RESEARCH ARTICLE

Comparison of height-diameter models based on geographically weighted regressions and linear mixed modelling applied to large scale forest inventory data

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Abstract

Aim of the study: The main objective of this study was to test Geographically Weighted Regression (GWR) for developing height-diameter curves for forests on a large scale and to compare it with Linear Mixed Models (LMM).

Area of study: Monospecific stands of *Pinus halepensis* Mill. located in the region of Murcia (Southeast Spain).

Materials and Methods: The dataset consisted of 230 sample plots (2582 trees) from the Third Spanish National Forest Inventory (SNFI) randomly split into training data (152 plots) and validation data (78 plots). Two different methodologies were used for modelling local (Petterson) and generalized height-diameter relationships (Cañadas I): GWR, with different bandwidths, and linear mixed models. Finally, the quality of the estimated models was compared throughout statistical analysis.

Main results: In general, both LMM and GWR provide better prediction capability when applied to a generalized height-diameter function than when applied to a local one, with R^2 values increasing from around 0.6 to 0.7 in the model validation. Bias and RMSE were also lower for the generalized function. However, error analysis showed that there were no large differences between these two methodologies, evidencing that GWR provides results which are as good as the more frequently used LMM methodology, at least when no additional measurements are available for calibrating.

Research highlights: GWR is a type of spatial analysis for exploring spatially heterogeneous processes. GWR can model spatial variation in tree height-diameter relationship and its regression quality is comparable to LMM. The advantage of GWR over LMM is the possibility to determine the spatial location of every parameter without additional measurements.

Keywords: Spatial analysis; *Pinus halepensis* Mill; forest modelling.

Abbreviations used: GWR (Geographically Weighted Regression); LMM (Linear Mixed Model); SNFI (Spanish National Forest Inventory).

Introduction

In forestry, spatial heterogeneity is theorized as one of the major drivers of biological diversity (Wiens, 1976). Spatial heterogeneity results from the spatial interactions between a number of biotic and abiotic factors and the differential responses of organisms to these factors (Milne, 1991). It may have significant influences on many ecosystem processes at multiple spatial scales (Turner, 1989). The spatial heterogene-ity of vegetation patterns (i.e., landscape heterogeneity) is a structural property of landscapes (Li & Reynolds, 1994) that can be defined by the complexity and variability of ecological system properties in space. The spatial heterogeneity of a tree variable (height, diameter, etc.) in a forest stand results from the complex historical and environmental mosaic imposed by competition and systematic environmental heterogeneity (Zhang *et al.*, 2004); which implies a spatio-temporal heterogeneity. Forest researchers have realized

that the spatial pattern of tree locations strongly affects (1) competition among neighbouring trees, (2) size variability and distribution, (3) growth and mortality and (4) crown structure (Zhang *et al*., 2004). Ignoring spatial heterogeneity in forest modelling causes biased parameter estimates, misleading significance tests, and sub-optimal prediction (Anselin & Griffith, 1988).

Global and local regression models have traditionally been used for studying spatially heterogeneous data (Fotheringham *et al.,* 2002). There are important differences between these models, for example, global regression models summarize the characteristics of the spatial pattern over the whole study area and attempt to identify homogeneity (i.e. Ordinary Least Squares). On the other hand, local regression models make explicit the differences in the pattern observed among parts in the study area (i.e. Geographically Weighted Regression) (Fortin & Dale, 2005). This aspect makes global models easy to compute. However, these models are not realistic and the results obtained may not be correct. Local models attempt to identify exceptions and are variable throughout the space. Hence, these models are suitable for specific sample plots but are of limited use for large populations (Fotheringham *et al.,* 2002).

There are several methods for solving the heterogeneity problem, for example, models based on heteroscedasticity specification, generalized additive models, classification and regression trees, linear mixed models (LMM) and geographically weighted regression (GWR). In this document the latter two methods were analysed.

LMM has been widely used for height-diameter modelling (Lappi, 1997; Mehtätalo, 2004; Calama & Montero, 2004). This method estimates fixed and random parameters simultaneously for the same model. The introduction of random parameters into the model, specific to each sampling unit, enables us to model the variability detected for a given phenomenon among different locations, after defining a common fixed functional structure (Lindstrom & Bates, 1990). Mixed models give an unbiased and efficient estimation of the fixed parameters of the model. Furthermore, to correctly apply mixed models in unsampled areas a calibration is required, which is of critical importance when making local predictions (Meng & Huang, 2009).

The second methodology analysed, GWR, is used to explore spatially heterogeneous processes (Brunsdon *et al*., 1996). The underlying idea of GWR is that parameters may be estimated anywhere in the study area given a dependent variable and a set of one or more independent variables which have been measured at known locations (Charlton & Fotheringham, 2009). Finally, no additional measurement is needed to predict the value of parameters for unsampled locations. GWR

extends ordinary least squares regression models by accounting for spatial structure and estimates a separate model for each geographic location within the studied area (Matthews & Yang, 2012). It can be said that GWR is similar to a '*spatial microscope*' in reference to its ability to measure and visualize variations in relationships, unobservable for non-spatial global models (Matthews & Yang, 2012). GWR was originally used in relation to the economy and civil engineering. In the field of ecology, GWR has been applied to remotely sensed data counts with some interesting results (Wang *et al*., 2005; Salas *et al*., 2010; Chen *et al*., 2012). However, in forest management, the GWR method has predominantly been used for small study areas although it was developed for use in large scale studies.

