License or entry with vertical differentiation in duopoly

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Abstract

We consider choice of options for a foreign innovating firm to license its technology for producing the high quality good to a domestic firm, or to enter the market of the domestic country with or without license. Under the assumption of uniform distribution about taste parameters of consumers; when cost functions are linear, if the low quality good's quality is sufficiently high, license without entry strategy is optimum; if the low quality good's quality is low, both of entry without license strategy and license without entry strategy are optimum; when cost functions are quadratic, if the high quality good's quality is high, license without entry strategy is optimum; if the high quality good's quality is low, entry with license strategy is optimum.

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1. Introduction

We examine a choice of options for a foreign innovating firm to license its technology for producing the high quality good to a firm in the domestic country without entry, or to enter the market of the domestic country, or to enter the market with license to the domestic firm. At present the domestic firm is a monopolist, and if the foreign firm enters, the market of the domestic country becomes a duopoly with or without vertical differentiation.

There are many references about technology adoption or R&D investment in duopoly or oligopoly. Lots of researches focus on the relation between technology licensor and licensee. The difference of means of contracts which are royalties, up-front fees, the combinations of these two and auction are well discussed (Katz and Shapiro (1985), Kamien and Tauman (1986), Sen and Tauman (2007)). Kamien and Tauman (1986) shows that if the licensor does not have production capacity, fixed fee is better than royalty and it is also better for consumers. This topic

under Stackelberg oligopoly is discussed in Kabiraj (2004) when the licensor does not have production capacity, and discussed in Wang and Yang (2004) and Filippini (2005), when the licensor has production capacity. A Cournot oligopoly with fixed fee under cost asymmetry is analyzed in La Manna (1993). He shows that if technologies can be replicated perfectly, a lower-cost firm always has the incentive to transfer its technology and hence a Cournot-Nash equilibrium cannot be fully asymmetric, but there exists no non-cooperative Nash equilibrium in pure strategies.

Some other studies are worthy of mention, for example, Watanabe and Muto (2008), Boone (2001), Matsumura, Matsushima and Cato (2013), Pal (2010), Elberfeld and Nti (2004), Zhang, Mei and Zhong (2014), Hattori and Tanaka (2014) and (2015) and Rebolledo and Sandonís (2012).

In particular, Sen and Tauman (2007) compared the license system when the licensor is an outsider and that when the licensor is an incumbent firm. In this paper we consider a problem of choice of options for the innovating firm as an outsider to license its technology for producing the high quality good to the other firm, or to enter the market *with or without license*.

We will show that under the assumption of uniform distribution about taste parameters of consumers the results depend on the form of the cost functions. When the cost functions are linear, if the level of the low quality good is sufficiently high, license without entry is the optimum strategy for the foreign innovating firm; if the level of the low quality good is low, both of entry without license and license without entry are the optimum strategies. When the cost functions are quadratic, if the level of the high quality good is sufficiently high, license without entry is the optimum strategy; if the level of the high quality good is low, entry with license is the optimum strategy.

2. The model

Our model of vertical product differentiation is according to Mussa and Rosen (1978), Bonanno and Haworth (1998) and Tanaka (2001). There are two firms, Firm A and B, in two countries, respectively, Country A and B. Firm A produces the high-quality good whose quality is k_H, and Firm B produces the low-quality good whose quality is k_L, where $k_H > k_L > 0$. k_H and k_L are fixed. At present each firm operates as a monopolist in each country. Both of the high-quality and low-quality goods are produced at the same cost.

In Country B there is a continuum of consumers with the same income, denoted by y, but different values of the taste parameter θ. Each consumer buys at most one unit of the good. If a consumer with parameter θ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \theta k$. If a consumer does not buy the good, his utility is equal to his income y. The parameter θ is distributed according to a smooth distribution function $\rho = F(\theta)$ in the interval $0 < \theta \leq 1$. ρ denotes the probability that the taste parameter is smaller than or equal to θ. The size of consumers is normalized as one. The inverse function of $F(\theta)$ is denoted by $G(\rho)$.

