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Abstract

Microteaching provides a valuable setting for preservice teachers to practice teaching prior to real classroom teaching experiences, as well as allowing teacher educators to observe the strengths and weaknesses of the preservice teachers' pedagogical content knowledge. Therefore, we studied the preservice mathematics teachers' microteachings, with an aim to investigate their pedagogical content knowledge as reflected in their microteaching. Twenty preservice secondary mathematics teachers participated in the study. The participants formed groups of two, three or four people of their choice, resulting in six groups in total. Each group designed and conducted a microteaching on a topic of their choice. The videotapes of the groups' microteaching and their lesson plans constituted the data set for the study. We analyzed the data per the components of the pedagogical content knowledge framework outlined by various researchers. We found that the preservice mathematics teachers in general were knowledgeable about different instructional strategies and the curriculum about the topic of their microteaching, but their knowledge of learners was relatively poor. Implications for teacher education are discussed.

Keywords: Pedagogical content knowledge, microteaching, lesson plan, preservice mathematics teachers

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Investigando Conocimiento Pedagógico de los Maestros en Formación a través de Microlecciones

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Resumen

El propósito de este estudio fue investigar cómo el conocimiento del contenido pedagógico de los maestros de matemáticas en formación se reflejó en sus microlecciones. Veinte maestros de matemáticas de secundaria de pregrado participaron en el estudio. Los participantes formaron grupos de dos, tres o cuatro personas de acuerdo con su elección, lo que dio como resultado seis grupos en total. Cada grupo diseñó y condujo una microlección sobre un tema de su elección. Las cintas de video de las microlecciones de los grupos y sus unidades didácticas constituyeron el conjunto de datos para el estudio. Analizamos los datos por los componentes del marco de conocimiento de contenido pedagógico esbozado por varios investigadores. Descubrimos que los profesores de matemáticas en formación en general conocían las diferentes estrategias de instrucción y el plan de estudios sobre el tema de su microlección, pero el conocimiento que tenían de sus alumnos era relativamente pobre. Se discuten implicaciones para la formación docente.

Palabras clave: Conocimiento del contenido pedagógico, microlecciones, unidades didácticas, maestros de matemáticas en formación

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The domains of knowledge that teachers need to possess have been introduced and examined in detail by Shulman (1987), outlining seven domains of knowledge that teachers need to have as follows: (a) the general pedagogical knowledge (GPK); (b) the knowledge of learners and their characteristics; (c) the knowledge of educational context; (d) the knowledge of educational ends, purposes and values, and their philosophical and historical grounds; (e) the content/subject matter knowledge; (f) the pedagogical content knowledge (PCK); and (g) the curriculum knowledge (CK). The first four domains of knowledge are common knowledge that all teachers need to have regardless of their subject (i.e., *generic knowledge)*, while the rest of three domains are specific to subject area, (i.e., *content-specific knowledge)*. Shulman describes the subject matter knowledge (SMK) as what teachers know about their subject, to what extent they know it and what they need to know, while describing the PCK as teachers' capacity to transform their SMK to forms that are accessible to students (Ball & McDiarmid, 1990; Leavit, 2008; Shulman, 1986, 1987). Additionally, Shulman (1986) defines CK as being aware of all components of the curriculum designed for teaching of a topic or a special area at a particular level, being aware of the diversity of existing instructional tools related to that curriculum, and being aware of both appropriate and inappropriate conditions for the use of a particular instructional tool in a specific situation.

With the introduction of the notion of the PCK, the belief that one who knows mathematics well teaches mathematics the best –once a widespread belief– has started to change. Ball (1988) argues that mathematics teachers should be able to distinguish the difference between knowing mathematics for oneself and teaching mathematics to someone else, indicating that knowing mathematics well is not enough for teaching mathematics. Hence, this view that mere subject knowledge is inadequate for the teaching of it has contributed to the emergence and development of the notion of PCK as an important construct in the field of education. Accordingly, supporting preservice teachers in developing their PCK is an essential goal for teacher educators.

Several definitions of the PCK have been proposed in different studies, emphasizing similar or different aspects. Shulman (1986) defines the PCK as knowing how to present a topic in a way that is accessible to others and understanding the approaches that facilitate or hinder the learning of a topic.

While Shulman (1987) originally proposed the curriculum knowledge as a distinct domain of knowledge that teachers need to have, it has been later considered as a component of the PCK in almost all of the subsequent studies (An, Kulm, & Wu, 2004; Chick, Baker, Pham, & Cheng, 2006; Grossman, 1990; Hill, Ball, & Schilling, 2008; Leavit, 2008; Magnusson, Krajcik, & Borko, 1999; Marks, 1990; Schoenfeld, 1998; Tamir, 1988). For instance, in a study introducing a framework for teacher knowledge, Tamir (1988) described the components of the PCK as follows: (a) orientation to teaching, (b) knowledge about students' understandings, (c) curriculum knowledge, (d) knowledge of assessment, and (e) knowledge of teaching strategies.

Similarly, Grossman (1990) characterized the PCK as following:

- teachers' knowledge and beliefs about the purposes for teaching a subject to students of different levels, including their conceptions regarding the nature of the subject and what topics are important for students to learn,
- knowledge of students' prior knowledge, preconceptions, possible misconceptions and alternative conceptions,
- knowledge of curriculum and curricular materials, including knowledge of the relationships within a subject as well as between subjects,
- knowledge of different instructional strategies and representations.