Tree height is an important variable for forest management in several ways: (1) total height together with diameter at breast height allows us to estimate the total volume of the tree, (2) in regular stands, dominant height together with age are used to estimate the site index and (3) the distribution of tree heights in the stand is very important to describe the vertical structure.

Nevertheless, height measurement can be time consuming and consequently expensive, therefore it is common to use height-diameter models instead. There are two kinds of height-diameter models: local models and generalized models.

Local models are designed specifically for forest stands and are the most precise curves for a particular forest stand (Diéguez-Aranda *et al*., 2009). However, they have several disadvantages, for example: (1) they require the heights of many trees in the stand to be measured, therefore are very expensive and (2) due to forest heterogeneity (age, quality, density) and the silvicultural state of stands within a given forest, a local function cannot be adjusted to all situations (Diéguez-Aranda *et al*., 2009). Generalized height-diameter models could improve the results in such forests. These models include stand variables for obtaining precise and unbiased estimations (Gaffrey, 1988). Dominant height is usually included because it is easy to measure and is independent of silvicultural treatments. Other variables such as basal area, density or quadratic mean diameter could also be included (Sloboda *et al*., 1993; Páscoa, 1987).

The main aim of this paper is to analyse the suitability of GWR for modelling height-diameter relationships in a large area and to compare it with LMM, which is one of the most frequently used methods for including spatial heterogeneity in these models.

To test whether GWR provides as good results as the more frequently used LMM, we applied both methodologies to two different functions: the Petterson (1955) local function and the Cañadas *et al*., (1999)

generalized function, and compared their ability to predict heights when applied to an independent dataset.

Material and methods

Study area

To study GWR, we selected mono-specific stands of Aleppo pine (*Pinus halepensis* Mill.) in the Region of Murcia, in South-East Spain (between latitude 37º 47' 36'' N and 38º 6' 11'' N, and longitude 1º 53' 48'' W and 1º 26' 17'' W). The study area covered around 1 300 km² in Espuña, Cambrón, Burete, Lavia and Quipar mountain ranges (Fig. 1). In this region, Aleppo pine was growing in low-density stands with relatively low dominant heights, probably reflecting medium to poor site qualities (Condés & Sterba, 2008).

Data were obtained from 230 sample plots with 2582 trees belonging to the Third Spanish National Forest Inventory (SNFI). SNFI design is based on permanent sample plots located at the nodes of one kilometre square grid, re-measured in an inventory cycle of ten years. The sample plots consisted of four concentric circles of 5, 10, 15 and 25 meter radius, in which the diameters and heights of all trees with a breast height diameter over 7.5, 12.5, 22.5 and 42.5 cm respectively were measured. Furthermore, plot centre coordinates, together with the polar coordinates of each measured tree, i.e. distance and azimuth form the plot centre, were recorded (MAGRAMA, 2015).

Data in the studied area were randomly split into two subsets. The first consisted of 152 plots with 1697 trees and was used to develop the model (training data). The second was used to validate the model (validation data) and consisted of 78 plots with 885 trees (Fig. 1). Table 1 shows the main stand variables for both subsets estimated by weighting individual tree data according to the area of the concentric SNFI subplots.

Modelling height-diameter relationships

Two different height-diameter functions were selected for the study. The first of these was the local height-diameter Petterson's function (Petterson, 1955), (eq. 1), which has been successfully used in similar, previously published studies (e.g. Juárez de Galíndez *et al.*, 2007; Drápela, 2011; Adamec, 2014; Adamec & Drápela, 2015)

$$h = 1.3 + \frac{1}{\left(a_0 + \dfrac{a_1}{d}\right)^3} \tag{1}$$

where h is the total height (m), d is the diameter at breast height (cm) and a_0 and a_1 are the parameters to be estimated.