We consider the following two stage game:

(1) In the first stage Firm A decides whether it enters the market of Country B or not, and whether it sells a license to use new technology for producing the high-quality good to Firm B or not.
   Firm A have three options. The first option is to enter the market of Country B without license to Firm B, the second option is to license its technology to Firm B without entry to the market of Country B, and the third option is to enter the market of Country B with license to Firm B. If Firm A enters, the market of Country B becomes a duopoly with or without vertical differentiation. If Firm A enters with license, both firms produce the

high-quality good. If it enters without license, Firm A produces the high-quality good, and Firm B produces the low-quality good.

(2) In the second stage, when Firm A enters the market of Country B in the previous stage, two firms determine their outputs; when Firm A does not enter, only Firm B determines its output.

Firm B cannot enter the market of Country A. The markets of two countries are separated. The cost function of the high-quality and low-quality goods is $c(\cdot)$.

Let p_L be the price of the good of quality k_L and p_H be the price of the good of quality k_H; and let q_A and q_B be the supplies of Firm A and B in the market of Country B. The supply of Firm A in Country A is ignored.

3. Entry case

3.1. General model

First suppose that Firm A enters the market of Country B without license to Firm B. Then, in the market of Country B Firm A supplies the high-quality good and Firm B supplies the low-quality good. Let θ_L be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\theta_L = \frac{p_L}{k_L}.$$

Let θ_H be the value of θ for which the corresponding consumer is indifferent between buying the low-quality good and the high-quality good. Then

$$\theta_H = \frac{p_H - p_L}{k_H - k_L}.$$

We assume $0 < \theta_L < \theta_H < 1$. The direct demand function of the high-quality good is

$$q_H = 1 - F\left(\frac{p_H - p_L}{k_H - k_L}\right), \tag{1}$$

and the direct demand function of the low-quality good is

$$q_L = F\left(\frac{p_H - p_L}{k_H - k_L}\right) - F\left(\frac{p_L}{k_L}\right). \tag{2}$$

We have $0 < q_L < 1$, $0 < q_H < 1$, and

$$q_A = q_H, q_B = q_L.$$

From equations (1) and (2) the inverse demand functions are

$$p_H = (k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B),$$
$$p_L = k_L G(1 - q_A - q_B).$$

Since $G(1 - q_A - q_B) < G(1 - q_A) < 1$, we have $p_L < k_L$ and $p_H < k_H$. The profits of Firm A and B are written as

$$\pi_A = [(k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B)]q_A - c(q_A),$$
$$\pi_B = k_L G(1 - q_A - q_B)q_B - c(q_B).$$

The first order conditions for profit maximization of Firm A and B are

$$\frac{\partial \pi_A}{\partial q_A} = (k_H - k_L)G(1 - q_A) + k_L G(1 - q_A - q_B)$$

$$- [(k_H - k_L)G'(1 - q_A) + k_L G'(1 - q_A - q_B)]q_A - c' = 0,$$

$$\frac{\partial \pi_B}{\partial q_B} = k_L G(1 - q_A - q_B) - k_L G'(1 - q_A - q_B)q_B - c' = 0.$$

The second order conditions are

$$\frac{\partial^2 \pi_A}{\partial q_A^2} = -2[(k_H - k_L)G'(1 - q_A) + k_L G'(1 - q_A - q_B)]$$

$$+ [(k_H - k_L)G''(1 - q_A) + k_L G''(1 - q_A - q_B)]q_A - c'' < 0,$$

$$\frac{\partial^2 \pi_B}{\partial q_B^2} = -k_L[2G'(1 - q_A - q_B) - G''(1 - q_A - q_B)q_B] - c'' < 0.$$

Denote the equilibrium values of the outputs of Firm A and B, the prices of the high-quality and low-quality goods, and the profits of the firms by q_A^e, q_B^e, p_H^e, p_L^e, π_A^e, and π_B^e.

In the following subsections, we will show that if the cost functions are linear and k_L is small, Firm B cannot operate with non-negative profit and drops out from the market; however if k_L is sufficiently large, it can get positive profit. On the other hand, if the cost functions are quadratic, Firm B always gets positive profit.

