WATER QUALITY INDEX BASED ON FUZZY LOGIC APPLIED TO THE ABURRA RIVER BASIN IN THE JURISDICTION OF THE METROPOLITAN AREA

UN ÍNDICE DE CALIDAD DE AGUA BASADO EN LÓGICA DIFUSA, APLICADO A LA CUENCA HIDROGRÁFICA DEL RÍO ABURRÁ, EN LA JURISDICCIÓN DEL ÁREA METROPOLITANA

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ABSTRACT: The inherent uncertainties, subjectivity, and engineering challenges in environmental problems are increasingly being worked upon computing methods based on artificial intelligence (AI), including computerized tools that allow an analyst to use approximate reasoning with incomplete and inaccurate information and with the support of an expert in the field. In order to evaluate the applicability of this tool, this paper presents the development of a new water quality index based on fuzzy logic called the "fuzzy water quality index" (FWQI). This index is compared to the NSF water quality index (NSFWQI) and with the water quality index (ARWQI) obtained for the project "Red Río Fase I" for the Medellin metropolitan area environmental authority. The new FWQI provided reasonable correlations and results in comparison to the other two reference indexes. Finally, the FWQI could be used as a decision maker in the water management of Aburra River.

KEYWORDS: water quality indices (WQIs), fuzzy-logic, fuzzy inference system, bioindicators

RESUMEN: Las incertidumbres, las subjetividades y los grandes retos ingenieriles en los problemas ambientales se trabajan cada vez más con el uso de métodos de cálculo basados en la Inteligencia Artificial (IA), incluyendo herramientas computarizadas que permiten usar razonamientos aproximados con información incompleta e inexacta y con el apoyo de expertos. Con el fin de evaluar la aplicabilidad de esta herramienta, este estudio propone la aplicación de un Índice de Calidad del Agua, basado en la Lógica Difusa denominado "Índice de Calidad de Agua Difuso" (FQWI), en el río Aburrá. EL FQWI se comparó con el Índice de Calidad de Agua NSF (NSFWQI y con el Índice de Calidad de Agua elaborado para el proyecto "Red Río Fase I" (ARWQI)) para la autoridad ambiental del Área Metropolitana de Medellín. Este nuevo índice (FQWI) presentó resultados y correlaciones razonables, con relación a los otros índices propuestos. Finalmente, se espera que este nuevo índice FQWI pueda ser utilizado para la toma de decisiones en la gestión hídrica del río Aburrá.

PALABRAS CLAVE: índices de calidad de agua, lógica difusa, sistema de inferencia difuso, bioindicadores

1. INTRODUCTION

The urban development of the Aburra River Valley has increased the levels of water pollution in the Aburra River in such a way that the use of water as a source for various human activities and as a recipient of waste water has promoted the study and development of water quality indicators. As a contribution to this development, the aim of this paper is to compare three different water quality indices (WQIs) and to analyze the spatial and temporal variability of Aburra River water quality in the jurisdiction of the Aburra River Metropolitan Area.

A water quality index allows the analyst to combine information from different water quality parameters

into a single value for a better interpretation. The main objective of these indices is to evaluate the general state of water, depending on a range of a group of established water quality parameters. Additionally, the information that they provide can easily be interpreted from a list of numerical values and be useful in taking decisions for environmental analysis and for compliance with environmental regulations.

The limitations of WQIs demonstrate the need to develop techniques and more advanced assessment methods which enable the analyst to include and interpret qualitative and quantitative information. The methods based on Artificial Intelligence (AI) can combine the advantages of the traditional methods with the advantages provided by the AI.

In this paper, the FWQI [1,2,3] is estimated along the Aburra River for different sampling campaigns from 2004 to 2007. The results are compared with water quality indices of the U.S. National Sanitation Foundation (NSFWQI) and the water quality index estimated in the study of the Aburra River Phase I (ARWQI) [4].