The second selected function was the Cañadas I generalized height-diameter function (Cañadas *et al.*, 1999), (eq. 2). This function was selected because it offered the possibility to perform the computation with GWR together with the fact that it had previously provided good results for Aleppo pine forests (Cabanillas, 2010).
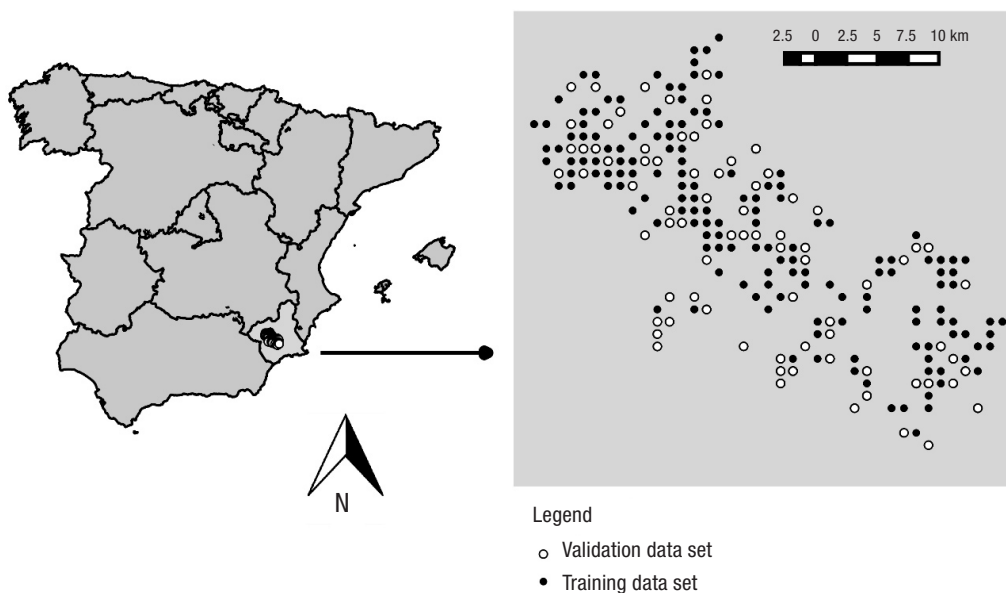

Figure 1. Study area & subsamples.

Table 1. Statistical analysis of data

	Training dataset 152 plots / 1697 trees						
	d	**h**	**N**	**G**	**dg**	**H$_o$**	**D$_o$**
Min	7.5	2.0	14	0.6	10.2	4.0	11.1
Max	55.7	21.5	1351	27.3	38.3	16.3	39.0
Mean	21.4	8.6	361	9.1	20.2	8.8	24.3
Standard Deviation	2.6	8.3	283	5.6	6.3	2.4	5.9
	Validation dataset 78 plots / 885 trees						
Min	17.8	2.4	19	1.3	9.2	3.7	12.5
Max	53.1	15.5	1489	32.4	40.4	12.7	41.0
Mean	20.0	8.0	420	9.3	18.9	8.3	23.4
Standard Deviation	7.4	2.1	334	6.2	6.0	1.9	5.4

d - Diameter at breast height (cm); h - Total height (meters); N - Number of stems per hectare (stems/ha); G - Plot basal area (m2/ha); d$_g$ - Quadratic mean diameter (cm); H$_o$ - Dominant height (m); D$_o$ - Dominant diameter (cm).

$$h = 1.3 + (H_0 - 1.3) \cdot \left(\frac{d}{D_0} \right)^{b_0} \quad (2)$$

where h is total height (m), d is diameter at breast height (cm), H_o is dominant height (m) obtained according to the Hart criteria (Hart, 1928), D_0 is dominant diameter, calculated as the average of the 100 thickest trees per hectare, and b_0 is the parameter to be estimated.

As GWR is basically a technique developed for linear regressions (Fotheringham *et al.*, 2002), to fit these equations it was necessary to linearize them. Consequently, to fit these equations it was necessary to linearize them, which was done as follows in equations 3 and 4:

Petterson:
$$\frac{1}{\sqrt[3]{h - 1.3}} = a_0 + a_1 \cdot \frac{1}{d} \quad (3)$$

Cañadas I:
$$\ln \left(\frac{h - 1.3}{H_0 - 1.3} \right) = b_0 \ln \left(\frac{d}{D_0} \right) \quad (4)$$

These two height-diameter functions were fitted following the two different methodologies detailed below.

Linear Mixed Models

In a first step, linear mixed models were used for fitting the height-diameter equations. As data came from a hierarchical structure, with several trees measured in the same plot, a mixed model was used with the plot as the grouping structure of the random effects. We included random effects in the a_0 and a_1 term of Petterson local model (eq. 3) and in b_0 for the Cañadas I (eq. 4).

The models were fitted using the maximum likelihood method of the lme procedure (R Development Core Team, 2014) to allow the comparison of results. A level of $p = 0.05$ was used for significance testing of the variables in the model, and Akaike's information criterion (AIC) was used to compare results. The significance of random effects was tested by comparing the models with similar ones developed using a constant as grouping variable. Goodness-of-fit measures for linear mixed models were calculated using the lmmR2 procedure (R Development Core Team, 2014).

Geographically Weighted Regression

In a second step, the models were fitted using the GWR with three different bandwidths and an optimal number of trees.

Two of these bandwidths, calculated taking into account the SNFI grid, were 1000 m and 500 m. These two values were used to determine whether the SNFI design influenced the GWR results. The third bandwidth was calculated according to the calibration recommended by Fotheringham *et al.*, (2002), which was based on the optimization of the studied formula according to a kernel and a weighted function. In this case, the use of a fixed kernel plus a gaussian weighted function was recommended. A fixed kernel assumes that the bandwidth at each centre i is a constant across the study area. The gaussian weighted function (eq. 5) is a continuous function of the distance between two observation points (Gollini *et al.*, 2015).