Fernandez-Balboa and Stiehl (1995) offered another categorization of the PCK, including four components as follows: (a) subject-matter knowledge, (b) knowledge of learners, (c) knowledge of teaching strategies, and (d) knowledge of content and goals of instruction. In short, various definitions of the PCK exist, with different researchers adopting different definitions and highlighting different components of the PCK. Thus, by building on an extensive literature review, Bukova-Güzel (2010) developed a comprehensive framework for PCK, consisting of three main categories as shown in Table 1.

While an extensive body of research in mathematics education focuses on SMK needed for teaching various mathematical topics (e.g., Ball, 1990, 1991; Ball, Thames, & Phelps, 2008), within the last decade there is a growing body of research on PCK of teachers. In many studies, however, teachers' PCK has been investigated through survey and interview methods. As the Research and Development Corporation stressed, those methods generate weak indicators for the assessment of the knowledge needed for teaching (RAND, 2003, as cited in Bütün, 2005).

Table 1
A framework for the components of PCK

Knowledge of teaching strategies and multiple representations	Knowledge of learners	Knowledge of curriculum
*Using appropriate instructional activities	* Knowing students' prior knowledge	*Being aware of the elements of the mathematics curriculum
*Using real life examples and analogies	* Knowing potential student difficulties	*Being aware of the variety of instructional tools presented in the mathematics curriculum and knowing how to use them
*Using different instructional strategies	* Knowing potential student misconceptions	*Being aware of the instruments to assess student learning and knowing how to use them
*Making use of different representations	* Knowing student differences	*Having horizontal and vertical program knowledge of a topic

Since the PCK is typically developed with teaching experience, it is important to create various opportunities for preservice teachers to practice teaching. Microteaching provides a valuable setting for preservice teachers to practice teaching prior to real classroom teaching experiences. Microteachings allow preservice teachers to share the lessons they designed with other preservice teachers and teacher educators, enabling them to get feedback, gain different perspectives and develop their PCK. Furthermore, preservice teachers' lesson plans and the instructional activities they design for their lessons provide a fruitful context for studying their PCK, allowing teacher educators to observe the strengths and weaknesses of the preservice teachers' PCK. Although we acknowledge that implementing their lessons in real classrooms would be even more beneficial, microteaching offers a valuable alternative when the opportunities for classroom implementations are limited. Therefore, we studied the preservice mathematics teachers' microteachings, with an aim to investigate their PCK as reflected in their microteaching. In particular, we examined to what extent the preservice mathematics teachers exhibited evidence of their PCK in the context of microteaching.

Methods

We conducted a case study to do an in-depth analysis of how the preservice teachers' PCK was reflected in their microteaching.

Microteaching

Microteaching provides preservice teachers with initial experience and practice in teaching, allows teacher educators to assess preservice teachers' competencies and weaknesses in teaching, and supports in-service teachers' professional development (Allen, 1967). By reducing the complexities of regular classroom teaching in terms of the number of students, time, and management, microteaching offers a controlled setting for practicing teaching, and thus, is considered as a useful method for training preservice teachers (Allen & Clark, 1967; Kpanja, 2001). By focusing on certain aspects of instruction or strategies, preservice teachers practice teaching a lesson or a particular topic and then receive immediate feedback from supervisors and students (Allen & Clark, 1967).

Participants

The participants of the study were 20 (ten female, ten male) preservice mathematics teachers who were in their final year of a five-year secondary mathematics teacher education program in a large state university in Turkey. During the first three-and-half years of this program preservice teachers take an extensive array of content courses such as Calculus, Analytic Geometry, Abstract Algebra, Discrete Mathematics, Differential Equations, Differential

Geometry, Complex Analysis, and Topology. In addition, they take other courses (such as Mathematical Problem Solving; History of Mathematics; Mathematical Modeling; Mathematics and Art; Mathematics and Games) that can further support preservice teachers' content knowledge. During the last one-and-half years, preservice teachers take courses aimed at developing their GPK (such as Introduction to Educational Sciences; Classroom Management; Guidance; Curriculum Development; Assessment and Evaluation) and PCK (such as Teaching Methods in Mathematics, Alternative Assessment and Evaluation Methods in Mathematics Education, Examination of Mathematics Textbook). Additionally, during the last three semesters preservice teachers have field experiences; they first take School Practicum for two semesters in which they mainly observe mathematics classes, and during the last semester they practice teaching in classrooms (Student Teaching). At the time of this study, the participant preservice teachers were taking School Practicum-II, and thus have not yet had teaching experience in classrooms.

Procedures

The study was conducted as part of the School Practicum-II course. The course consisted of two components: (a) field experience at schools whereby the preservice teachers observed mathematics classes for four hours per week and (b) seminar at the university whereby they prepared for and discussed their field experiences (one hour per week). During the first month of the seminars, we informed the preservice teachers about developing a lesson plan and designing instructional activities, by offering and discussing various examples. The preservice teachers were then asked to develop their own lesson plans and instructional activities for a mathematics topic of their choice in the context of a collaborative microteaching with their peers. They were asked to form groups of three or four people of their choice to collaborate in microteaching. Six groups were formed in total, resulting in three groups of 4-, two groups of 3-, and one group of 2-preservice teachers. Each group was considered as a case.

The groups submitted their lesson plan for two class-periods on the selected topic prior to implementing them through microteaching. The microteachings occurred over two consecutive days during the last month of the seminars. During the microteachings the preservice teachers conducting the microteaching acted as teachers, while the remaining preservice teachers acted as students at the particular grade level relevant to the mathematics topic being taught. Thus, when referring to the preservice teachers acting as students, we italicize *students* to clarify that we do not refer to (high school) students in general. Each microteaching was followed up by a discussion in which the preservice teachers and the faculty supervisors (the first and third authors of the paper) provided feedback and suggestions. The microteachings were videotaped, field notes were taken, and lesson plans and other instructional materials were collected.

