Foreword:
Consistency, Contradiction, and Consequence

Abílio Rodrigues

Departamento de Filosofia, Universidade Federal de Minas Gerais, Brazil
abilio.rodrigues@gmail.com

Federico Matías Pailos

CONICET, University of Buenos Aires, Argentina
fpailos@hotmail.com

The classical notions of consistency and logical consequence are different sides of the same coin. From the point of view of classical logic, a set of propositions Γ is said to be consistent when it does not imply a pair of contradictory propositions A and $\neg A$, or equivalently when there is at least one proposition A that is not implied by Γ. The second condition is the same as saying that Γ is not trivial. This is because the principle of explosion, according to which anything follows from a contradiction, formally $A, \neg A \vDash B$ for any A and B, holds in classical logic. So, it is easy to see that if a set of propositions Γ implies a contradiction, then Γ implies any proposition whatsoever; and conversely, if Γ implies any proposition, obviously Γ implies a pair of contradictory propositions. Classical logic, thus, collapses the concepts of triviality and contradictoriness.

The classical notion of consistency appears even in the pre-theoretical notion of logical consequence: a conclusion A follows from a set of premises Γ if and only if it is impossible that all the propositions in Γ are true but A false. This impossibility is formally expressed by saying that the set formed with all premises and the negation of the conclusion, i.e. $\Gamma \cup \{\neg A\}$, is inconsistent. If A does follow from Γ, then the set $\Gamma \cup \{\neg A\}$ has no models, precisely because it has among its consequences a pair of contradictory propositions, and an essential feature of classical negation is that two contradictory propositions cannot be simultaneously true. The standard notion of consistency is so much connected with the standard notion of consequence that saying that logic is concerned with 'what follows from what' is exactly the same as saying that logic is concerned with consistent sets of propositions — and this holds, of course, for classical logic.

If the validity of $\Gamma \vDash A$ depends on being impossible that all premises in Γ are true but A false, then if it is impossible that all premises in Γ are true, it follows that it is impossible that all premises in Γ are true *and A* is false — the falsity of one conjunct

is enough for the falsity of the whole conjunction. Thus, the principle of explosion is founded on the most basic idea of classical logic, namely, that preservation of truth is not only a necessary condition but also a sufficient condition for logical consequence. Truth-preservation and the corresponding concepts of contradiction, consistency and consequence form a unity upon which classical logic has been built.

So far so good, but paraconsistent logics reject the principle of explosion. What now? What happens with these classical concepts when explosion no longer holds?

A paraconsistent negation accepts pairs of propositions A and $\neg A$ such that both hold — which does not need to mean that both are true. So, a set of propositions can be contradictory without being trivial. The classical concept of consistency, thus, has been splitted into two non-equivalent concepts: contradictoriness and triviality. And clearly the identification between logical consequence and consistency does not hold anymore, since it is not the case now that A follows from Γ only if the set $\Gamma \cup \{\neg A\}$ has no models. From the semantical point of view, rejecting explosion means that there exists a model M and two propositions A and B such that both A and $\neg A$ hold in M but B does not hold in M. If we keep the idea that logic is concerned with truth-preservation, it seems that there is no escape but to endorse the controversial dialetheist thesis that true contradictions do exist. Since contradictions have always been a signal that something went wrong in Mathematics, empirical sciences and Philosophy, the concept of truth that has been guiding these inquiries has to be discarded. (Notice that Heraclitean and Hegelian contradictions are not contradictions in the strict sense of an object a and a predicate P such that a satisfies and does not satisfy P *at the same time and in the same respect*; so, they are not really counterexamples to explosion and non-contradiction).

Whether or not truth-preservation as the central concept of logical consequence has to be maintained, and how to deal with the notions of consistency, consequence and contradiction from the point of view of paraconsistent logics are question about which there is no final agreement. Attempts are made, as usual in Philosophy, to conciliate positions at first sight irreconcilable, to introduce subtle changes in concepts in order to make them suitable to each other. It is fair to say that the disagreement is the rule rather than the exception. But it is precisely this kind of exciting and fruitful disagreement that is the motivation and the content of the discussions that originated this volume.

Papers

1. Henrique Antunes: On Existence, Inconsistency and Indispensability

In this paper, the author discusses some lines of response to Mark Colyvan's indispensability arguments for the existence of inconsistent objects, being mainly concerned

with the indispensability of inconsistent mathematics. By drawing on some of Jody Azzouni's views in the philosophy of mathematics he attempts to block Colyvan's arguments, specially when inconsistent mathematical objects are concerned. Moreover, the author presents a logical framework based on the logic *LFI1* which makes use of an existence predicate in order to formally express the metaphysical claim that no existent object is allowed to have contradictory properties.

2. Guilherme Araújo Cardoso: Situations, Liar Paradoxes and the Revenge Problem

This paper outlines a critical introduction to Situation Theory as an approach to the liar paradoxes. The author shows how Situation Theory is capable of solving the semantic paradoxes while blocking revenges. Nevertheless, the theories are no longer universally expressive.

3. Jonas Arenhart and Ederson Melo: Dialetheists' Lies about the Liar

This paper presents a battery of arguments purported to show that the dialetheist solution to the Liar paradox, and specifically, the adoption of the Logic of Paradox, is unsatisfactory to deal with the Liar. The authors argue that one cannot coherently adopt a paraconsistent solution and preserve the pre-theoretic intuition that the Liar delivers genuine contradictions. Preservation of this intuition is one of the desiderata advanced by dialetheists themselves on an adequate solution of the paradoxes. It is argued that the limitations of the formal account presented by Graham Priest make it unable to satisfy this requirement to represent the contradiction obtained in the conclusions of the simple and of the extended Liar.