Gaussian weighted function:

$$w_{ik} = e^{\left(-\left(d_{ik}/b \right)^2 \right)} \quad (5)$$

where w_{ik} is the k-*th* element of the diagonal of the matrix of geographical weights, and d_{ik} is the distance between observations i and k (in our case observations were trees) and b is the bandwidth.

Finally, the optimal number of trees was calculated according to the calibration recommended by Fotheringham *et al.*, (2002). This was based on the optimization of the number of trees, which is to be applied in every height-diameter formula. As in the case of bandwidth, this optimization is based on the type of kernel and weighted function. For this situation, an adaptive kernel together with a bi-square weighted function was recommended. This kernel adapts itself in the size to the data density. The bi-square weighted function (eq. 6), is a discontinuous function, giving null weights to observations with a distance greater than a specific point (Gollini *et al.*, 2015).

Bisquare weighted function:

$$w_{ij} = \left[1 - \left(d_{ij} / n\right)^2\right]^2 \quad \text{When } d_{ij} \leq n \quad (6)$$

$$w_{ij} = 0 \quad\quad\quad\quad\quad \text{When } d_{ij} \leq n$$

where w_{ij} is the *j-th* element of the diagonal of the matrix of geographical weights, and d_{ij} is the distance between observations i and j, and n is the number of neighbourhoods (number of trees).

GWR provides a range of coefficients geographically located in the study area, and a continuous GWR map of coefficients can be obtained by the coefficients interpolation. The Inverse Distance Weighting (Watson & Philip, 1985) was selected as interpolation methodology because this interpolation is widely used in natural sciences (Chen *et al.*, 2012) and is easily applicable to GWR (Tardanico, 2006). See for instance Fotheringham *et al.*, (2002) for further description of GWR.

GWR was fitted using the GWModel package in R language (Binbin *et al.*, 2015; R Development Core Team, 2014). A level of $p = 0.05$ was used for significance testing of the variables in the models, and Akaike's information criterion was used to compare results.

Model validation

Models were validated by applying GWR as well as LMM to the validation dataset, obtaining a set of predicted heights. In the case of GWR, the predicted values were obtained through the GWR map of coefficients.

In order to validate LMM, it was necessary to calibrate the model. Very few measurements can be used in the calibration, even less than the number of random parameters to be predicted (Calama & Montero, 2004). Four trees per plot were used for calibrating the model in this study, this is a very common practice in forest inventories in Spain (Calama & Montero, 2004). The technique of best linear unbiased predictor (BLUP) (Robinson, 1991) was used to estimate of random parts of the parameters.

The validation was performed by comparing observed and predicted heights through a statistical analysis of the residuals. Mean error (ME), mean absolute error (MAE), root mean square error (RMSE) and coefficient of determination (R^2) were calculated according to the formula in Table 2.

Results

Table 3 shows the estimated coefficients for equations 3 and 4, firstly for the LMM and GWR when

Table 2. Residual formulas

Mean error	$ME = \dfrac{\sum \left(y_i - \hat{y}_i\right)}{n}$
Root Mean Square Error	$RMSE = \sqrt{\dfrac{\sum \left(y_i - \hat{y}_i\right)^2}{n}}$
Mean Absolute Error	$MAE = \dfrac{\sum \left|y_i - \hat{y}_i\right|}{n}$
Coefficient of determination	$R^2 = 1 - \dfrac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{n}\left(y_i - \overline{y}_i\right)^2}$

Where n is the sample size; y_i and $\hat{y}_i$ are respectively the observed and predicted tree heights and $\overline{y}_i$ the average of individual tree heights.

applied to the Petterson height-diameter function, and secondly when they were applied to the Cañadas I height-diameter function. In this table, the minimum, mean, standard deviation and maximum values of the coefficients obtained for the whole study area (considering the map of coefficients for GWR and the fixed and random effects for LMM) were detailed together with R-squared (R^2) and the Akaike information criterion (AIC).

For the Petterson height-diameter function, the coefficient a_0 mean value varied between 0.37 and 0.55 for LMM and also for GWR independently of the bandwidth (between 0.34 and 0.52 for a bandwidth of 1000, between 0.32 and 0.69 for a bandwidth of 500, between 0.34 and 0.71 for a Fixed Kernel bandwidth, and between -0.19 and 0.64 for an adaptive kernel bandwidth). The range of this parameter was lowest when obtained by LMM or by GWR with a bandwidth of 1000 m, but increased for the bandwidth of 500 and the fixed kernel, and reached a maximum for the adaptive kernel. The coefficient a_1, i.e. the slope of the curve, showed higher variation with larger standard deviations and wider ranges than the a_0. So, for LMM the standard deviation of a_1 was 0.48 while for GWR were from 0.65 to 1.39. However, mean values along the study area were similar for all the methodologies, from 1.79 to 2.06. Once again, the largest variations were found for the GWR fitted with fixed or adaptive kernel, proving to be more sensitive to the spatial variability of the data. Moreover, the smallest AIC was obtained with the fixed kernel, representing an optimal bandwidth of 363 m; and with the adaptive kernel, representing an optimal number of trees of 26. However, all GWR methods showed certain incongruent results such as negative or very large parameter values. An example

of estimation using the Petterson map of coefficients is presented in Fig. 2, which provides a graphical representation of the previously explained parameter distribution for GWR with a predefined bandwidth of 1000 m.