Data Collection

The participants developed lesson plans for a topic of their choice from the national mathematics curriculum (The Ministry of National Education [MoNE], 2006) and designed instructional activities to be used in their microteaching. Table 2 presents the topics and the corresponding objectives that each group chose from the national curriculum for their microteaching. The lesson plans and the videotapes of the microteachings constituted the data corpus for the study.

Table 2
The topics and objectives of the microteachings

Cases	Topics	Objectives*
Case A	Periodic Functions	Explains period and periodic functions and finds the period of trigonometric functions.
Case B	Inverse Functions	Understands the concept of inverse function, constructs the rules about inverse functions, and applies knowledge about inverse functions.
Case C	Odd-Even Functions	Explains even and odd functions and interprets their graphs.
Case D	Operations	Explains binary operations and the properties of binary operations.
Case E	One-to-One, Onto, Constant, Identity, and Linear Functions	Explains the concepts of one-to-one, onto, constant, identity, and linear functions.

Table 2
The topics and objectives of the microteachings (…/…)

Cases	Topics	Objectives*
Case F	Infinite Integral	Explains the Riemann sum and definite integral by means of the area under a curve. Describes the difference between integral and derivative, and between definite and indefinite integral. Explains indefinite integral of a function.

* As stated in the national high school mathematics curriculum

Data Analysis

We first developed descriptive summaries of each microteaching as we passed through the videotapes of the microteachings. These summaries included descriptions of how the instruction began, how it proceeded, and the participants' instructional practices and goals. Subsequently, we coded each microteaching (including the videotapes, lesson plans, and the instructional materials) per the PCK framework (as seen in Table 1) individually, and then compared our coding and discussed any differences. Final conclusions were made in consensus. In particular, we analyzed each case's microteaching per the components of the PCK framework based on the following measures: (0) if the group did not demonstrate an appropriate approach or did not attempt to demonstrate any approach, (1) if the group demonstrated an appropriate approach to some extent, and (2) if the group demonstrated a fully appropriate approach. For example, consider *Using Real Life Examples and Analogies*, the sub-component of the Teaching Strategies and Multiple Representations component of the PCK. If a group did not offer any real-life example of the related mathematical concept or did not attempt to make any connection to real life, we coded it as (0); if the group provided a real-life example or analogy pertinent to the mathematics concepts, but contained a mathematical inaccuracy or shortcoming, which may potentially lead to misconceptions (such as presenting the mom-child relation as an example of function), we coded it as (1). Lastly, we coded (2) if the group offered a mathematically accurate and appropriate real-life example or analogy for the relevant mathematics concept.

Results

We present the results for each case respectively. We begin by briefly summarizing the participants' microteachings, with images taken from the video recordings of their microteaching. We then provide a discussion of each case according to the PCK framework.

The microteachings

Case A

For their microteaching, the preservice teachers in Case A developed a lesson plan for teaching period, periodic function, and finding periods of trigonometric functions. They identified the key elements of their lesson plan as follows: (a) showing a video clip to motivate students to the lesson, (b) bringing materials that support student learning and designing new materials related to the topic, (c) doing individual applications to ensure that each student actively participates, and (d) using student responses to the questions in the activity sheets as a formative assessment. Hence, they began their microteaching with a video clip of a TV commercial, which they considered it to be reminiscent of the concept of period encountered in everyday life.

The commercial in question was designed to draw attention to the daily recurring routines by showing errands made in certain times of the day. The group then shifted to examining the movement of a model train on the rail and continued the activity by focusing on the fact that the train passes by the same spots on its whole rotations, which happens at certain intervals. The participants marked those spots on the rail with a green paper strip to draw attention that the train passes on those spots on its full rotations at certain periods (see Figure 1). Next, they folded papers and cut them to create a particular figure. Then, they unfolded the papers to show that the created figure repeats along the paper. In addition, they designed activity sheets with questions related to the concept of period in everyday situations such as using antibiotic every 12 hours, the location of service areas on highways being a constant distance apart from each other, etc. With such questions, the preservice teachers aimed at revealing students' thoughts about the concept of period. Through these activities, the group aimed at supporting students' understanding of the concept of period, by drawing on everyday situations for which students can make sense of and make inferences about the phenomena. Another goal of the group was to help students understand the periods of trigonometric functions. To achieve this goal, they designed materials with paperboards, glue pads, construction papers and weather strips. First, the group had *the students* construct the graphs of functions such as $sinx$, $cosx$, $sin2x$ and so on, by using weather strips. Next, the class discussed at what interval each function was repeating and then determined the period of each function from its graph. Similarly, the class discussed and determined the period of $tanx$ and $cotx$. Below we present some images captured from the video recording of Case A's microteaching.