2. STUDY ZONE

The Aburra River Valley area includes the following municipalities in the province of Antioquia, Colombia: Caldas, La Estrella, Sabaneta, Envigado, Itagüi, Medellin, Bello, Copacabana, Girardota, and Barbosa (Fig. 1). The Aburra Valley covers an area of 1,152 Km^2, with an urban area of 176.1 Km^2 with a population density of 14,214 inhabitants/Km^2 and a rural area of 975.9 Km^2 with a population density of 191 inhabitants/Km^2. The Aburra River runs 105 Km along the valley, beginning in the Alto de San Miguel and flowing into the Nechi River.

Industry and services are the dominant economic activities in the urban area, whilst family agriculture is the main economic activity in the rural area.

The climate is a typical equatorial mountain system, with a mean annual temperature of 22 ºC, and a bimodal rainfall regime with an annual average of 1500 mm.

The water in the area is used primarily for domestic activities, followed by industrial activity. The pollution is mainly generated by the dumping of solid, sewage, industrial, and domestic wastes [5].

Information of water quality in five sampling stations along the main channel were used in this study (see Fig. 1 and Table 1) and were obtained from previous studies conducted between 2004 and 2007 [4].

3. WATER QUALITY INDEXES

The problem of understanding WQIs lies primarily in the definition of the concept of water quality. Water quality can be understood as the intrinsic capacity of a water body to respond to the use external agents make of it so a balance with the ecosystem is obtained, satisfying some water quality standards. This definition helps analysts to have a clearer idea of a balanced ecosystem in which water carries out certain quality objectives and different indexes measuring them. The indexes have the advantage of being easy to use and provide a quick and intuitive idea of the water quality. The use of more than one index allows analysts to have a more comprehensive idea about water quality.

In the construction of an index it is required to define the variables and weights according to the parameters established by current regulations. In the Colombian case, the evaluation of watersheds for the protection of flora and fauna is ruled by Decree 1594/84 of the Ministry of Health. This decree regulates dumping in relation to the natural water potential uses [5,6].

3.1 The National Sanitation Foundation index (NSFWQI)

In the United States, the National Sanitation Foundation (NSF), developed the NSFWQI in 1970 through the use of the Delphi survey technique of the Rand Corporation. This index has the characteristic of being a multiparameter index based on three studies. The first study adopted 35 pollution variables and included the opinion of recognized experts in the field. The second study recognized nine variables identified as the most important ones like dissolved oxygen (DO), fecal coliform, pH, biochemical oxygen demand (BOD$_5$), nitrate, phosphate, turbidity and total solids (TS). Finally, the third study [7] developed a series of rating curves for each variable under analysis.
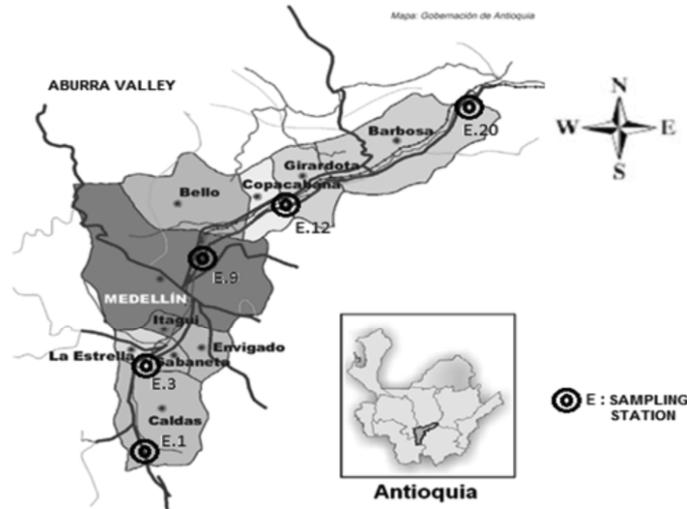
Figure 1. Location of Aburra River valley

