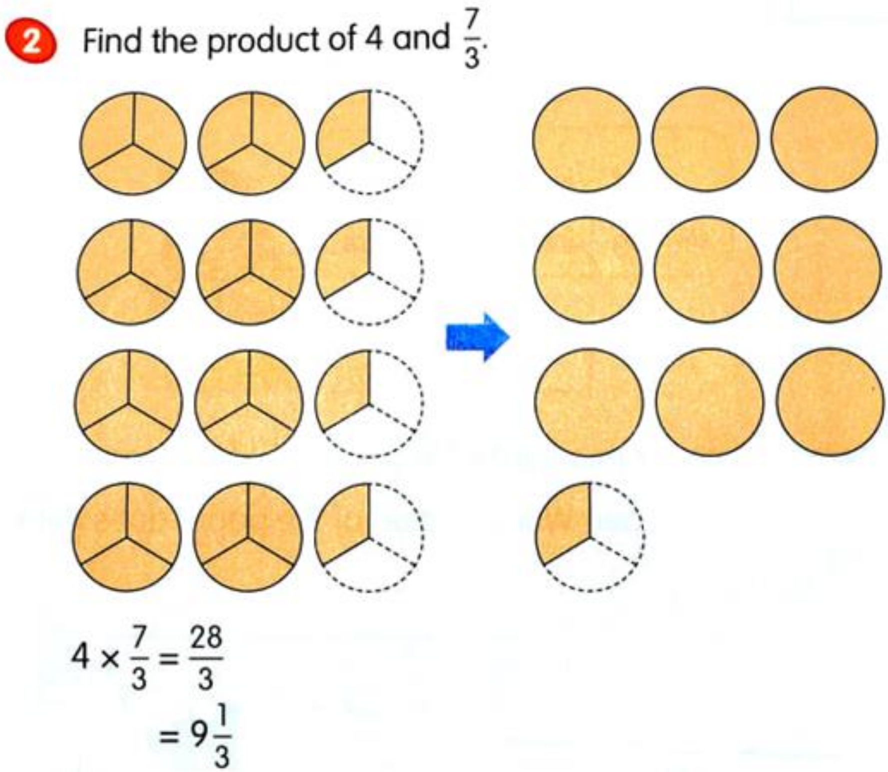


Figure 6. The multiplication of a positive whole number and an improper fraction in the Singaporean textbook.

In summary, the multiplication of a positive whole number by a fraction was calculated only through the meaning of repetitive addition in multiplication in the Turkish textbook, while the Singaporean textbook included the multiplication of a fraction by a positive whole number as well as the multiplication of a positive whole number by a fraction. Therefore, besides the meaning of repetitive addition in multiplication, the operator meaning of multiplication was included in the Singaporean textbook. The multiplication of two fractions in the Turkish and Singaporean textbooks was made through the meaning of "taking part of a part" and visual representations. In the Turkish textbook, the multiplication algorithm was used to multiply two fractions in symbolic form, but there was no mention that a simplification was possible. In the Singaporean textbook, on the contrary, after the multiplication algorithm was presented, it was emphasized that the multiplication of two fractions could be simplified. Moreover, while the Turkish textbook modeled only how to multiply two proper fractions, the Singaporean textbook included details of how to multiply a proper fraction by an improper fraction. Figures 7 and 8 illustrate how the multiplication of two fractions in both textbooks was calculated through the meaning of "taking part of a part."

Birlikte Öğrenelim

Fatma Hanım oğlu ve arkadaşları için yarısı çikolatalı, yarısı fındıklı olan bir tepsi kurabiye yapmıştır. Çocuklar fındıklı kurabiyelerin $\frac{3}{4}$'ünü yemişlerdir. Buna göre çocukların tüm kurabiyelerin kaçta kaçını yediğini bulalım.

Bir bütünün önce $\frac{1}{2}$'ini daha sonra $\frac{1}{2}$'inin $\frac{3}{4}$'ünü modelleme yardımıyla hesaplayalım.

Önce bütünü iki parçaya ayıralım.

Sonra $\frac{1}{2}$'lik kısımları 4 parçaya ayıralım.

Daha sonra fındıklı kısmın yenen 3 parçasını mavi renge boyayalım.

Mavi renkli kısım tüm şeklin $\frac{3}{8}$'üdür.