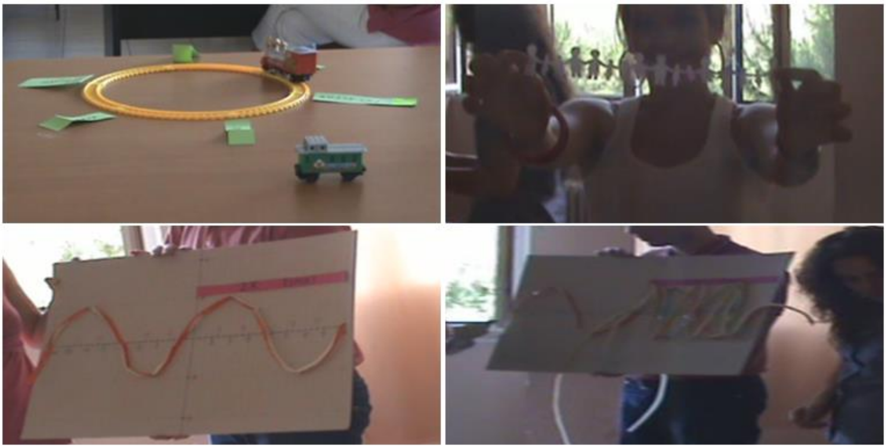

Figure 1. Images from Case A's microteaching

Case B

The preservice teachers in Case B chose to design a microteaching about the concept of inverse function, the rules for finding the inverse of some functions, and some applications of inverse functions. They indicated that they would follow a rather traditional approach to teaching; mainly a teacher-centered approach with some teacher questioning involved. Throughout their microteaching they relied on the PowerPoint presentation they had created.

They began with reminding the *students* the definition of function, and then followed with an example of a function as a relation between two sets.

They identified two sets to construct the inverse function and discussed the notion of inverse function only via set representation of function. However, they did not attempt to relate the notion of inverse function to everyday life situations. Instead, they relied on one activity (see Fig. 2) throughout their microteaching, in which they worked with the function of $f(x) = ax$. When shifting to its inverse, they focused on the operations: multiplying by a and its inverse operation– dividing by a for the inserve function. Similarly, they demonstrated how to find the inverse of the following functions: $f(x) = x \pm a$ and $f(x) = ax \pm b$. Following this sequence, the participant group explained the inverse of the functions.

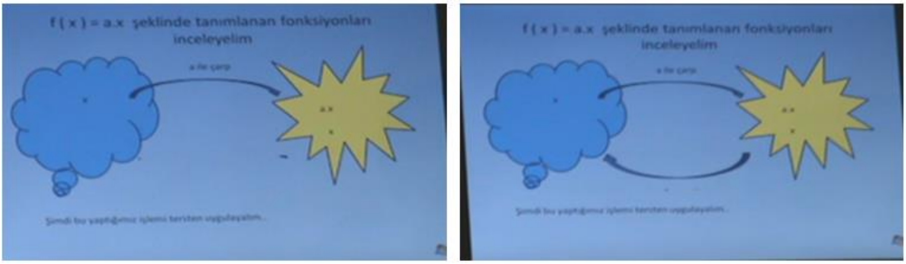

Figure 2. Images from Case B's microteaching

Case C

The preservice teachers in Case C designed a lesson plan for teaching even and odd functions and interpreting the graphs of those functions. They identified the following points critical for their microteaching: (a) attending to students' prior knowledge about function, and domain and range of functions, (b) constructing graph of a function, (c) reminding and applying the notion of symmetry, (d) constructing algebraic representations of even and odd functions, (e) providing examples of even and odd functions from real life situations to motivate students, and (f) asking thought-provoking questions to students. Accordingly, the group designed an activity involving a problem about printing five different documents that were saved on a desktop in two forms: one as a white-and-black copy and the other as a color copy. The preservice teachers made connections between even-odd functions and the condition whether the color printer had sufficient cartridge. In other words, the group suggested that if the color printer had sufficient color cartridges, then the document would be printed in color, but if the printer run out of color cartridges, then the document would be printed in black-and-white (B&W). The preservice teachers discussed the inputs, operation, and the outputs, and then represented them algebraically as well as drawing the corresponding graphs. Thus, through this activity the preservice teachers attempted to construct and define the notion of even and odd functions.

Additionally, with those graphs they aimed to show students that odd functions are symmetric to the origin and even functions are symmetric to the y-axes. The problem given in the activity for the condition that the printer run out of color cartridges is provided below. A similar scenario was given for the condition that the printer had sufficient cartridge. Figure 3 presents images taken from the Case C's lesson plan, with translations into English.

There are five different documents saved on a computer desktop in two forms: one in black-and white and another copy in color. When attempted to print those documents, we realized that the color cartridges were blank. Given that there is no ink in color cartidges, determine how each document would be printed if those ten different documents are to be printed?

A′, B′, C′, D′, E′ → Documents in B&W **B&W Document** = $\{A', B', C', D', E'\}$
$f(A'), f(B'), f(C'), f(D'), f(E')$ → B&W prints
B&W Print = $\{f(A'), f(B'), f(C'), f(D'), f(E')\}$

A, B, C, D, E → Documents in color **Color Print** = $\{A, B, C, D, E\}$
$f(A), f(B), f(C), f(D), f(E)$ → Color Prints
Color Print = $\{f(A), f(B), f(C), f(D), f(E)\}$