Table 1. Location of monitoring stations

Sampling Station	Station	Coordinates		Abscissa	Municipality
		North	**West**		
E.1	San Miguel	6° 02' 7.20"	75° 37' 1.22"	K2+000	Caldas
E.3	Ancón Sur	6° 09' 7.95"	75° 37' 5.48"	K22+000	La Estrella
E.9	Puente Acevedo	6° 18' 35.99"	75° 33' 23.88"	K42+100	Medellin
E.12	Metro Mezclas	6° 22' 16.21"	75° 29' 21.29"	K55+000	Copacabana
E.20	Puente Gabino	6° 33' 33.99"	75° 12' 25.64"	K103+600	Barbosa

To calculate this WQI, a weighted arithmetic average is used:

$$NSF\,WQI = \sum_{i=1}^{n} SI_i W_i \qquad (1)$$

Where:
NSF WQI: NSF Water Quality Index
SI_i: curve function value "q".
W_i: weighting factor for the subscript i

The result of the application of this index is a

number between 0, poor water quality and, 100, excellent water quality [8].

3.2 The ARWQI index

ARWQI index [4] aims to estimate a value between 0 and 1 that defines the quality level of

the water body, the value of one being an index of very good water quality (low pollution).

According to the results obtained in the different samples taken during the Aburra River Project Phase I [4], the selected parameters of water quality along the river were: DO, BOD_5, TS, total fixed solid (TFS), total suspended solids (TSS), electrical conductivity (EC), total nitrogen (TN), total phosphorus (TP) and aquatic macro-invertebrates (MI). These variables were gathered in five groups, namely: group 1, DO and BOD_5; Group 2 consists of TN and TP; Group 3 includes only EC; group 4 includes TS, TFS and TSS; and group 5 includes only MI.

The description of the MI parameter is defined by assigning a fixed value in each category as follows: to the first category, which includes the stations that presented the best water quality conditions a value of 1.0 was assigned; in the second category, stations with good water quality to moderately contaminated waters where grouped with a value of 0.7; the third category included stations with moderately polluted waters to heavily contaminated with a value of 0.45; and the fourth category represents highly polluted

waters with a value of 0.15. In this study, this grading criterion is established according to the degree of water saprophyte and pollution [9]. Additionally, weighting factors were defined for every group under consideration, which assigned the highest value to the most influential variable along the river. The maximum sum of the weighting factors is 10 corresponding to the highest index and indicates the best water quality in the channel. In this way, five weighting factors are assigned and these are represented by the letters A, B, C, D, and E, which have values of 3.0, 2.0, 2.0, 1.5, and 1.5 respectively, in relation to the river water quality [4].

The ARWQI is the addition over all groups of the multiplication of the values of each group of variables and their respective weighting factor, as is shown in the following expression:

WQI=A(OQD•OBD$_5$)+B(TN•TP)+C(EC)+

D(TS•TFS•TSS)+E(MI)   (2)

3.3 Fuzzy Logic water quality index (FWQI)

The fuzzy logic methodology allows an analyst to use a set of qualitative conditional expressions and to increase the initial awareness, through processes of inference or "learning". The integration of fuzzy logic and fuzzy inference systems [1–3,10–14] to the variables of environmental monitoring require a conceptual change. The fuzzy sets were defined in terms of a membership function that represents the shape of the fuzzy sets that divide the universe of each linguistic variable, which in turn is assigned a domain of interest for the interval [0,1], ($\mu: X \rightarrow [0,1]$). Figure 2 shows the membership functions used in this study.
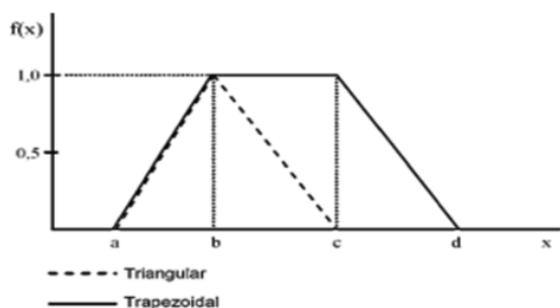
Figure 2. Membership function in trapezoidal and triangular shape

For this study the following linguistic variables and groups were defined: Group 1: Temperature (T) and pH; Group 2: DO and BOD$_5$; Group 3: MI; Group 4: TN and TP; Group 5: TS and Turbidity. The linguistic values defined were: More than excellent (VE), Excellent (E), Very Good (VG), Good (G), Fair/Good (FG), Average (R), Average/Bad (RB), Poor (P) and Very Poor (VP).