3.2. Uniform distribution and linear cost function case

Now assume that $\rho = F(\theta)$ has a uniform distribution, the (common) marginal cost is constant and there is no fixed cost. Denote the marginal cost by c. We assume $k_H > k_L > c$. Then, $\rho = \theta$, $\theta = G(\rho) = \rho$, $F'(\theta) = G'(\rho) = 1$ and $F''(\theta) = G''(\rho) = 0$. There are the following two cases.

(1) When $k_L > \frac{2ck_H}{k_H + c}$ (or $k_H < \frac{ck_L}{2c - k_L}$), the equilibrium values of the variables are

$$q_A^e = \frac{2k_H - k_L - c}{4k_H - k_L}, q_B^e = \frac{k_H k_L + ck_L - 2ck_H}{k_L(4k_H - k_L)},$$

$$p_H^e = \frac{2k_H^2 + 3ck_H - k_H k_L - ck_L}{4k_H - k_L}, p_L^e = \frac{k_H(k_L + 2c)}{4k_H - k_L},$$

$$\pi_A^e = \frac{k_H(k_L - 2k_H + c)^2}{(4k_H - k_L)^2}, \pi_B^e = \frac{(k_H k_L + ck_L - 2ck_H)^2}{k_L(4k_H - k_L)^2}.$$

(2) When $k_L \leq \frac{2ck_H}{k_H + c}$ (or $k_H \geq \frac{ck_L}{2c - k_L}$), $q_B = 0$ and the equilibrium values of the variables are

$$q_A^e = \frac{k_H - c}{2k_H}, q_B^e = 0,$$

$$p_H^e = \frac{k_H + c}{2}, p_L^e = c,$$

$$\pi_A^e = \frac{(k_H - c)^2}{4k_H}, \pi_B^e = 0.$$

3.3. Uniform distribution and quadratic cost function case

Now assume that $F(\theta)$ has a uniform distribution, the (common) cost function is quadratic. Let q be an output of a firm; then the cost function is cq^2. The equilibrium values of the variables are

$$q_A^e = \frac{2k_H k_L + 2ck_H - k_L^2}{4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2},$$

$$q_B^e = \frac{k_L(k_H + 2c)}{4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2},$$

$$p_H^e = \frac{(k_H + 2c)(2k_H k_L - k_L^2 + 2ck_H)}{4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2},$$

$$p_L^e = \frac{k_L(k_H + 2c)(k_L + 2c)}{4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2},$$

$$\pi_A^e = \frac{(k_H + c)(2k_H k_L + 2ck_H - k_L^2)^2}{(4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2},$$

$$\pi_B^e = \frac{k_L^2(k_H + 2c)^2(k_L + c)}{(4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2}.$$

4. License case

4.1. General model

Next suppose that Firm A licenses its technology for producing the high-quality good to Firm B at a fixed license fee, and does not enter the market of Country B. Then, Firm B gives up the low quality good, and produces the high quality good as a monopolist paying the license fee. Denote the license fee by L. Suppose that the licensor can take all of the increase in the profit of Firm B due to adoption of the new high-quality good.

Let θ_H be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then

$$\theta_H = \frac{p_H}{k_H}.$$

The direct demand function is

$$q_H = 1 - F\left(\frac{p_H}{k_H}\right). \tag{3}$$

q_H denotes the supply of the good of quality k_H in the market. We have $0 < q_H < 1$, and

$$q_B = q_H.$$

From equation (3) the inverse demand function is

$$p_H = k_H G(1 - q_B).$$

Since $0 < G(1 - q_B) < 1$, we have $0 < p_H < k_H$. The profit of Firm B is

$$\pi_B = k_H G(1 - q_B)q_B - c(q_B) - L.$$

The first order condition for profit maximization of Firm B is

$$\frac{\partial \pi_B}{\partial q_B} = k_H G(1 - q_B) - k_H G'(1 - q_B)q_B - c' = 0.$$

The second order condition is

$$\frac{\partial^2 \pi_B}{\partial q_B^2} = -k_H[2G'(1 - q_B) - G''(1 - q_B)q_B] - c'' < 0.$$

Denote the equilibrium values of the output of Firm B, the price and its profit by q_B^l, p_H^l and π_B^l.