$f(A) = f(A')$ $f(B) = f(B')$ $f(C) = f(C')$ $f(D) = f(D')$ $f(E) = f(E')$

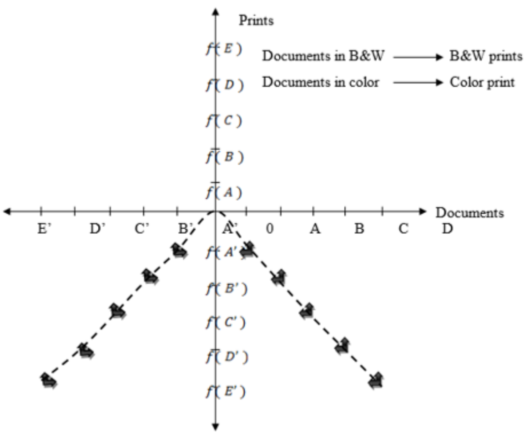

Figure 3. Images from Case C's lesson plan

Case D

The preservice teachers in Case D designed a lesson plan for teaching binary operations and the properties of binary operations. Their lesson plan included sections on drawing attention of students, motivating students, reviewing prerequisite knowledge, launching the activity related to the topic, and assessment and evaluation. They identified the following points critical for their microteaching: (a) building on students' prior knowledge, (b) making connections to real life situations, (c) encouraging students to reason and discuss about the concepts, (d) using multiple representations, (e) constructing the notion of operation, and (f) making connections between concepts. For their microteaching the preservice teachers came up with a scenario about a food chain, consisting of mice, snakes, and eagles. The scenario specified that eagles eat mice and snakes, and snakes eat mice. Given those rules, the *students* were asked to find which animal would have survived if the animals were to put into a cage in pairs (e.g., a mouse and a snake).

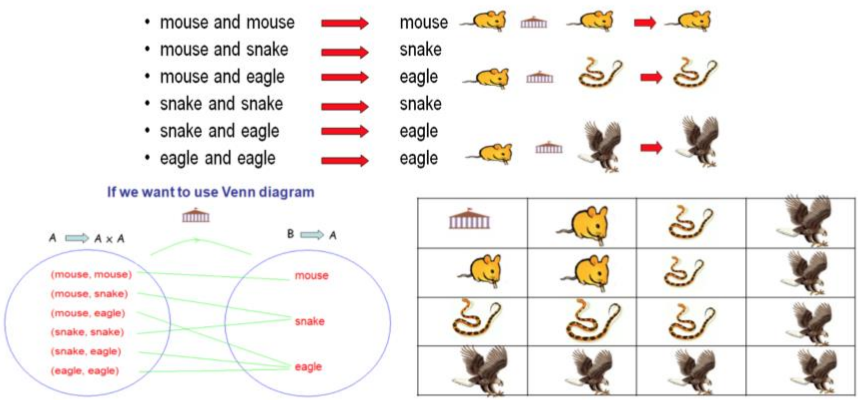

Figure 4. Images from Case D's microteaching

Through this scenario the preservice teachers attempted to construct the critical aspects of the notion of binary operation. They highlighted that binary operation is a special function and focused on the relation between its domain and range. They also discussed the properties of operations, by examining the closure property, associative property, commutative property, identity element, and inverse element. During discussion the preservice teachers reviewed the *students*' prior knowledge about binary operations with number sets such as natural numbers and integers and discussed which properties those operations satisfy. In these activities, the preservice teachers aimed to construct the notion of operation by means of examples from everyday life and multiple representations (e.g. operation table, set representation, and algebraic representation). Figure 4 presents some images showcasing different representations illustrated during the Case D's microteaching.

Case E

The preservice teachers in Case E designed a lesson plan for teaching the concepts of one-to-one, onto, constant, identity, and linear functions. They identified the following points critical for their teaching: (a) making connections to everyday life, (b) presenting video clips that illustrate the function types in question, (c) using mathematical software, and (d) asking questions via worksheet. The preservice teachers began their microteaching by presenting selected video clips from a movie, titled *Beautiful Mind,* to make connections to everyday life. They then discussed the parts of the clips that were associated with one-to-one, onto, constant, identity, and linear functions.

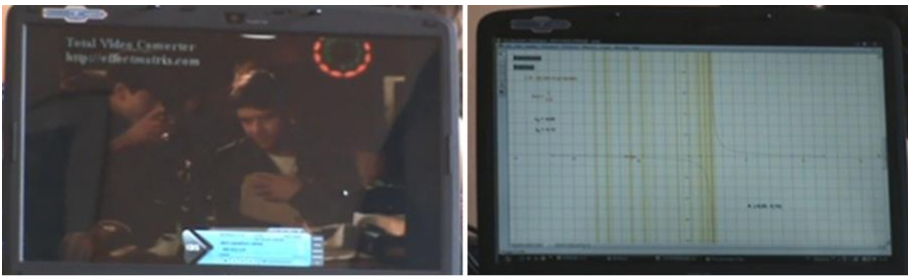

Figure 5. Images from Case E's microteaching

Through guiding questions, the group tried to help the *students* discern the rules of the functions in question. By means of these rules they identified the domain and range of the functions and drew the mappings between the domain and the range for each function. They continued their microteaching with further discussions on the graphs created with mathematical software. Figure 5 presents two images captured from the videotapes of Case E's microteaching.

Case F

The preservice teachers in Case F designed a lesson plan for teaching the Reimann sum, derivative, and definite and indefinite integral. They identified the key elements for their microteaching as following: (a) drawing students' interests to motivate them for the lesson, (b) reviewing students' prior knowledge, (c) illustrating the mathematical ideas with real life examples, (d) informing students of the historical development of the mathematical concepts, (e) using mathematics software (e.g., Geometer's sketchpad, Geogebra) to illustrate the concepts, (f) using concept cartoons to create a discussion and to add fun to the topic, and (g) making connection between the area under a curve and the area of regular polygons. Hence, the preservice teachers began their microteaching with describing the historical development of the concept of integral. Specifically, they highlighted the particular needs that led to the emergence of the concept of integral, its historical development, and how it relates to the concept of derivative. They then tried to illustrate the infinitesimal calculus for finding the area under a curve by dividing the area under curve into various rectangles and observing the behavior of the sum of the area of the rectangles as the number of rectangles approaches to infinity. The group used a graphic analysis program to illustrate how increasing the number of rectangles affects the sum of the area of the rectangles. The preservice teachers continued with a discussion on finding the area of a circular region as the sum of the area of regular polygons that were inscribed in the circle, and then shared a concept cartoon about finding the area of a field whose shape is not a regular polygon to spark further discussion. Figure 6 below presents some images captured from Case F's microteaching, including the concept cartoon (translated into English).