Tables 3 and 4 shows the physicochemical and biological parameters taken into account in the study, with their input fuzzy sets, and following the membership functions or curves described above. Figure 3 shows the flow chart of the process, where individual quality variables are processed by inference systems producing several groups normalized between 0 and 100. The groups are then processed (step 2) using a new inference, the parameters are normalized and grouped by the fuzzy inference systems using the fuzzy sets in Fig. 4 and as a result the FWQI is obtained[1].

A rule in the inference system is a mathematical formalism which expresses the opinion of experts in linguistic and therefore is an important factor in the subjective and qualitative inference block, for example: Rule 1: If fecal coliforms are high and the pH is excellent then the index is very poor, Rule 2: If fecal coliforms are very high and the pH is average then the index is poor. These rules are designed by an expert system and a subjective and qualitative weighting factor based on expert of the field's opinion is introduced in the process.

The rules for standardization and aggregation of the logic used in this work are described below (Step 1) and, therefore, always operate under requirement of a minimum operator: If the first parameter (FP) is More than excellent (VE) and the second parameter (SP) is More than Excellent (VE) then the output group (OG) is More than excellent (VE). There are 804 inference rules for this case.

The FWQI was developed from the fuzzy inference rules that were applied to groups 1 to 5 as input. The input sets or history (groups 01–05) and all consequential or output groups were created as trapezoidal functions for all excellent and as a triangular function for the other qualifications, as shown in Table 2.

In this paper, 3125 fuzzy inference rules were developed for Step 2, and as an example we

present the following: Rule 1: If Gr01 to Gr05

are E, then FWQI is E; Rule 1250: if GR01 is G,

Gr02 is P, Gr03 is P, Gr04 is P and Gr05 is P,

then FWQI is B; Rule 2630: IfGr01 is P, Gr02 is G, Gr03 is E, Gr04 is E and Gr05 is P, then FWQI is R. Above, Gr0n refers to Group n.

Table 2. Output Fuzzy sets for step 2

	a	b	c	D
Excellent	65	90	100	100
Good	44	65	90	
Average	28	44	65	
Bad	8	28	44	
Poor	0	0	9	28

All rules were computed and processed in the program MATLAB fuzzy logic toolbox of 2009 ® [15].

Table 3. Fuzzy sets and linguistic terms for input parameters. Groups 01, 02, and 03

	Group 01						Group 02						Group 03		
PARAMETER	Temperature			pH			DO			BOD5			Macro- invert.		
Unit	°C						mg/L			mg/L			Gender/Family		
Interval	-6 to 45			1-14			0-9			0-170			0-170		
Linguistic variable	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c
More than excellent	15	16	21	6.8	6.9	7.1	7.0	7.5	9.0	0	0	3	145	155	170
Excellent	14	15	16	7.1	7.8	8.3	6.5	7.0	7.5	3	11	17	131,5	145	155
Low Excellent ↓	21	22	24	6.6	6.8	6.9									
Very Good	13	14	15	7.8	8.3	8.5	6.0	6.5	7.0	11	17	23	115	131.5	145
Low very Good↓	22	24	26	6.3	6.6	6.8									
Good	10	13	14	8.3	8.5	8.8	5.0	6.0	6.5	17	23	29	100	115	131.5
Low Good ↓	24	26	28	6.1	6.3	6.6									
Fair/Good	5	10	13	8.5	8.8	9.0	4.0	5.0	6.0	23	29	34	82	100	115
Fair/Low Good	26	28	30	5.9	6.1	6.3									
Average	0	5	10	8.8	9.0	9.2	3.5	4.0	5.0	29	34	46	66	82	100
Low Average ↓	28	30	32	5.6	5.9	6.1									
Average/Bad	-2	0	5	9.0	9.2	9.6	3.0	3.5	4.0	34	46	68	52	66	82
Average/Low Bad ↓	30	32	36	5.2	5.6	5.9									
Bad	-4	-2	0	9.2	9.6	10.0	2.0	3.0	3.5	46	68	85	35	52	66
Low Bad ↓	32	36	40	4.8	5.2	5.6									
Very Bad	-6	-4	-2	9.6	10.0	10.5	1.0	2.0	3.0	68	85	125	22	35	52
Very low Bad ↓	36	40	45	4.0	4.8	5.2									
Poor	-6	-6	-4	10.0	10.5	12.0	0.0	1.0	2.0	85	125	170	12	22	35
Low Poor ↓	40	45	45	2.0	4.0	4.8									
Very Poor	-6	-6	-6	10.5	14.0	14.0	0.0	0.0	1.0	125	170	170	0	12	22
Very Low Poor↓	45	45	45	1.0	1.0	4.0									