Bir tepsinin yarısının $\frac{3}{4}$'ünü gösterelim: $\frac{1}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}$

Çocuklar tüm kurabiyelerin $\frac{3}{8}$'ünü yemiştir.

Figure 7. The solution of the multiplication of two fractions through the meaning of "taking part of a part" in the Turkish textbook.[2]

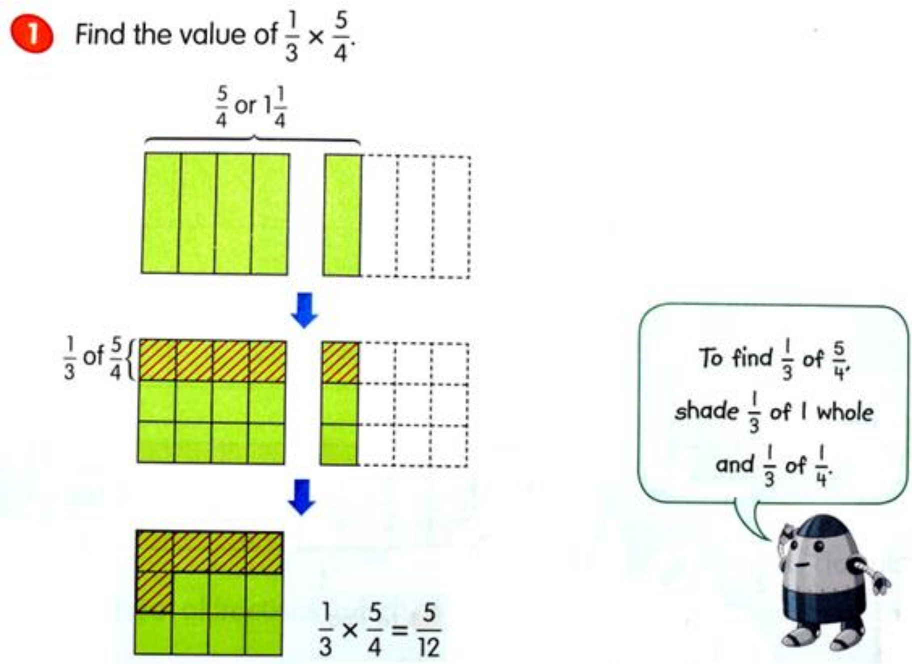


Figure 8. The modeling of the multiplication of a proper fraction by an improper fraction.

The evaluation of the textbooks of both countries in terms of the presentation of multiplying fractions indicated that both Singaporean and Turkish textbooks were prepared considering the synchronous development of conceptual learning and procedural fluency. In the Turkish textbook, the teaching of multiplying fractions began on the basis of the relationship between multiplication and repeated addition (conceptual learning) (Figure 3); then, the multiplication algorithm $(ax\frac{b}{c} = \frac{a}{1}x\frac{b}{c} = \frac{axb}{1xc})$ (procedural fluency) was presented by highlighting the need to write 1 in the denominator of the positive whole number when a positive whole number and a fraction were multiplied. The Singaporean textbook introduces and develops fraction multiplication as a fractional part of a whole (conceptual learning), and it gives an algorithm $\left(\frac{a}{b}xc = \frac{axc}{b}\right)$ (procedural fluency). The set (Figure 4) and area (Figure 6) models were used when the meanings of multiplying fractions were presented in the Singaporean textbook. The Turkish textbook, on the contrary, included only the area model (Figure 3). Thus, it can be said that the length model was used indirectly in multiplying fractions in the Singaporean textbook. Figure 9 exemplifies such a use. However, the fact that the set model was not used in the Turkish textbook, that the number line model was not used in the textbooks of either country, and only one representation form was used in modeling the problems of multiplying fractions in the textbooks of both countries can be considered as major drawbacks.

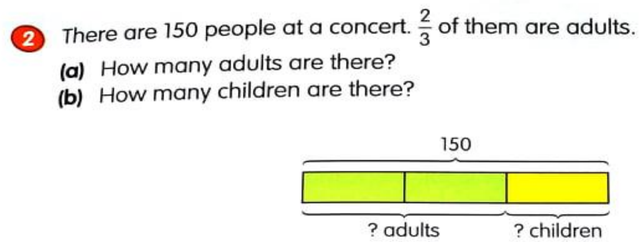


Figure 9. The use of the length model in multiplying fractions in the Singaporean textbook.