   If the negotiation between the foreign firm and the domestic firm about the license fee breaks down, the foreign firm can enter the market of Country B without license. Therefore, the domestic firm must pay the difference between its profit excluding the license fee and its profit in the previous entry case. Thus, the license fee is equal to

$$(\pi_B^l + L) - \pi_B^e.$$

Denote it by L^l.

4.2. Uniform distribution and linear cost function case

In the uniform distribution and linear cost function case the equilibrium values of the variables are

$$q_B^l = \frac{k_H - c}{2k_H}, \; p_H^l = \frac{k_H + c}{2},$$

$$\pi_B^l = \frac{(k_H - c)^2}{4k_H} - L^l.$$

About the license fee there are the following two cases.

   (1) When $k_L > \frac{2ck_H}{k_H + c}$ (or $k_H < \frac{Ck_L}{2c - k_L}$),

$$L^l = \frac{A}{4k_H k_L (4k_H - k_L)^2},$$

   where

$$A = k_H^2 k_L^3 - 2ck_H k_L^3 + c^2 k_L^3 - 12k_H^3 k_L^2 + 8ck_H^2 k_L^2 - 12c^2 k_H k_L^2 + 16k_H^4 k_L$$
$$- 16ck_H^3 k_L + 32c^2 k_H^2 k_L - 16c^2 k_H^3.$$

   (2) When $k_L \leq \frac{2ck_H}{k_H + c}$ (or $k_H \geq \frac{Ck_L}{2c - k_L}$), since $\pi_B^e = 0$,

$$L^l = \frac{(k_H - c)^2}{4k_H}.$$

4.3. Uniform distribution and quadratic cost function case

In the uniform distribution and quadratic cost function case the equilibrium values of the variables are

$$q_B^l = \frac{k_H}{2(k_H + c)}, \; p_H^l = \frac{k_H(k_H + 2c)}{2(k_H + c)},$$

$$\pi_B^l = \frac{k_H^2}{4(k_H + c)} - L^l.$$

$$L^l = \frac{B}{4(k_H + c)(k_L^2 - 4k_H k_L - 4ck_L - 4ck_H - 4c^2)^2},$$

where

$$B = k_H^2 k_L^4 - 12k_H^3 k_L^3 - 28ck_H^2 k_L^3 - 32c^2 k_H k_L^3 - 16c^3 k_L^3 + 16k_H^4 k_L^2$$
$$+ 20ck_H^3 k_L^2 - 12c^2 k_H^2 k_L^2 - 32c^3 k_H k_L^2 - 16c^4 k_L^2 + 32ck_H^4 k_L + 64c^2 k_H^3 k_L$$
$$+ 32c^3 k_H^2 k_L + 16c^2 k_H^4 + 32c^3 k_H^3 + 16c^4 k_H^2.$$

5. Entry with license case

5.1. General model

Suppose that Firm A enters the market of Country B and at the same time licenses its technology for producing the high-quality good to Firm B at a fixed license fee. Then, Firm B gives up the low quality good, and produces the high quality good paying the license fee. Denote the license fee by L. Suppose that the licensor can take all of the increase in the profit of Firm B due to adoption of the new high-quality good. Both firms produce the high-quality good.

Let θ_H be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then

$$\theta_H = \frac{p_H}{k_H}.$$

The direct demand function is

$$q_H = 1 - F\left(\frac{p_H}{k_H}\right). \tag{4}$$

q_H denotes the supply of the good of quality k_H in the market. We have $0 < q_H < 1$, and

$$q_H = q_A + q_B.$$

From equation (4) the inverse demand function is

$$p_H = k_H G(1 - q_A - q_B).$$

Since $0 < G(1 - q_B) < 1$, we have $0 < p_H < k_H$.