The d values of each parameter used to build the membership function are: for T = 22 °C, potential of hydrogen = 7.75, DO = 9.0 mg/L, BOD_5 =11 mg/L, and MI = 170.

Table 4. Fuzzy sets and linguistic terms for input parameters. Groups 04 and 05 and output parameters for all groups.

	Group 04						Group 05						Output group		
PARAMETER	**TN**			**TP**			**TS**			**Turbidity**			**Output**		
Unit	mg/L			mg/L			mg/L			mg/L					
Interval	0-20			0-10			0-880			0-350			0-100		
Linguistic variable	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c
More than excellent	0	0	0	0.0	0.0	0.1	0.0	0.0	10.0	0.0	0.0	2	0	0	1
Excellent	0.3	0.8	2	0.1	0.2	0.3	10.0	30.0	50.0	2.0	6.0	17,5	0	10	20
Very Good	0.8	1.5	3	0.2	0.3	0.4	30.0	50.0	170.0	6.0	17.5	29	10	20	30
Good	1.5	3	6	0.3	0.4	0.6	50.0	170.0	320.0	17.5	29.0	52,5	20	30	40
Fair/Good	3	5.8	8	0.4	0.6	0.8	170.0	320.0	470.0	29.0	52.5	82	30	40	50
Average	5.8	8	11	0.6	0.8	1.0	320.0	470.0	600.0	52.5	82.0	117	40	50	60
Average/Bad	8	10.8	13	0.8	1.0	1.5	470.0	600.0	685.0	82.0	117.0	163	50	60	70
Bad	10.8	13	16	1.0	1.5	3.0	600.0	6850	750.0	117.0	163.0	222	60	70	80
Very Bad	13	15.8	18	1.5	3.0	6.0	685.0	750.0	815.0	163.0	222.0	280	70	80	90
Poor	15.8	17.5	20	3.0	6.0	10.0	750.0	815.0	880.0	222.0	280.0	350	80	90	100
Very Poor	17.5	20	20	6.0	10.0	10.0	815.0	880.0	880.0	280.0	350.0	350	90	100	100

The d values of each parameter used to build the membership function are for TN =1 mg/L, TP = 0.2 mg/L, TS = 30, turbidity = 6 and output = 10.
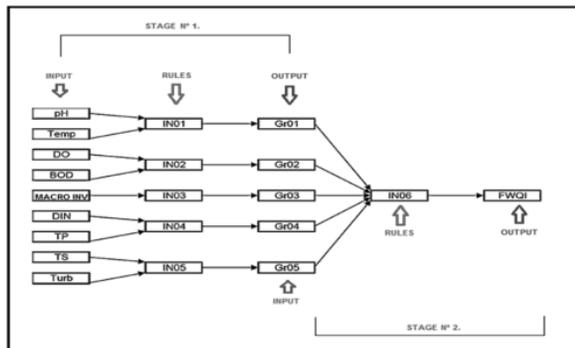
Figure 3. Graphic flow of process
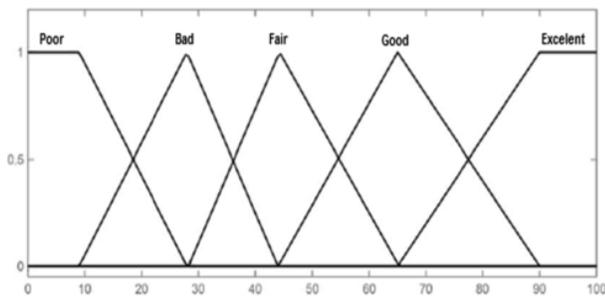
Figure 4. Graphs of the Fuzzy sets functions