In the case of the Cañadas-I height-diameter function it can be seen that the coefficient b_0 had slightly larger standard deviations and ranges for GWR than for LMM. Additionally, the mean values were almost the same between both methods. The AIC for this function was smaller when GWR was fitted with a predefined bandwidth of 500 m. For this generalized height-diameter function, the optimal bandwidth calculated by fixed kernel was 755 m, and the optimal number of trees in the adaptive kernel was 65. Again, some incongruent minimum values can be seen for the GWR methodology in Table 3. A representative map of coefficients for the Cañadas I estimation is shown in Fig. 3, and the parameter distribution of GWR is presented with a predefined bandwidth of 1000m.

As response variables in the Petterson and Cañadas-I functions were different, the AIC and R^2 values were presented in Table 3. However, AIC were not comparable between models.

Model validation

Table 4 shows the errors obtained for the validation dataset when different methodologies were used. Pre-

dicted values were estimated according different assumptions: (1) LMM with fixed effects, (2) LMM with random effects and (3) GWR, including all the analysed variations.

The validation revealed that the Petterson height-diameter function had poorer predictive capacity than the Cañadas I generalized function. Consequently, it can be seen that the R^2 for the LMM with only fixed effects increased from 0.27 to 0.71, when calibration the improvement was from 0.63 to 0.72 and for GWR the R^2 ranged between 0.29 and 0.35 for Petterson while for Cañadas was 0.71.

When analysing the errors obtained from Petterson model, it was observed that, independently of bandwidth or kernel, GWR resulted in lower bias than LMM with only fixed effects (ME=0.6) or in similar than the calibrated (ME=0.1). Moreover, MAE and RMSE were very similar for GWR and LMM with only fixed effects (MAE=1.3 and RMSE=1.7 for LMM without calibration; and MAE=1.3 - 1.4 and RMSE=1.7 for GWR). However both MAE and RMSE were smaller when calibrating LMM (MAE= 0.9 and RMSE=1.3). Regarding the Cañadas-I height-diameter function, the error analysis showed similar results for all the GWR and LMM methods with and without calibration (Table 4).

The standardized residual plots showed that residuals are symmetrically distributed in a random pattern around the horizontal axis for any of the functions and fitting methodologies (Figures 4 and 5). Judging from the errors, it can be stated that GWR applied to generalized function works as well as the more frequently used LMM methodology.

Discussion

Tree height is an important variable which is used for estimating stand volume or site quality (Diéguez-Aranda *et al.*, 2009). Forest variables are typically highly heterogeneous and in this paper, two different methods were presented for modelling height-diameter relationships and their spatial heterogeneity. The first method was LMM, in which the height-diameter curves use fixed and random coefficients and which gives very accurate predictions when random effects can be predicted (Nanos *et al.*, 2004). However, to calibrate the LMM prior to use, additional height measurements of trees within the stand are required (Lappi, 1997). The second method was GWR, which is a geostatistical method that can predict height-diameter models without the necessity for additional measurements. This effect is very interesting in case of large inventory areas such as national forest inventories, because the only available data are those collected in the inventory plots.
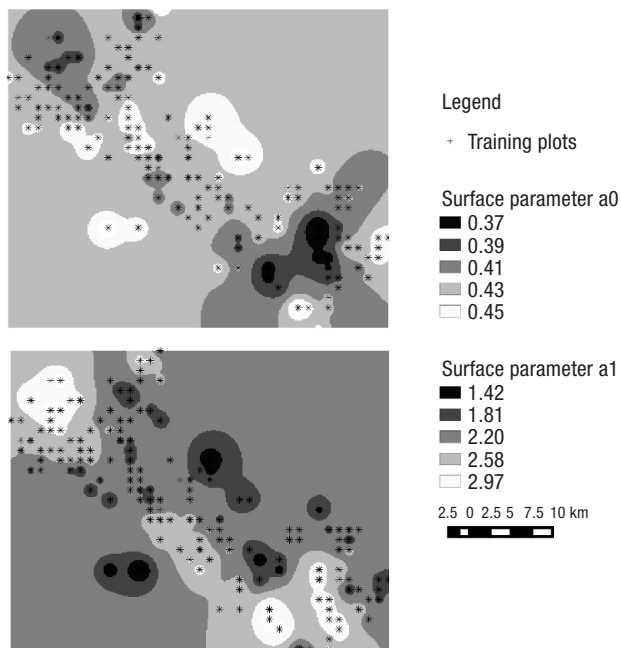
Figure 2. GWR map of regression coefficients. Example of Petterson height-diameter model (1000 m predefined bandwidth), coefficient a_0 and a_1.