The profits of Firm A and B are

$$\pi_A = k_H G(1 - q_A - q_B)q_A - c(q_A),$$
$$\pi_B = k_H G(1 - q_A - q_B)q_B - c(q_B) - L.$$

The first order conditions for profit maximization of Firm A and B are

$$\frac{\partial \pi_A}{\partial q_A} = k_H G(1 - q_A - q_B) - k_H G'(1 - q_A - q_B)q_A - c' = 0,$$

$$\frac{\partial \pi_B}{\partial q_B} = k_H G(1 - q_A - q_B) - k_H G'(1 - q_A - q_B)q_B - c' = 0.$$

The second order conditions are

$$\frac{\partial^2 \pi_A}{\partial q_A^2} = -k_H[2G'(1 - q_A - q_B) - G''(1 - q_A - q_B)q_A] - c'' < 0,$$

$$\frac{\partial^2 \pi_B}{\partial q_B^2} = -k_H[2G'(1 - q_A - q_B) - G''(1 - q_A - q_B)q_B] - c'' < 0.$$

Denote the equilibrium values of the outputs of Firm A and B, the price and the profits of the firms by q_A^{el}, q_B^{el}, p_H^{el}, π_A^{el}, and π_B^{el}. Similarly to the previous section, if the negotiation between the foreign firm and the domestic firm about the license fee breaks down, the foreign firm can enter the market of Country B without license. Therefore, the domestic firm must pay the difference between its profit excluding the license fee and its profit in the entry (without license) case. Thus, the license fee is equal to

$$(\pi_B^{el} + L) - \pi_B^e.$$

Denote it by L^{el}. The payoff of the foreign innovating firm is the sum of the license fee and its profit as a firm in the duopoly. It is equal to

$$L^{el} + \pi_A^{el}.$$

5.2. Uniform distribution and linear cost function case

In the uniform distribution and linear cost function case the equilibrium values of the variables are

$$q_A^{el} = q_B^{el} = \frac{k_H - c}{3k_H}, \; p_H^{el} = \frac{k_H + 2c}{3},$$

$$\pi_A^{el} = \frac{(k_H - c)^2}{9k_H}, \; \pi_B^{el} = \frac{(k_H - c)^2}{9k_H} - L^{el}.$$

About the license fee there are the following two cases.

(1) When $k_L > \frac{2ck_H}{k_H + c}$ (or $k_H < \frac{ck_L}{2c - k_L}$),

$$L^{el} = \frac{(k_H - k_L)(2ck_H k_L^2 - k_H^2 k_L^2 - c^2 k_L^2 + 16k_H^3 k_L + 4ck_H^2 k_L + 16c^2 k_H k_L - 36c^2 k_H^2)}{9k_H k_L (4k_H - k_L)^2}.$$

$$L^{el} + \pi_A^{el} = \frac{C}{9k_H k_L (4k_H - k_L)^2},$$

where

$$C = 2k_H^2 k_L^3 - 4ck_H k_L^3 + 2c^2 k_L^3 - 25k_H^3 k_L^2 + 14ck_H^2 k_L^2 - 25c^2 k_H k_L^2 + 32k_H^4 k_L$$
$$-28ck_H^3 k_L + 68c^2 k_H^2 k_L - 36c^2 k_H^3.$$

(2) When $k_L \le \frac{2ck_H}{k_H + c}$ (or $k_H \ge \frac{ck_L}{2c - k_L}$), since $\pi_B^e = 0$,

$$L^{el} + \pi_A^{el} = \frac{2(k_H - c)^2}{9k_H}.$$

5.3. Uniform distribution and quadratic cost function case

In the uniform distribution and quadratic cost function case the equilibrium values of the variables are

$$q_A^{el} = q_B^{el} = \frac{k_H}{3k_H + 2c}, \; p_H^{el} = \frac{k_H(k_H + 2c)}{3k_H + 2c},$$

$$\pi_A^{el} = \frac{k_H^2(k_H + c)}{(3k_H + 2c)^2}, \; \pi_B^{el} = \frac{k_H^2(k_H + c)}{(3k_H + 2c)^2} - L^{el},$$

$$L^{el} = \frac{D}{(3k_H + 2c)^2 (k_L^2 - 4k_H k_L - 4ck_L - 4ck_H - 4c^2)^2},$$

and

$$L^{el} + \pi_A^e = \frac{E}{(3k_H + 2c)^2(k_L^2 - 4k_H k_L - 4ck_L - 4ck_H - 4c^2)^2},$$