Analysis of the Problems Presented in the Textbooks

The problems in the textbooks were analyzed in terms of the number of steps, the answer type, and context characteristics. The findings are presented in Table 3.

Table 3
The distribution of the problems in the textbooks

Categories	TR f(%) $[N = 16]$	SI f(%) $[N = 65]$
Number of Steps		
Single Computational Calculation (S)	14	60
Multiple Computational Calculation (M)	2	5
Answer Type		
Numerical Answer only (NA)	14	60

Table 3 (continue)
The distribution of the problems in the textbooks

Categories	TR f(%) $[N = 16]$	SI f(%) $[N = 65]$
Numerical Expression required (NE)	0	1
Explanation or Solution required (ES)	2	4
Context		
Daily Life (DL)	6	13
Irrelevant to Daily Life (IDL)	10	52
In Symbolic Form (ISF)	5	45
In Verbal Form (IVeF)	11	20
In Visual Form (IViF)	0	0
In Multiple Forms (e.g., Visual + Verbal) (IMF)	0	2

The analysis of the textbooks in relation to the cognitive expectation characteristics indicated that most of the problems measured procedural knowledge. Although the Singaporean textbook was more abundant in terms of the number of problems in the mathematical reasoning category compared to the Turkish textbook, both textbooks were significantly weak in terms of conceptual knowledge and including problems in representation categories. In the problem-posing category, no problems were included in the textbooks of both countries.

Discussion and Conclusion

The first difference in multiplying fractions in the Turkish and Singaporean textbooks is the grade level in which the subject is presented. The teaching of multiplying fractions takes place at the 5th-grade level in Singapore, whereas it occurs at the 6th-grade level in Turkey. Earlier comparative studies focusing on various mathematical subjects have shown that countries succeeding in international examinations start the teaching of mathematics subjects earlier than other countries (Ding & Li, 2010; Hong & Choi, 2014). For instance, the multiplication of fractions in Taiwanese textbooks begins in the 4th grade; and in Japanese and Korean textbooks, it begins in the 5th grade (Watanabe et al., 2017). On the other hand, Grade-5 textbook coded TM covers the subject of "multiplication of fractions" on 5.42% of all its pages whereas the Turkish mathematics textbook coded TR covers the subject on 2.08 of all its pages. This finding can be interpreted as that the subject of "multiplication of fractions" is attached more importance in Singaporean textbooks. A study conducted by Li et al. (2009) has reported that Japanese and Chinese textbooks allocated more pages to the subject of "division of fractions" than American textbooks. The Turkish mathematics textbook allocates a unit to the subjects of addition, subtraction, multiplication, and division of fractions, where these subjects are handled consecutively. In addition, in Singapore, addition and subtraction with fractions are taught in 4th grade, and multiplication with fractions is taught in 5th grade. Similarly, the teaching of addition and subtraction with fractions in Japanese, Korean, and Taiwanese mathematics textbooks begin at the 3rd-grade level. Teaching the subject of multiplication by fractions is taught in the next grade levels.

The second difference in multiplying fractions in Turkish and Singaporean textbooks relates to how the subject is introduced. The introduction of multiplying fractions was communicated through the meaning of repetitive addition in multiplication in the Turkish textbook, whereas the introduction of multiplying fractions was taught through the operator meaning of multiplication in the Singaporean textbook. Son and Senk (2010) found that the Korean textbooks introduced the teaching of multiplying fractions through the meaning of repetitive addition in multiplication. It has been recommended in the previous literature that multiplying fractions should start with the meaning of repetitive collection and continue with taking part of a part of a positive whole number and a fraction (Musser, Peterson & Burger, 2014; Wu, 2001).