4. Eduardo Barrio: Models and Proof: *LFIs* without a canonical interpretation

The author defends the thesis that pure logics do not have canonical (standard) interpretations. As a special case, the author analyses the paraconsistent logics *BLE* and LET_j, proposed by Carnielli and Rodrigues in [2] and motivated by the idea of capturing contradictions as conflicting evidence and rejecting true contradictions (dialetheias). He shows that, despite such motivation, they are not incompatible with true contradictions. Moreover, the author shows how to recover classical logic inside LET_j, and argues that accepting or rejecting true contradictions, in the end, depends upon our philosophical views.

5. Thomas Ferguson: Axiom $(cc)_0$ and Verifiability in Two Extracanonical Logics of Formal Inconsistency

In this paper, the author analyzes the axiom $(cc)_0$ by considering its interpretation in contexts in which consistency is understood as a type of verifiability. The idea is

that such an interpretation is implicit in two extracanonical *LFIs*: Hallden's nonsense-logic *C* and Priest's cointuitionistic logic *daC*. The paper discusses both interpretations drawing some substantial conclusions concerning the status of $(cc)_0$.

6. Samir Gorsky: Information, Contradiction and the Bar-Hillel-Carnap Paradox

This paper focus on the logical structure of information, and present a new way to measure information (from contradictory scenarios in the system *LFI1*) that is presented as an alternative to the Bar-Hillel-Carnap Paradox. According to the results presented, if the logic is paraconsistent, then the amount of information in a contradictory scenario is not infinite, and therefore the theoretical framework of the theory of semantic information guarantees that such a measure is computable. Thus, the so-called Bar-Hillel-Carnap Paradox vanishes.

7. Nicolas Lo Guercio and Damian Szmuc: Remarks on the Epistemic Intrepretation of Paraconsistent Logic

The author explore the relation between ω-inconsistency and plain inconsistency, in the context of *LPTT* and *STTT*. She shows that both theories are ω-inconsistent. Finally, she presents some thoughts about if it is possible to have an inconsistent, but ω-consistent theory of truth, restricting the analysis to substructural theories.

8. Bruno Da Ré: Inconsistency, Paraconsistency and Omega-inconsistency

The author explore the relation between ω-inconsistency and plain inconsistency, in the context of *LPTT* and *STTT*. She shows that both theories are ω-inconsistent. Finally, she presents some thoughts about if it is possible to have an inconsistent, but ω-consistent theory of truth, restricting the analysis to substructural theories.

9. Ariel Roffe and Mariela Rubin: Against a metaphysical understanding of rejection

The authors defend that incorporating a rejection operator into a paraconsistent language involves fully specifying its inferential characteristics within the logic. In this context, they criticize Berto's proposal in [1] for a paraconsistent rejection, which intend to avoid paradox, claiming that it is too incomplete. They defend that the inferential characteristics of the new operator are left unspecified. Finally, they show that when completing this proposal with some plausible rules for the rejection operator, paradoxes arise.

Acknowledgements

The Buenos Aires Logic Group (BA Logic) is a collective of research that works on logic and philosophy of logic under the guidance of Eduardo Barrio (University of Buenos Aires and CONICET, Argentina), and its home is the Argentinean Society of Philosophical Analysis (SADAF). The Centre for Logic, Epistemology and the History of Science (CLE) is based at the State University of Campinas (UNICAMP), Brazil, and its faculty members include scholars from several Brazilian and foreign universities. The close friendship and fruitful collaboration between CLE and BA Logic started more than ten years ago when both organized a visit of Saul Kripke to South America. Since then, four meetings have been organized, the latest one in Brazil, April 2018. A collaborative agreement has been recently signed between Walter Carnielli, Marcelo Coniglio and Alberto Moretti, on behalf of CLE and SADAF that, we hope, will further promote the research and friendship between colleagues of CLE and BA Logic.

The idea of organizing a volume dedicated to the notions of consistency, contradiction and consequence from the viewpoint of paraconsistent logics, and related topics, came up in the 3rd Workshop CLE-BA Logic, held in Buenos Aires at SADAF, in April 2016. The present volume, however, does not contain only papers presented at the 2016 workshop, it also includes the results of dialogues initiated there as well as contributions from authors who, while not present at the event, share common interests.

Lastly, we thank all those without whom this volume would not have been possible:

- Eduardo Barrio, Natália Buacar, Walter Carnielli, Alberto Moretti, and Lucas Rosenblatt for organizing the 3rd Workshop CLE-BA Logic, and CONICET, MinCyT (Ministerio de Ciencia, Tecnología e Innovació Productiva, Argentina) and SADAF for financial and logistical support;
- Jonas Arenhart, Jaimir Conte, Ivan Ferreira da Cunha, and Cezar Mortari from *Principia* for all the work in preparing the volume for publication;
- the anonymous referees, for their helping in reviewing the papers;
- and last but not least, very special thanks to Henrique Antunes, Guilherme Araújo, Jonas Arenhart, Ederson Melo, Eduardo Barrio, Thomas Ferguson, Samir Gorsky, Nicolas Lo Guercio, Damian Szmuc, Bruno Da Ré, Ariel Roffe, and Mariela Rubin for their valuable submissions.

The editors.

References

[1] Berto, F. 2014. Absolute contradiction, dialetheism, and revenge. *The Review of Symbolic Logic* **7**(2): 193–207. DOI:10.1017/S175502031400001X

[2] Carnielli, W. A. & Rodrigues, A. 2017. An epistemic approach to paraconsistency: a logic of evidence and truth. *Synthese*. DOI: 10.1007/s11229-017-1621-7. Preprint available in `http://bit.ly/syntletj`.