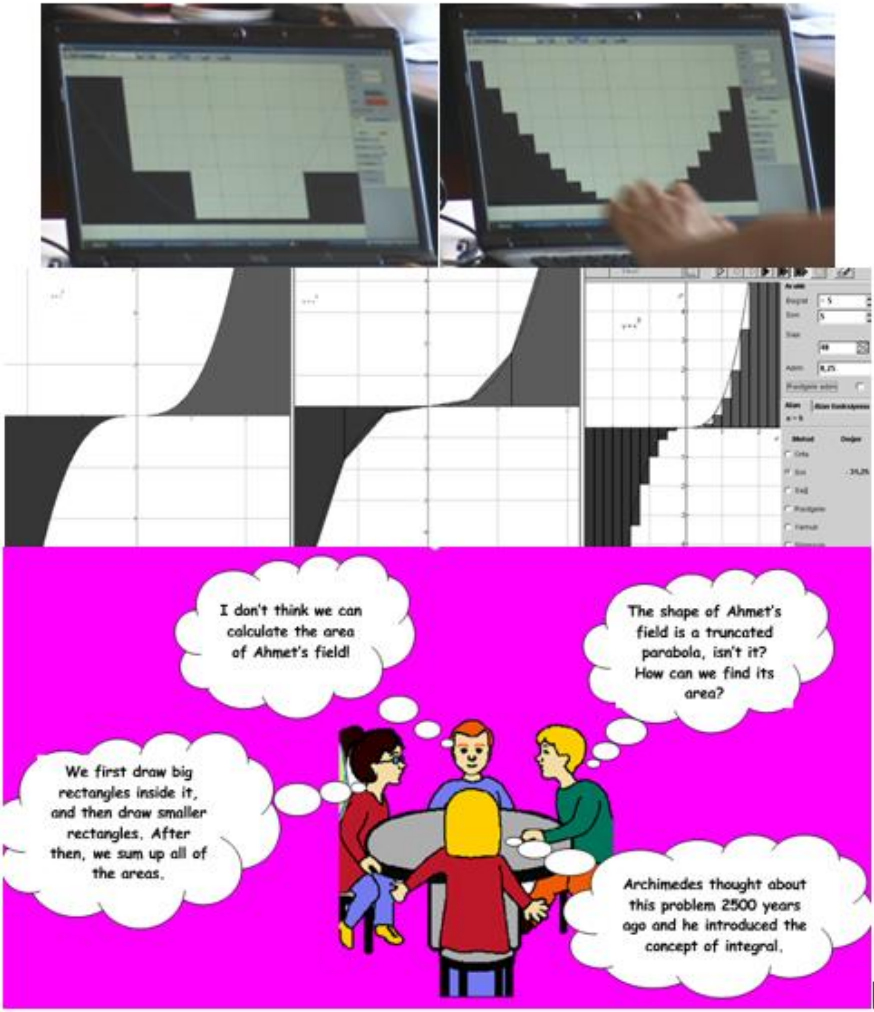

Figure 6. Images from Case F's microteaching

Evaluation of the Participants' PCK Through Their Microteaching

We present the findings regarding each component of the PCK framework (knowledge of teaching strategies and multiple representations; knowledge of learner; and knowledge of curriculum). Table 3 presents the evaluation of each group's microteaching in terms of the participants' knowledge of teaching strategies and multiple representations. To reiterate, (0) indicates that the group did not demonstrate an appropriate approach or did not attempt to demonstrate any approach, (1) indicates that the group exhibited a somewhat appropriate approach, while (2) indicates that the group demonstrated a fully appropriate approach.

Table 3
Knowledge of teaching strategies and multiple representations

Knowledge of teaching strategies and multiple representations	Cases					
	A	B	C	D	E	F
Using appropriate instructional activities	2	1	2	2	2	2
Using real life examples and analogies	2	0	2	2	2	2
Using different instructional strategies	2	1	2	2	2	2
Making use of different representations	2	1	2	2	2	2

Overall, the preservice teachers demonstrated *knowledge of teaching strategies and multiple representations* for the mathematics topics they chose for their microteaching. Five groups made use of appropriate activities for presenting the concepts in question, and two groups tried to construct the concept through one activity. Except Case B, all groups provided appropriate real-life examples and analogies in their microteaching. For instance, Case A tried to exemplify the concept of period via a video clip of a TV commercial in which daily recurring themes displayed, the movement of a model train on the rail, and the folding-and-cutting paper activity. All groups, but Case B, designed their lesson plans based on a constructivist approach to learning and tried to implement their microteaching with that perspective in mind. Also, the groups made use of different teaching strategies in their microteachings.

Three groups incorporated instructional technology into their microteaching as suggested by the national secondary mathematics curriculum (MoNE, 2006). In general, the groups also made use of different representations of the concepts they aimed to teach; the preservice teachers drew on various forms of representations, including algebraic, graphic, verbal, table, and Venn diagram. For instance, Case F made use of graphic and verbal representations, while Case B used Venn diagram, algebraic and verbal representations.