where

$$\begin{aligned}D = (k_H - k_L)(k_H k_L + ck_L + ck_H)(16k_H^3 k_L - k_H^2 k_L^2 + 48ck_H^2 k_L + 48c^2 k_H k_L \\ + 16c^3 k_L + 16ck_H^3 + 48c^2 k_H^2 + 48c^3 k_H + 16c^4),\end{aligned}$$

and

$$\begin{aligned}E = & \ 2k_H^3 k_L^4 + 2ck_H^2 k_L^4 - 25k_H^4 k_L^3 - 80ck_H^3 k_L^3 - 104c^2 k_H^2 k_L^3 - 64c^3 k_H k_L^3 \\ & - 16c^4 k_L^3 + 32k_H^5 k_L^2 + 71ck_H^4 k_L^2 + 16c^2 k_H^3 k_L^2 - 72c^3 k_H^2 k_L^2 - 64c^4 k_H k_L^2 \\ & - 16c^5 k_L^2 + 64ck_H^5 k_L + 192c^2 k_H^4 k_L + 192c^3 k_H^3 k_L + 64c^4 k_H^2 k_L + 32c^2 k_H^5 \\ & + 96c^3 k_H^4 + 96c^4 k_H^3 + 32c^5 k_H^2.\end{aligned}$$

6. The optimum strategy for the foreign innovator

To obtain the optimum strategy for the foreign innovating firm let us compare the profit of Firm A when it licenses its technology to Firm B without entry and its profit when it enters the market of Country B with license to Firm B. The difference between them is

$$L^l - (L^{el} + \pi_A^{el}).$$

Similarly, the difference between its profit when it licenses its technology to Firm B without entry and its profit when it enters the market of Country B without license, and the difference between its profit when it enters the market of Country B with license to Firm B and its profit when it enters the market of Country B without license are

$$L^l - \pi_A^e,$$

and

$$(L^{el} + \pi_A^{el}) - \pi_A^e.$$

For example, if both of $L^l - (L^{el} + \pi_A^{el})$ and $L^l + \pi_A^e$ are positive, license only (without entry) strategy is the optimum strategy.

We consider two specific cases.

6.1. Uniform distribution and linear cost function case

There are two cases.

(1) When $k_L > \frac{2ck_H}{k_H + c}$ (or $k_H < \frac{ck_L}{2c - k_L}$), we have

$$L^l - (L^{el} + \pi_A^{el}) = \frac{(k_H - c)^2}{36k_H} > 0.$$

Thus, license only strategy is more beneficial than entry with license strategy for the foreign innovating firm. Also we have

$$L^l - \pi_A^e = \frac{[k_L(k_H + c) - 2ck_H](4k_H^2 k_L - 10ck_H k_L + 8ck_H^2 - 3k_H k_L^2 + ck_L^2)}{4k_H k_L(4k_H - k_L)^2}.$$

The denominator is positive, and $k_L(k_H + c) - 2ck_H > 0$. Let

$$\lambda = 4k_H^2 k_L - 10ck_H k_L + 8ck_H^2 - 3k_H k_L^2 + ck_L^2.$$

When $k_H = k_L$, $\lambda = k_L^3 - ck_L^2 > 0$ and

$$\frac{d\lambda}{dk_H} = 8k_H k_L - 10ck_L + 16ck_H - 3k_L^2 > 0.$$

Since $k_H > k_L$, we have $L^l > \pi_A^e$. Therefore, license only strategy is the optimum strategy for the foreign innovating firm.

(2) When $k_L \leq \frac{2ck_H}{k_H + c}$ (or $k_H \geq \frac{ck_L}{2c - k_L}$), we have

$$L^l - (L^{el} + \pi_A^{el}) = \frac{(k_H - c)^2}{36k_H} > 0, L^l - \pi_A^e = 0.$$

Thus, both of license only strategy and entry without license strategy are the optimum strategies for the foreign innovating firm.

Summarizing the results;

Proposition 1. In the uniform distribution and linear cost function case;
1. If k_L is sufficiently large, the license only (without entry) strategy is the optimum strategy for the foreign innovating firm.
2. If k_L is small, both of the license only strategy and the entry only (without license) strategy are the optimum strategies.