4. RESULTS

This study used the results of physicochemical and biological parameters collected between 2004 and 2007 in five sampling stations [4]. Figure 5 shows the results obtained from the ARWQI, FWQI and NSFWQI indexes and their spatial and temporal variability in the different measuring campaigns conducted by the Medellin Metropolitan Area environmental authority in partnership with the universities of the city. Figures 6 and 7, present the relationship between FWQI and the other two indexes and Fig. 8 presents bar charts for the three indices calculated at 5 stations along the Aburra River.

5. ANALYSIS OF THE RESULTS

From Figure 5 it is clear that the three indexes show the same tendency (notice that the scale for the ARWQI index is 10 times smaller than that for the other two indexes). In San Miguel Station (Km 0) all indexes show good water quality with values between 70

and 90. Downstream and throughout the urban area, water quality decreases to a minimum at Ancon Norte Station, Km 55, with values from 20 to 50. Farther downstream, the river showed an improvement in the water quality due to the dilution effect of the water discharges from "La Tasajera" Hydroelectric power plant and the re-aeration of the river reaching values from 50 to 60 at Puente Gabino Station (Km 100). The results showed that the FWQI is stricter than the NSFWQI and ARWQI.
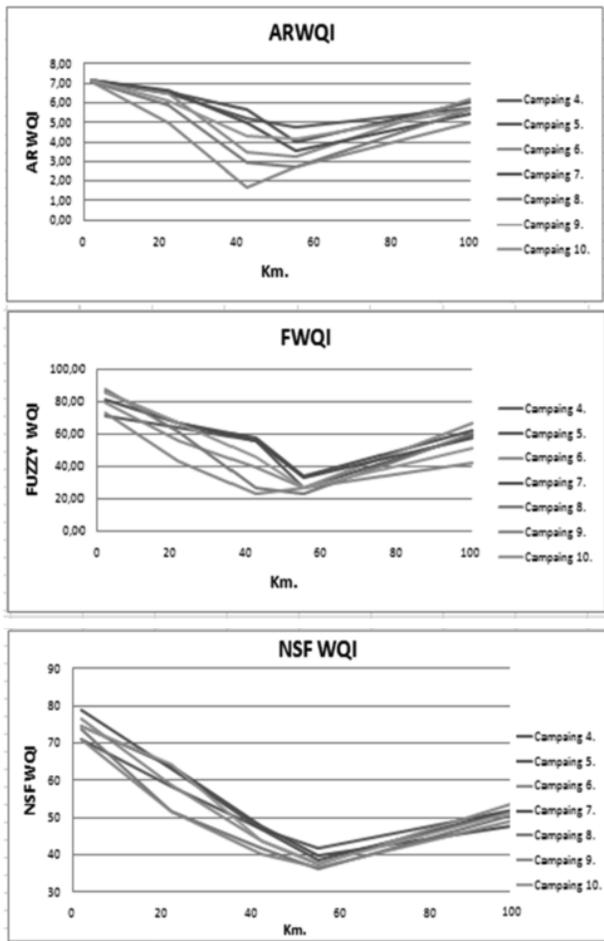
Figure 5. WQI's Spacial-temporal variability

Figures 6 and 7 show the linear relationship between FWQI and the other two indexes. Both figures show a good linear relationship with coefficients of determination of 0.87 and 0.86, respectively, with most data falling inside the 95 % confidence interval. The

FWQI can be calculated using this statistic relationship with the NSFWQI or the ARWQI as predictors.