Table 4 displays the evaluation of each group's microteaching in terms of knowledge of learners. Our analysis revealed that the preservice teachers did not demonstrate adequate *knowledge of learners* in the domain of mathematics topics on which they conducted their microteachings. However, all groups attempted to make connections to students' prior knowledge around the topics they focused on, and five groups could make fully appropriate connections. For example, Case E reminded the *students* of the properties of functions through a series of guiding questions, identified the domain and range of the functions and drew the mappings between two sets (domain and range) for each function to help students understand one-to-one, onto, constant, identity, and linear functions. However, the preservice teachers appeared not to have sufficient knowledge of possible student difficulties about the concepts that they designed lessons for. In general, they tried to make connections to real life situations and students' prior knowledge, and their microteaching usually followed a progression of ideas from easy to more complex ideas. By doing so, they might have aimed at overcoming possible student difficulties. However, the groups, except Case F, fell short in addressing possible student difficulties in their microteaching.

The preservice teachers in Case F believed that students would have difficulties in understanding the concept of integral if taught via traditional approaches in which computational meaning of integral is stressed. Thus, they attempted to prevent potential student difficulties by emphasizing the conceptual meaning of integral by introducing the notion of integral via infinitesimal calculus.

Table 4
Knowledge of learners

Knowledge of learners	Cases					
	A	B	C	D	E	F
Knowing students' prior knowledge	2	1	2	2	2	2
Knowing potential student difficulties	1	0	1	1	1	2
Knowing potential student misconceptions	0	0	1	1	0	1

Knowing student differences	1	0	1	2	1	2

Additionally, it appeared that the preservice teachers lacked a thorough understanding of possible student misconceptions about the concepts they taught. They did not pay much attention to possible student misconceptions in their microteaching; only three groups partially addressed the possible misconceptions about the concepts in question. For instance, Case C drew on the symmetry of the graphs of even and odd functions in order to help students achieve the learning goal of explaining even and odd functions and interpreting their graphs; but, they did not emphasize that a function does not need to be either odd or even function. On the other hand, Case F drew on graphic, algebraic, verbal, and table representations along with mathematics software to construct the concept of integral. Instead of introducing the notion of integral as a process of finding the function whose derivative is given, Case F foregrounded the geometric meaning of integral and related the concept of integral to the area under a curve, and thus attempted to prevent students from developing a possible concept image of integral as an algebraic computation. Hence, they provided students an opportunity to develop a conceptual understanding of integral, in addition to the procedural knowledge of integral. Lastly, the preservice teachers did not seem to have ample knowledge of student differences. For instance, Case B did not make use of multiple representations, neither did they make connections to real life or provide examples that might draw students' attention. Instead, they followed a traditional instruction approach, disregarding student differences and needs.

Table 5 presents the evaluation of each group's microteaching in terms of the knowledge of curriculum. Overall, the preservice teachers appeared to possess necessary *knowledge of curriculum*. Almost all groups demonstrated knowledge of the elements of the mathematics curriculum in their microteaching. While Case A drew on real life examples and multiple representations, for example, Case F drew on computer-assisted instruction coupled with real life examples. All but one group conducted their microteaching by considering the various elements of the national mathematics curriculum and made use of appropriate instructional tools for the learning goals they set. In general, the groups seemed to be aware of various instruments for assessing student learning and appeared to know how to use them. Generally, they assessed the *students'* understanding through probing questions. But Case B did not include any of the evaluation and assessment approaches that are recommended by the national mathematics curriculum. For instance, Case B did not take time for assessing the *students'* understanding during their microteaching; instead they used their entire time for lecturing. The other groups, on the other hand, assessed the *students'* understanding through follow-up questions and worksheets. Additionally, Case F used concept cartoons as another tool for assessing student understanding.

Table 5
Knowledge of curriculum

Knowledge of curriculum	Cases					
	A	B	C	D	E	F
Being aware of the elements of the mathematics curriculum	2	0	2	2	2	2
Being aware of the variety of instructional tools presented in the mathematics curriculum and knowing how to use them	2	0	2	2	2	2
Being aware of the instruments to assess student learning and knowing how to use them	2	0	2	2	2	2
Having horizontal and vertical program knowledge of a topic	1	1	2	2	1	2

Overall, the preservice teachers seemed to have horizontal curriculum knowledge. In their microteachings, the preservice teachers often chose to make connections only to the pertinent prior knowledge rather than making connections across all mathematics topics covered at that grade level. For example, Case B made connections to sets when teaching the inverse function, and similarly Case E made connections to sets when teaching one-on-one, onto, constant, identity and linear functions. Furthermore, only three groups appeared to have vertical curriculum knowledge. For instance, when introducing the binary operations, Case D made connections not only to the concept of function but also to the operations with real numbers and their properties, which has potential to help students build on their mathematics knowledge they developed in the previous grade levels. Similarly, Case F made connections among the concept of integral and the concept of limit, continuity, and the area of polygons.

Discussion and Conclusion

We analyzed the preservice mathematics teachers' PCK as reflected in their microteaching and found that the participants in general were knowledgeable about different instructional strategies and curriculum about the topic of their microteaching, but their knowledge of learners was relatively poor. While some groups drew on multiple activities for introducing mathematical concepts, some groups relied on one instructional activity in their microteaching. Although the activity may be appropriate for teaching the targeted concept, relying on only one activity still runs the risk of being accessible to all students. All but one group included real life examples and analogies for the concepts they targeted in their microteaching. Thus, they offered the *students* some ideas about the applications of the mathematical concepts in real life. The groups also made use of computer-assisted instructional tools as recommended by the national mathematics curriculum.