Table 3. Coefficient distribution of models - In LMM and GWR using linearized equations (eq. 3 and 4)

			Petterson						Cañadas I						
	Method	Coefficient	Min	Mean	SD	Max	R2	AIC	Coefficient	Min	Mean	SD	Max	R2	AIC
LMM		a_0+a_{0j}	0.37	0.44	0.03	0.55	0.82	-6304	b_0+b_{0j}	0.30	0.66	0.14	1.31	0.59	-1161
		a_1+a_{1j}	1.80	1.96	0.48	2.76									
GWR	**1000**	a_0	0.34	0.42	0.03	0.52	0.68	-5923	b_0	0.18	0.66	0.16	1.35	0.57	-1260
		a_1	-0.23	2.06	0.65	4.07									
	500	a_0	0.32	0.43	0.05	0.69	0.81	-6676	b_0	-0.42	0.65	0.21	1.49	0.60	-1319
		a_1	-1.09	1.85	0.81	4.31									
	Fixed Kernel	a_0	0.34	0.44	0.05	0.71	0.83	-6884	b_0	0.01	0.66	0.18	1.37	0.58	-1287
		a_1	-2.46	1.79	0.88	4.76									
	Adaptive Kernel	a_0	-0.19	0.43	0.06	0.64	0.82	-6809	b_0	-0.04	0.66	0.17	1.34	0.57	-1267
		a_1	-0.48	1.87	1.39	21.88									

Where Min is the minimum value; SD is the standard deviation; Max is the maximum value of coefficient; R^2 is the coefficient of determination, in case of LMM is referred to conditional R^2; AIC the Akaike's information criterion; a_0, a_1 and b_0 are the fixed factor in the case of LMM models; a_{0j} and b_{0j} are random factors in the case of LMM. In GWR a_0, a_1 and b_0 are the parameters estimated by the model.

These two methodologies were tested according to a local (Petterson) and a generalized (Cañadas I) height-diameter function as well as with different bandwidths in the case of GWR.

In the case of the Petterson model, it was observed that this method had insufficient predictive quality when GWR or uncalibrated LMM was applied. It is possible that the large heterogeneity of the studied area made it difficult for the Petterson function to reach the necessary quality. This could be due to the fact that the Petterson formula only had diameter at breast height as an independent variable and would appear to be too simple to reproduce the forest heterogeneity. Nevertheless, when Petterson equation was validated with calibrated LMM, a high improvement in the predictive quality was observed. In the case of height-diameter model, random component represents the specific conditions of every stand in the model. It causes better precision of the model for particular location in comparison with the model with fixed parameters. This

could be important in a local model such as Petterson, designed specifically for giving precise prediction in individual forest stands (Diéguez-Aranda *et al.*, 2009).

In contrast, the Cañadas I model revealed high stability in the model fitting and high predictive quality in the validation in both LMM and GWR. In these cases, R^2 values in the validation improve from local to generalized regressions from 0.63 to 0.72 in calibrated LMM. Similarly, MAE and RMSE were lower for the generalized models. The comparison between local and generalized equations is analogous with previous studies which concluded that the inclusion of stand-density variables such as dominant height or dominant diameter in the base height–diameter function increased the accuracy of prediction (Temesgen & Gadow, 2004; Diéguez-Aranda *et al.*, 2009).

The predictive quality of the model was improved when few additional height measurements were used for estimating the random parameters for the validation dataset. Calibration using small subsamples of trees is one of the main utilities of mixed models in forest applications (Calama & Montero, 2004). However, when additional measurements are not available GWR provides predictions with higher quality than the LMM.

In some of the GWR models the presence of odd, extreme values was observed. With the Petterson equation these values appear in every GWR model and did not correspond to specific plots but rather, were scattered randomly in the study area. This could be another indicator of the weakness of these models. However, in the case of Cañadas I, these odd values only appeared in the models with a bandwidth of 500 m and adaptive kernel. Specifically, this odd value was located in plot number 1473, which contained only 3 trees and was located in a relatively isolated area. Odd values have also been observed in similar height-diam-

2.5 0 2.5 5 7.5 10 km
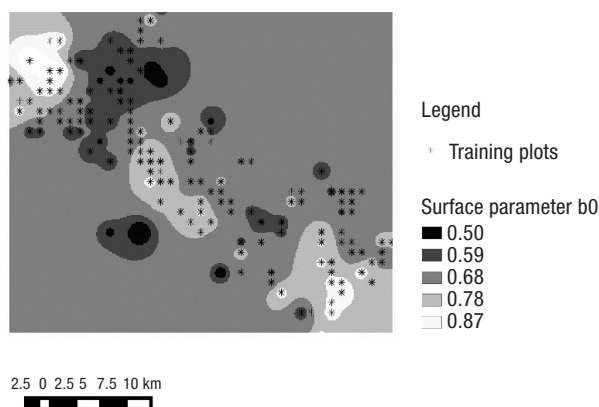
Figure 3. GWR map of regression coefficients. Example of Cañadas I height-diameter model (1000 m predefined bandwidth), coefficient b_0.