While only the meaning of repetitive addition in multiplication was highlighted in the multiplication of a positive whole number by a fraction in the Turkish textbook, the operator and repetitive addition meanings in multiplication were used in the Singaporean textbook. Therefore, in the Singaporean textbook, in addition to multiplication operations such as $ax\frac{b}{c}$, the solutions of operations such as $\frac{a}{b}xc$ were also included. However, the appropriate symbolic form corresponding to the visual representations was not used in a number of problems in the Singaporean textbook. The visual representation used in solving the problem in the textbook that states "There is $\frac{2}{5}\,lt$ water in 12 identical glasses. The water in each glass is poured into a jug. What is the amount of water in the jug?" indicates the meaning of repetitive addition in multiplication. Therefore, the fact that the symbolic equivalent of the visual representation in the textbook was given as $\frac{2}{5}x12$ ($\frac{2}{5}$ of 12) can be considered as a drawback regardless of the fact that the symbolic equivalent of the visual representation is $12x\frac{2}{5}$. Although multiplication has a commutative property, it is crucial that students learn that the meaning of each condition is represented in different symbolic forms (Van de Walle et al., 2013). In addition, $\frac{3}{5}$ of 10 was expressed as $\frac{3}{5}x10$ in the introduction of the subject in the Singaporean textbook; however, the presence of the commutative property of multiplication was not modeled through conceptual problems using various representation forms. This issue in the Singaporean textbook could have been resolved by stating that the results of $3x\frac{1}{4}$ and $\frac{1}{4}x3$ are equal by modeling the operations using different forms of representation. For example, the problem that states "if Ali traveled one-fourth of a 3-km road by car, how far did he travel?" could have been modeled through a number line, which would enable students to understand what $\frac{1}{4}x3$ means (Figure 10). Additionally, in the symbolic form of the problem that states "if Ali traveled one-fourth of the road he travels every day, what proportion of the road would Ali have traveled after three days?" $3x\frac{1}{4}$ means the sum of 3 $\frac{1}{4}$s, which is calculated as $\frac{3}{4}$. Thus, owing to this and similar examples, students can observe that the results of $3x\frac{1}{4}$ and $\frac{1}{4}x3$ are equal. Therefore, it will be appropriate to write the symbolic form of $\frac{2}{5}x12$ instead of $12x\frac{2}{5}$ after providing such an example and similar ones. The fact that the Turkish textbook highlights only the meaning of repetitive addition in multiplication for the multiplication of a positive whole number and a fraction can be regarded as a drawback because, as can be understood from Figure 3, the meaning of repetitive addition in multiplication does not exist in every multiplication operation by fractions (Musser et al., 2014). For instance, when converting $\frac{3}{4}x20$ into 3/4 pieces of 20, it will be useful to refer to the operator meaning of multiplication (3/4 of 20; Son, 2012, p. 390). In this respect, in the Turkish textbook, the operator meaning of multiplication should also be presented in the multiplication of a positive whole number and a fraction in addition to the meaning of repetitive addition in multiplication. In both the Turkish and Singaporean textbooks, the multiplication of two fractions was based on the idea of "taking part of a part." While the multiplication of two proper fractions was solved through modeling in the Turkish textbook, the multiplication of a proper fraction and an improper fraction was modeled in addition to the multiplication of two proper fractions in the Singaporean textbook.

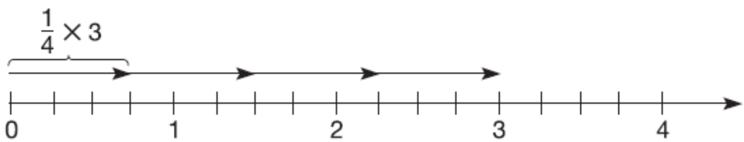


Figure 10. The model relating to the operator meaning (Musser et al., 2014).

In the Turkish textbook, the teaching of multiplying fractions began on the basis of the relationship between multiplication and repeated addition (conceptual learning) (Figure 3); then, the multiplication algorithm ($ax\frac{b}{c} = \frac{a}{1}x\frac{b}{c} = \frac{axb}{1xc}$) (procedural fluency) was presented by highlighting the need to write 1 in the denominator of the positive whole number when a positive whole number and a fraction were multiplied. The Singaporean textbook introduced and developed fraction multiplication as a fractional part of a whole (conceptual learning), and it gave an algorithm $\left(\frac{a}{b}xc = \frac{axc}{b}\right)$ (procedural fluency). Therefore, the content of multiplying fractions in the textbooks of both countries was based on the synchronous development of procedural and conceptual understanding. The teaching of multiplying fractions in the Turkish textbook was communicated through repetitive addition and the idea of taking part of a part; in the Singaporean textbook, this was done through repetitive addition, the operator meaning, and the idea of taking part of a part.