6.2. Uniform distribution and quadratic cost function case

In the uniform distribution and quadratic cost function case we have

$$L^l - (L^{el} + \pi_A^{el}) = \frac{k_H^2(k_H^2 - 4ck_H - 4c^2)}{4(k_H + c)(3k_H + 2c)^2}.$$

When $k_H > 2(\sqrt{2} + 1)c$, it is positive; and when $k_H < 2(\sqrt{2} + 1)c$, it is negative. Thus, when k_H is relatively large to c, license only strategy is more beneficial than license with entry strategy for the foreign innovating firm; when k_H is small, entry with license strategy is more beneficial.

Also we have

$$L^l - \pi_A^e = \frac{k_L^2(k_H + 2c)\varphi}{4(k_H + c)(4k_H k_L + 4ck_L + 4ck_H + 4c^2 - k_L^2)^2},$$

$$(L^{el} + \pi_A^{el}) - \pi_A^e = \frac{(k_H - k_L)(k_H k_L + ck_L + ck_H)\psi}{(3k_H - 2c)^2(k_L^2 + 4k_H k_L + 4ck_L + 4ck_H + 4c^2)^2},$$

where

$$\varphi = 4k_H^2 k_L - 3k_H k_L^2 - 2ck_L^2 - 4ck_H k_L - 8c^2 k_L + 4ck_H^2 - 4c^2 k_H - 8c^3,$$

$$\psi = 7k_H^2 k_L^2 + 12ck_H k_L^2 + 4c^2 k_L^2 - 4k_H^3 k_L + 12ck_H^2 k_L + 32c^2 k_H k_L + 16c^3 k_L - 4ck_H^3$$
$$+ 12c^2 k_H^2 + 32c^3 k_H + 16c^4.$$

Since $k_H > k_L$, we see that $k_L^2(k_H + 2c) > 0$, $(k_H - k_L)(k_H k_L + ck_L + ck_H) > 0$ and the denominators of them are positive. About φ and ψ we find

$$k_H \varphi + \psi = 2(k_H + 2c)(2k_H k_L^2 + ck_L^2 + 4ck_H k_L + 4c^2 k_L + 4c^2 k_H + 4c^3) > 0.$$

Thus, at least one of φ and ψ is positive, and hence at least one of $L^l - \pi_A^e$ and $(L^{el} + \pi_A^{el}) - \pi_A^e$ is positive. Therefore, entry without license strategy never be the optimum strategy.

Summarizing the results;

Proposition 2. In the uniform distribution and quadratic cost function case;
1. If k_H is sufficiently large (or c is small), license only strategy is the optimum strategies

for the foreign innovating firm.

2. If k_H is small, entry with license strategy is the optimum strategy.

The reason for the difference between proposition 1 and 2 is as follows. The profit of the foreign innovating firm under license only strategy is

$$\pi_B^l + L^l - \pi_B^e.$$

And the profit of the foreign innovating firm under entry with license strategy is

$$\pi_A^{el} + \pi_B^{el} + L^{el} - \pi_B^e.$$

So the difference between two strategies is

$$\pi_B^l + L^l - \pi_A^{el} + \pi_B^{el} + L^{el}.$$

$\pi_B^l + L^l$ is monopoly profit with the high-quality good and $\pi_A^{el} + \pi_B^{el} + L^{el}$ is total profit under duopoly where both firms produce the high-quality good. Therefore, when monopoly profit is larger (smaller) than total profit under duopoly, license only strategy is more profitable (unprofitable) than entry with license strategy.

In the linear cost functions case, monopoly profit always bigger than total profit under duopoly so the foreign innovating firm uses license only strategy. If the domestic firm is weak and drops out when the foreign innovating firm enters the domestic market, license only strategy and entry only strategy become optimal (proposition 1-2). The foreign innovating firm always gets monopoly profit.