Figure 8 shows the behavior of the three indexes at every station and for all measuring campaigns. A low variability of all three WQIs is noticeable in San Miguel Station and for all campaigns, showing values of good water quality with the FWQI index presenting the highest values. Except for the first campaign, the FWQI calculates higher values than the NSFWQI and ARWQI. In our case, FWQI includes macro-invertebrates, which improves the qualification meaning of this index.
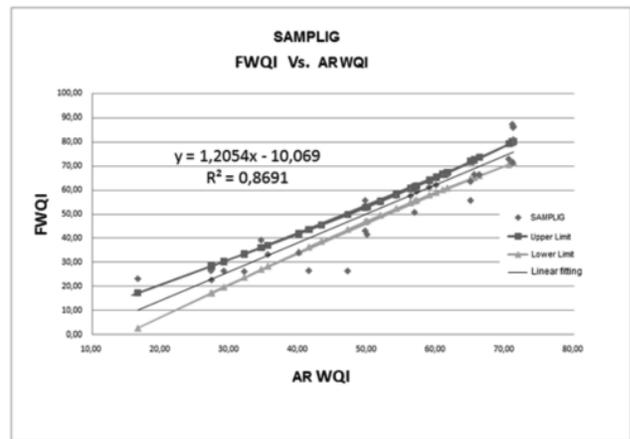
Figure 6. Linear relationship FWQI vs. ARWQI
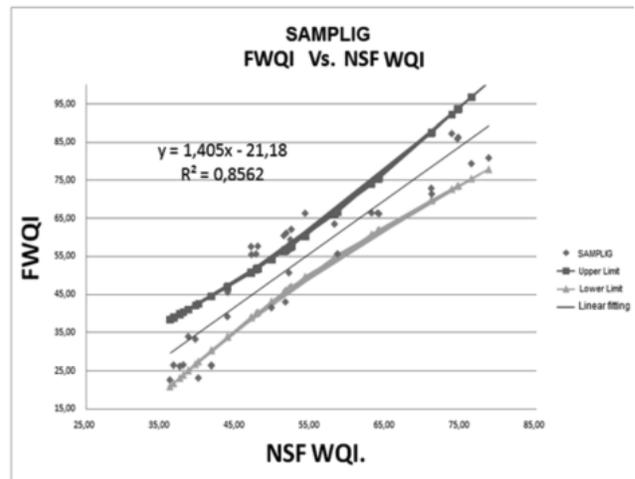
Figure 7. Linear relationship FWQI vs. NSFWQI
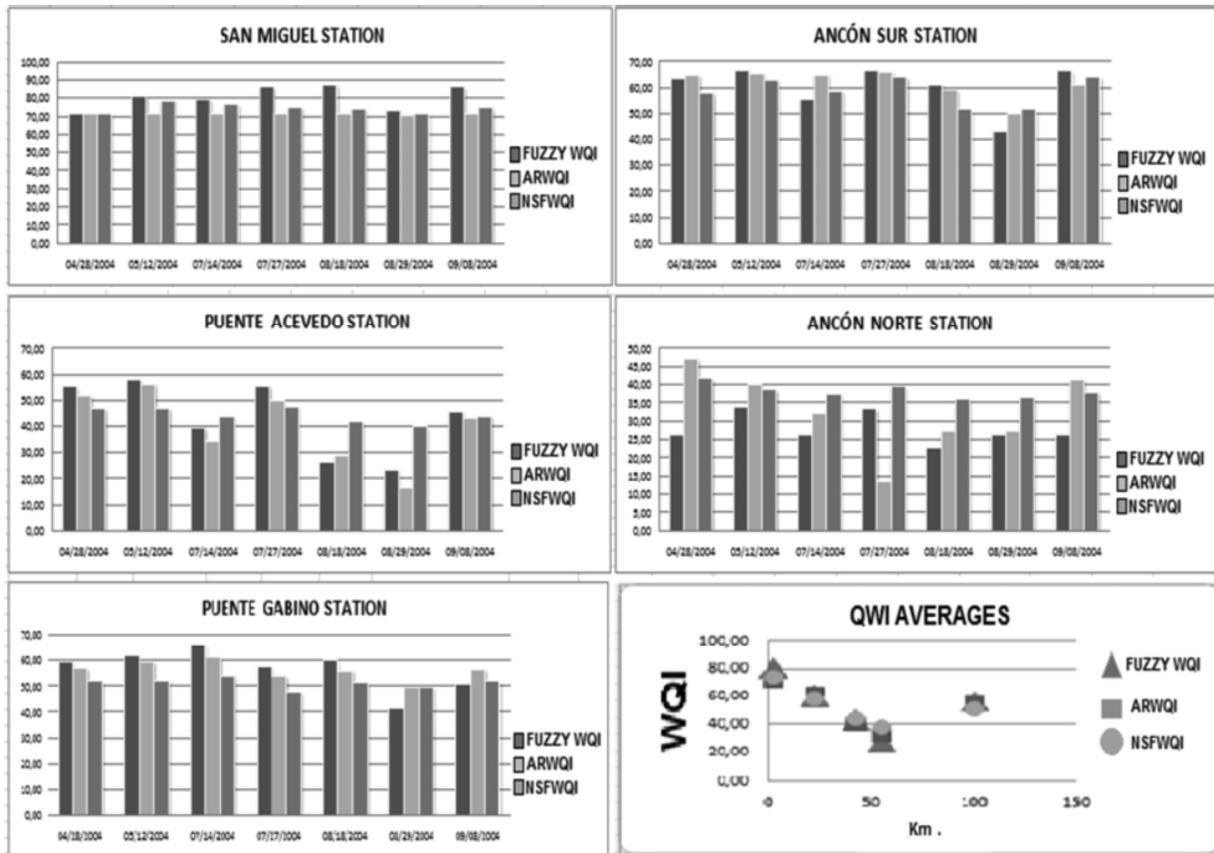
Figure 8. Bar diagrams of results and comparison among indices