Legend

+ Training plots

Surface parameter b0
- 0.50
- 0.59
- 0.68
- 0.78
- 0.87

Table 4. Comparison of errors obtained for the validation dataset

	VALIDATION	ME	MAE	RMSE	R²
PETTERSON	LMM fixed effect	0.6	1.4	1.7	0.27
	LMM random effect	0.1	0.9	1.3	0.63
	GWR	0.1	1.3-1.4	1.7	0.29-0.35
CAÑADAS I	LMM fixed effect	-0.1	0.8	1.1	0.71
	LMM random effect	-0.2	0.8	1.1	0.72
	GWR	-0.1	0.8	1.1	0.71

Where ME is the mean error; MAE is the mean absolute error; RMSE is the root mean squared error; R^2 is the coefficient of determination.

eter modelling studies (Zhang *et al.*, 2004; Zhang *et al.*, 2008; Zhang *et al.*, 2009). It is important to be aware of the presence of these odd values in order to select the correct model. The inclusion of these values in the interpolation could lead to erroneous results in the GWR coefficient map.

In our study, the observed GWR models gave the same residuals regardless of the calibration used. However, according to Zhang *et al.*, (2009), the GWR with adaptive kernel was expected to produce more desirable model residuals in terms of spatial autocorrelation and heterogeneity than the GWR with fixed kernel model for the plots. There are a wide range of calibration options so it is possible that future modifications of settings could improve the current results. For instance, it would be possible to test different bandwidths, or to add robustness to GWR (Fotheringham *et al.*, 2002), or even to calculate the optimal bandwidth differently or the optimal number of trees with different kinds of weighted functions (Gollini *et al.*, 2015).

The maps of coefficient in Figures 2 and 3 illustrate the spatial heterogeneity of the height-diameter relationship. In both figures it can be seen that there are greater slopes in both models in the upper-left area and in the bottom-right area. Additionally, more flat areas exist in the centre of the studied areas. In Fig. 2, two areas can be observed where a_0 takes the highest values and a_1 the lowest; therefore in Fig. 3, only the left side is visible. It would be interesting to compare these contours with local terrain layers (slopes, kind of soil, vegetation), in order to determine whether there is a relationship between fittings and local conditions. Since the spatial variation of local conditions and competition were taken into account in the GWR model, it showed better fitting results for the height-diameter regression relationship (Zhang *et al.*, 2004).

When comparing the LMM and GWR models in this study, it was found that GWR estimation produces similar errors to the LMM when generalized equation was applied. Analogous results have been found in other studies, supporting our finding that GWR gives similar quality estimations to the LMM methodology and predicts the response variable accurately (Zhang *et al.*, 2005; Zhang *et al.*, 2008, Zhang *et al.*, 2009). However, GWR showed not so good predictive quality with the Petterson function, showing no important improvement over LMM without calibration. We suppose that GWR quality could be improved by more sophisticated calibration, which needs a deeper investigation.