On the other hand, in the quadratic cost functions case, total profit under duopoly may become larger than monopoly profit so the foreign innovating firm uses entry with license strategy (proposition 2-2). In this case, marginal cost becomes large as production becomes large, so the foreign innovating firm enters the domestic market and produce with small marginal cost although the market becomes more competitive. But when c is small (or k_H is large), the result approaches to the result under linear cost functions, so license only strategy is optimal like proposition 1.

7. A note on extension to endogenous quality model

Nguyen (2014) and Nguyen, Sgro and Nabin (2014) presented models of vertical product differentiation with endogenous quality choice by firms. In particular, Nguyen, Sgro and Nabin (2014) analyzed a licensing problem in a duopoly with a foreign innovating firm which has a new technology to produce a higher quality good.

Our analysis can be extended to a case of endogenous quality choice. For example, let us consider the entry without license case. Assume uniform distribution of consumers' taste parameter. Suppose that Firm A produces a good of quality k_A, and Firm B produces a good of quality k_B. $k_A > k_B$. Let θ_B be the value of θ for which the corresponding consumer is indifferent between buying nothing and buying Firm B's good. Then, $\theta_B = p_B/k_B$. Let θ_A be the value of θ for which the corresponding consumer is indifferent between buying Firm A's good and buying Firm B's good. Then, $\theta_B = (p_A - p_B)/(k_A - k_B)$. p_A and p_B are the prices of the goods. Assume that the constant marginal cost of Firm A is $0.5k_A^2$, and that of Firm B is k_B^2. Let q_A and q_B be the outputs of Firm A and B. Then, the inverse demand functions are written as follows.

$$p_A = (k_A - k_B)(1 - q_A) + k_B(1 - q_A - q_B), \quad p_B = k_B(1 - q_A - q_B).$$

The profits of Firm A and B are

$$\pi_A = [(k_A - k_B)(1 - q_A) + k_B(1 - q_A - q_B)]q_A - \frac{1}{2}k_A^2 q_A, \quad \pi_B = k_B(1 - q_A - q_B)q_B - k_B^2 q_B.$$

The first order conditions for profit maximization of Firm A and B with respect to the outputs are

$$(k_A - k_B)(1 - q_A) + k_B(1 - q_A - q_B) - k_A q_A - \frac{1}{2}k_A^2 = 0,$$

and

$$k_B(1 - q_A - q_B) - k_B q_B - k_B^2 = 0.$$

The equilibrium outputs of the goods and the profits of the firms are

$$q_A = \frac{2k_A - k_B - k_A^2 + k_B^2}{4k_A - k_B}, q_B = \frac{k_A(2 + k_A - 4k_B)}{2(4k_A - k_B)},$$

$$\pi_A = \frac{k_A(2k_A - k_B - k_A^2 + k_B^2)^2}{(4k_A - k_B)^2}, \pi_B = \frac{k_A^2 k_B(2 + k_A - 4k_B)^2}{4(4k_A - k_B)^2}.$$

Given these equilibrium profits the firms choose their quality. The conditions for profit maximization with respect to the quality are

$$k_B^3 + 4k_A k_B^2 - k_B^2 - 5k_A^2 k_B + 2k_A k_B + 12k_A^3 - 8k_A^2 = 0,$$

for Firm A, and

$$4k_B^2 - 47k_A k_B + 2k_B + 4k_A^2 + 8k_A = 0,$$

for Firm B. The firms choose the following quality

$$k_A \approx 0.6882, k_B \approx 0.2523.$$

The full analyses using the model of endogenous quality choice are future challenges.

8. Concluding remark

We have analyzed a choice of the foreign innovating firm to license its technology for producing the high quality good to a domestic firm or to enter the market, which is monopolistic, of the domestic country, with or without license. We have shown that the relative benefit of license and entry depends on the form of cost functions.

In the future research we want to study the problem in oligopoly, and public policy by the government of the domestic country to promote or prevent license or entry by the foreign firm. As we mentioned in the previous section, our analysis can be extended to a case of endogenous quality choice[1].
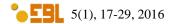
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[1] In the model of Nguyen, Sgro and Nabin (2014) the quality of the good produced by the new superior technology is fixed to be 1, and a firm, which does not buy a license, chooses the quality of its good between 0 and 1.

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