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Figure 8 also shows that downstream San Miguel Station and from Ancon Sur station to Ancon Norte station , which is the river branch that covers the urban area, the indexes show a relatively high variability considering the sampling campaigns, variability associated with the rainy-dry seasons and sewerage discharges. In general, all indexes show a better behavior during the winter season (April to May and October to December), taking into account the annual hydrologic cycle. At Puente Gabino station, the station in the downstream end of the domain, the indexes show low variability and slight recovery from the Ancon Norte station, the last station of the urban area.

The average value of each index at all stations shows similar values. The Ancon Norte station shows the lowest value and clearly shows the influence of sanitary discharges and contaminated urban streams that go into to the Aburra River.

6. CONCLUSIONS

The FWQI Index combines individual physico-chemical and biological variables, with the help of expert knowledge and rules of inference, to provide information about the real status of river water quality. Additionally, this index can be implemented to monitor water bodies. Furthermore, the FWQI provides essential support to environmental authorities in order to implement corrective measures and adjustments to the river environmental assessments because this index is more demanding, which implies to be stricter. This is a valuable aspect when environmental authority wants to recover a water stream.

Specifically from this study, it was found that the river branch from Puente Acevedo station to Ancon Norte station presents the highest level of deterioration of water quality, a result which can be explained by a deficiency in the sanitary infrastructure of the zone, providing a corridor of high water pollution levels that affects the population settled on both sides of the river.

The average of the three WQIs in each sampling station show that the FWQI is the strictest, one followed by ARWQI and NSFWQI, being the last one the most tolerant. General results show similarity on the three indexes in the sampling stations Ancon Sur, Puente Acevedo and Puente Gabino; in stations as San Miguel and Ancon Norte the variability of the three indexes is very low.

Estimation of a WQI based on physical-chemical parameters and bioindicators depends on budget limitations and on the scope of the monitoring campaign. The FWQI requires more sampling and more calculations than the ARQWI and NSFWQI. If the FWQI is taken as reference index, then the objectives of a study should be more demanding, for example, not just to recover the water body, but also to recover the aquatic ecosystem.

ACKNOWLEDGEMENTS

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