LMM and GWR provide different perspectives and insights for data analysis and model prediction. In LMM, the fixed component explains the impact of different variables as with the ordinary least squares regression (Yang & Huang, 2011), and the random component explains the heterogeneity and randomness by both known and unknown factors (Vonesh & Chinchilli, 1997). LMM is able to characterize the spatial covariance structures in the data with different geostatistic models (Zhang *et al.*, 2009), and produces accurate predictions for the response variable by accounting for the effects of spatial autocorrelation through the empirical best linear unbiased predictors (Littell *et al.*, 1996; Schabenberger & Pierce, 2002). However, GWR has several very important advantages over LMM, which include the following: (1) It is a useful tool for exploring spatial nonstationarity at different scales (Jetz *et al.*, 2005), which was achieved by changing the bandwidth of study. (2) Its capacity for testing the heterogeneity of the study area (Danlin & Yehua, 2013). The heterogeneity is shown in the coefficient maps (Figures 2 and 3) and can be tested numerically by applying the relative spatial heterogeneity index (Zhang *et al.*, 2009). (3) GWR is able to work with spatial data and every kind of attribute data (Danlin & Yehua, 2013); this is particularly useful in the case of Forest Inventories where attributes of measured trees and their location are known. (4) The residuals for GWR are at least similar as for LMM (Zhang *et al.*, 2004; Zhang & Shi, 2004; Zhang *et al.*, 2009), as confirmed in our research. (5) GWR results, unlike LMM model results, are mappable; these maps of coefficients facilitating the in-
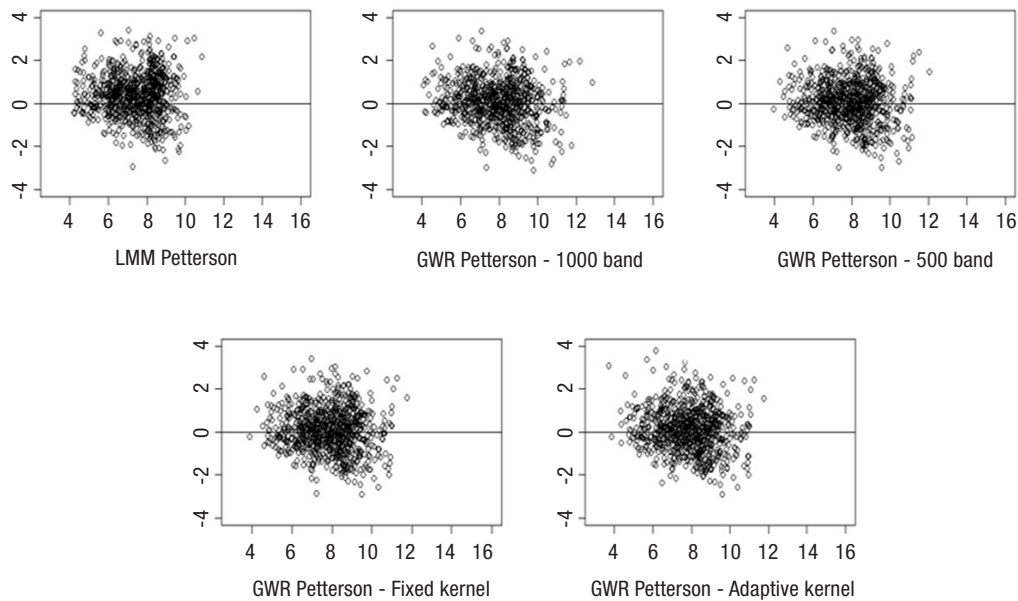
Figure 4. Standardized Residuals versus predicted height for the LMM and the different GWR methodologies when applied to Petterson equation.
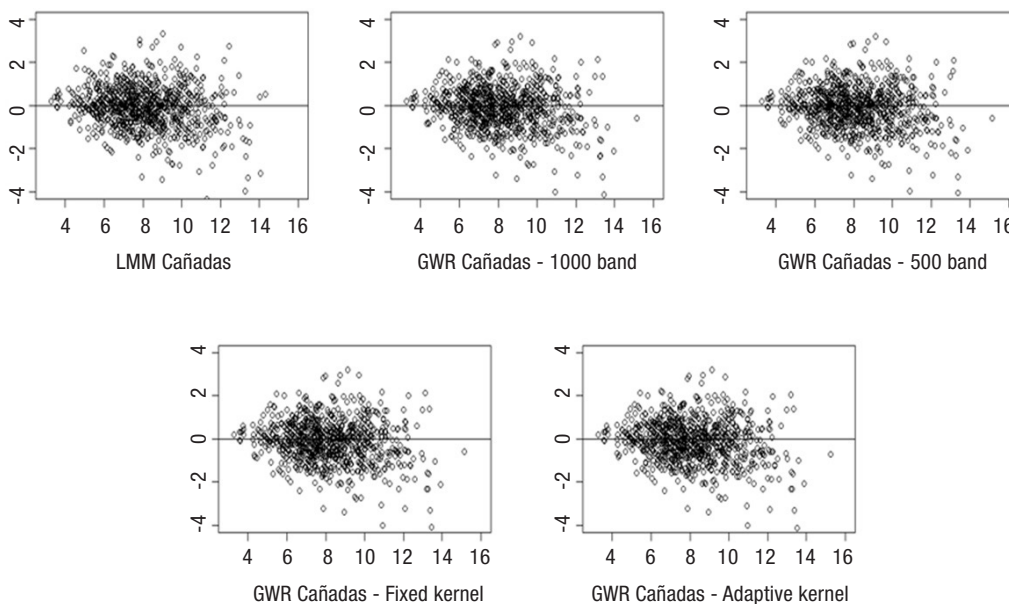
Figure 5. Standardized Residuals versus predicted height for the LMM and the different GWR methodologies when applied to Cañadas I equation.

terpretation based on spatial context and known characteristics of the study area (Goodchild & Janelle, 2004). The maps of coefficients in our study are shown in Figures 2 and 3. (6) In contrast to the LMM method, the GWR regression does not require any additional measurements or calibrations.

Conclusions

Two different methodologies were used to model local and generalized height-diameter relationships:

the geostatistical method GWR and LMM methodology. Additionally, two different height-diameter functions were selected to fit these models: a local function and a generalized function. Finally, the quality of the estimated models was compared through statistical analysis.

As expected, it was observed that in general, both LMM and GWR provide better prediction quality when applied to a generalized height-diameter equation. However, error analysis revealed that there were no great differences between these two methodologies, evidenc-

ing that GWR applied to a generalized function provides results which are at least as good as those obtained using the calibrated LMM methodology. However, GWR has a number of advantages over LMM. For instance, GWR provides information on a continuous map of regression coefficients within the studied area. Consequently, is possible to calculate tree heights in every single location without the need for additional measurements. Furthermore, the GWR method can help us to understand spatial processes and may have interesting applications in the area of forest yield and production.

Acknowledgements

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