Additionally, in their microteaching the preservice teachers drew on various instructional strategies such as using multiple representations, making connections to real life situations, highlighting the essential concepts related to the topic in question, and focusing on the issues that students may have difficulty with. As many researchers (e.g. Ball, 1990; Chang, 2005; Grossman, 1990; Marks; 1990; Shulman; 1987) point out, identifying the most appropriate instructional strategies is critical for effective teaching. Instructional strategies are also important for helping students develop conceptual knowledge and overcome their misconceptions (Elia, Gagatsis, Panaoura, Zachariades, & Zoulinaki, 2009). Thus, identifying appropriate instructional strategies and informing preservice teachers about those strategies are essential. The groups that included multiple representations in their microteachings made use of various forms (e.g., algebraic, graphic, verbal, table, set), and thus aimed to provide students with different representations of the concepts. Moru (2006) contends that learners cannot develop adequate conceptual knowledge if they are not presented with multiple representations of concepts. Likewise, NCTM (2000) advocate the use of multiple representations for more sophisticated mathematical thinking and learning.

The groups also attempted to incorporate various elements into their microteaching that are advocated by the national curriculum such as the use of real-life examples and a variety of instructional materials and instructional technologies. Some groups made use of appropriate instructional tools for the learning goals for which they designed their microteaching. While one group did not attempt to assess student understanding during their microteaching, the other groups assessed student understanding through probing questions and worksheets. Moreover, one group also made use of concept cartoons for assessment purpose. Furthermore, groups overall seemed to have horizontal curriculum knowledge, but only three groups showed evidence of vertical curriculum knowledge. For a robust learning of new concepts, however, it is equally important to be able to make connections to the prior knowledge attained in previous grades as well as to the prerequisite knowledge and concepts learned within the same grade level.

However, the preservice teachers demonstrated relatively poor *knowledge of learners* in the context of their microteachings and lesson plans. Overall, the groups attended to students' prior knowledge in their lesson plans, but mostly failed to take into account potential student difficulties, misconceptions, and differences. The groups that attempted to teach new concepts by making connections to students' prior knowledge often began their instruction with easier concepts and gradually moved to more advanced concepts. On the other hand, the preservice teachers in general seemed not to have adequate knowledge about possible student difficulties with and misconceptions about the concepts in question, as well as student differences. Yet, knowledge about learners constitutes an important knowledge domain that teachers need to possess. As Ball and her colleagues (2001) argue, teachers need to know the common misconceptions students hold or might hold in a specific subject. Hence, in addition to other domains of teacher knowledge, preservice teachers should be knowledgeable about possible student misconceptions and difficulties and draw on their knowledge of learners when designing their lessons.

A noteworthy finding of the study is the fact that the preservice teachers fared relatively poor in terms of knowledge of learners. We maintain that there might be two plausible explanations for this outcome. First, although the preservice teachers had some exposure to the common student misconceptions and students' prior knowledge during their coursework in the teacher education program, they had a limited school practicum experience and no teaching experience yet. Therefore, the preservice teachers' limited experience in student learning might have hindered their ability to take into account the potential student difficulties, misconceptions, and differences in their lesson plans and microteachings. Second, the preservice teachers might be unable to properly reflect their knowledge of learners during the microteachings as the other preservice teachers acted as high school students and, thus, might not have reacted to the instruction as high school students would normally do, preventing to observe a wide range of potential student difficulties or misconceptions that could arise in a real classroom setting.

In conclusion, although microteachings cannot substitute real teaching situations, they nevertheless provide helpful approximations for preservice teachers to practice teaching and for teacher educators to observe the preservice teachers' instructional practices in general and their PCK in particular. Specifically, we emphasize reflective discussions with preservice teachers following their microteaching as a critical feature that has great potential to support preservice teachers in becoming aware of the areas that they need to improve. Those discussions should begin with other preservice teachers' reflections on the observed microteaching and their suggestions for how to improve the lesson and the instruction, which should then be followed by the faculty supervisors' more extensive and detailed feedback. The focus of those reflections may include the mathematical content of the lesson (i.e., any flaw or inaccurate use of mathematical language), appropriateness of the activities, tasks, or materials for developing the targeted mathematical understanding, what the preservice teachers did – or did not – do in response to a student contribution, or any unclear or confusing situations that arose during the microteaching. Such reflective discussions are also likely to assist teacher educators in identifying the components of PCK that preservice teachers commonly have difficulty with, hence providing teacher educators a means to assess and revise their respective teacher education programs accordingly.

Finally, as we pointed out, conducting microteaching with preservice teachers acting as students has certain limitations in terms of eliciting and responding to student thinking, but we believe that its practical benefits outweigh the limitations. Therefore, we consider it to be an effective and helpful method, especially as an initial step for preservice teachers to get ready for and practice teaching. However, we also suggest these microteachings to be gradually advanced to include real students, instead of preservice teachers acting as students at a particular grade level, which should then be scaled up to teaching in real classrooms. Yet, considering that the preservice teachers have relatively poor knowledge of learners we recommend creating more opportunities for preservice teachers to explore student thinking. Having preservice teachers to conduct clinical interviews with students might be an effective method that can allow preservice teachers to investigate and analyze students' thinking in a particular mathematical concept.

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