In the Turkish textbook, only area models were used to present the multiplication of fractions; in the Singaporean textbook, set and area models were used. It was also found that the length model was used indirectly in the Singaporean textbook. However, the fact that the set model was not used in the Turkish textbook, that the number line model was not used in the textbooks of both countries, and that only one representation form was used in modeling the problems of multiplying fractions in the textbooks of both countries can be considered as major drawbacks. Ainsworth (2006) highlighted the importance of using different forms of representation, emphasizing that two representations are better. Similarly, Gagatsis and Shiakalli (2004) underlined that a single representation in the learning context reveals only certain aspects of the concept, and that multiple forms of representation have the potential to complement each other. Similarly, Japanese textbooks teach fractions by combining the area model and the number line model to provide students with the opportunity to apply proportional reasoning in multiplying fractions (Watanabe et al., 2017). In fact, a number line indicates that a fraction is both a number and possesses relative greatness compared to other numbers. In area models, this is not clear. More importantly, a number line reinforces the idea that there is always a fraction to be found between two fractions. The set model, on the other hand, helps students deal with plenty of objects as a whole and connect with the concept of ratio. Moreover, the set model paves the way for the use of fractions suitable for daily life (Van de Walle et al., 2013).

Additionally, the textbooks of both the countries illustrated how the multiplication algorithm can be used. Compared to the Turkish textbook, the Singaporean textbook included a higher number of solution strategies. In the Turkish textbook, the operation of $a x \frac{b}{c}$ was printed as $\frac{a}{1} x \frac{b}{c}$, and the result of $\frac{a x b}{1 x c}$ was calculated. The multiplication of two proper fractions was calculated as $\frac{a}{b} x \frac{c}{d} = \frac{a x c}{b x d}$. Unlike the Turkish textbook, the Singaporean textbook does not attempt to express the positive whole number “a” as $\frac{a}{1}$. The operations of $a x \frac{b}{c}$ or $\frac{b}{c} x a$ were expressed as $\frac{a x b}{c}$, and simplification was made if *a* and *c* had common divisors. The multiplication of two fractions was carried out through two techniques in the Singaporean textbook. In the first technique, the operation of $\frac{a}{b} x \frac{c}{d}$ was printed as $\frac{a x c}{b x d}$, and the result was calculated through simplification if the numerator and denominator in the fraction could be simplified within themselves. In the second solution technique, *a* and *d* and *b* and *c* in the operation of $\frac{a}{b} x \frac{c}{d}$ were simplified if possible, and the result was calculated by multiplying the numerators and writing them in the numerator part as well as by multiplying the denominators and writing them in the denominator part. Son (2012) stated that one of the noteworthy aspects of the Korean textbooks was the use of multiple solution strategies in multiplying fractions.

In the Turkish textbook, there were significantly fewer problems of multiplying fractions compared to the Singaporean textbook. Many of the problems in the textbooks were one-step problems that required numerical answers. In the Turkish textbook, there were no problems presented in multiple forms. In the Singaporean textbook, there were only two problems in multiple forms. While six (37%) of the 16 problems in the Turkish textbook were relevant to daily life, only 13 (20%) of the 65 problems in the Singaporean textbook were relevant to daily life. The analysis of the problems in the textbooks in accordance with cognitive expectation characteristics showed that most of the problems measured procedural knowledge. Although the Singaporean textbooks were more abundant in terms of the problems in the mathematical reasoning category compared to Turkish textbooks, the textbooks of both countries were significantly weak in terms of conceptual knowledge and including problems in representation categories. There were no problems that can be considered in the problem-posing category in the textbooks of both countries. Stein et al. (2007) emphasized that the problem types in the textbooks will reflect the performance of the students on international exams. On the TIMSS, there are questions from the cognitive domains of knowledge, application, and reasoning, two-thirds of which are at the level of application and reasoning. Students must know the meaning of the strategies and algorithms involved in multiplying fractions in addition to why they should use them to solve the questions of multiplying fractions at the application level. In addition, students are asked open-ended, multi-step questions at the level of reasoning. In such questions, students are expected to explain their solution and use different solution strategies, and the correct solution is marked with two points (Mullis et al., 2015). The results obtained from the problem analysis are not considered consistent with the fact that Singaporean students showed higher performance in the learning area of numbers compared to Turkish students in an international assessment. However, it is known that there are numerous factors influencing students' success in mathematics. Therefore, the findings of this study should be approached with caution because there are numerous factors influencing students' success in mathematics (i.e., the educational levels of parents, the number of educational resources at home, the teachers' role, and so on). A well-prepared and a not-well-prepared textbook will come to life at the hands of a well-equipped teacher. The findings of this study are listed below for a better understanding of readers.

- The teaching of multiplying fractions takes place at the 5th-grade level in Singapore, whereas it occurs at the 6th-grade level in Turkey.
- The introduction of multiplying fractions was communicated through the meaning of repetitive addition in multiplication in the Turkish textbook, whereas the introduction of multiplying fractions was taught through the operator meaning of multiplication in the Singaporean textbook.
- While the Singaporean textbook covers all the meanings of multiplying fractions (i.e., repeated addition, operator, and taking part of a part), the Turkish textbook does not refer to the procedural meaning of multiplying fractions.
- The content on multiplying fractions in the textbooks of both countries was created by considering the synchronous development of students' conceptual and procedural knowledge.
- While the Singaporean textbook utilizes the set and area models, the Turkish textbook uses only the area model.
- The number line model is not used in the textbooks of both countries, and only one representation format (i.e., only the set or area model) is used to model a fraction multiplication problem.
- The Singaporean textbook includes more solution strategies than the Turkish textbook.
- The Singaporean textbook includes more fraction multiplication problems than the Turkish textbook.
- Even though both textbooks attach importance to the synchronous conceptual and procedural development of students in multiplying fractions, most of the problems in the textbooks were one-step problems requiring numerical answers and were considered in the procedural knowledge category. This indicates an inconsistency between the teaching- and testing-assessment techniques.

The results of this study imply that the Singaporean textbooks provide better learning opportunities for grasping multiplication in fractions. Such conclusion is in line with Turkish and Singaporean students' performance in international exams.

Limitations and Future Research

The findings of this study reveal that a consensus should be reached on the need to further concentrate on the use of different meanings of multiplication and its different forms of representation in the preparation of Turkish textbooks. Textbooks should be enriched in terms of activities that enable students to learn real-life problems and the algorithm of multiplying fractions in a meaningful way and that are suitable for the use of all types of representations. Diverse types of problems should be distributed in a balanced way in the textbooks of both countries. Caution should be exercised with respect to the generalizability of the results obtained in this study, as it compared two textbooks from two countries. Also, considering the role of teachers in the instructional process (Wijaya, Van den Heuvel-Panhuizen, & Doorman, 2015; Yang, 2018), including full content in the textbook is not adequate on its own for effective mathematics instruction because students' achievement in mathematics can be influenced by many factors such as the educational qualification of parents, the quantity of learning resources at home and the role of the teacher, etc. Further studies might aim to discover how textbooks are used by Turkish and Singaporean teachers, whether they use additional teaching materials, how they teach the meaning of the concept of multiplication in fractions, whether they include problems and solution strategies other than those in the textbooks, and what effects this has on student learning.

Notes

[1] Problem: Ela feeds chickens with the same amount of feedstuff every day. Ela's grandfather has bought her some feedstuff. Ela has given 1/8 of the feedstuff to the chickens every day. Let's find out how much feedstuff Ela has fed the chickens for a week?. Solution 1: There are 7 days in a week. Let's add 7 times 1/8s. The shorter way of adding 7 times 1/8s is to multiply them. 7 1/8s = Ela gave 7/8 of the whole feedstuff to the chickens in a week. Solution 2: Day 1, Day 2, Day 3, Day 4, Day 5, Day 6, Day 7. The feedstuff given at the end of a week is 7 1/8s = 7/8).

[2] Problem: Ms. Fatma made a tray of cookies for her son and his friends. Half of the cookies had chocolate, and the other half had nuts. The children ate ¾ of the cookies with nuts. So, let's

find out what proportion of the cookies the children ate. Solution: Let's calculate 1/2 of the whole and then 3/4 of 1/2 through modeling. Cookies with nuts, Cookies with chocolate. Let's divide the whole into two. Next, let's divide the 1/2 parts into 4. Then, let's paint the 3 pieces of the cookie with nuts into blue. The blue part is 3/8 of the whole. Let's highlight 3/4 of half a tray.

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Suphi Önder Bütüner is Associate Professor of Math Education at Faculty of Education, Yozgat Bozok University, Turkey.

Contact Address: Direct correspondence concerning this article, should be addressed to the author. **Postal Address:** Yozgat Bozok University, Erdogan Akdag Campus, Faculty of Education, 66900, Yozgat, Turkey
Email: s.onder.butuner@bozok.edu.tr