Second Best Optimal Taxation in the Uzawa-Lucas Model with Externality in Human Capital*

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Abstract

In this paper we look for the second best optimal policy in the Uzawa-Lucas model with externality in human capital and labor-leisure choice. We study a setup where lump sum taxes are not available. Given that the authorities should aim at increasing time spent studying, we explore which instruments can help government conduct the economy to the highest possible welfare. Our results suggest that both taxes on capital and labor income should be used as instruments to raise revenues for financing the education subsidy. Welfare losses due to different tax policies are illustrated by means of numerical simulations.

Keywords: optimal policy, two-sector model, endogenous growth, indeterminacy;
JEL classification: O41, E62, H31

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1 Introduction

In this paper we look for the second best optimal policy in the Uzawa-Lucas model with externality in human capital, with labor-leisure choice. We study the setup where lump sum taxes are not available and explore which combination of fiscal instruments can help government to conduct the economy to the highest possible welfare.

In the setup that builds on Uzawa (1965) and Lucas (1988), Gomez (2003) shows that lump sum taxation can be avoided and that the optimal solution can be achieved by taxing labor income and subsidizing investment into human capital. Gorostiaga et al. (2013) include leisure-labor decisions into the model of Gomez (2003). They show that obtaining revenues through lump sum taxes and using them to subsidize forgone earnings while studying leads to optimal solution. However, in the absence of lump sum taxes, revenues for the subsidy will have to be raised by some other means. Seminal contributions of Judd (1985) or Chamley (1986) recommend not to tax capital in the long run. However, it may not be valid in all scenarios, see Conesa et al. (2009). We explore here what should be the optimal combination of labor and capital income taxes to reach the second best solution.

Equilibrium dynamics in the Uzawa-Lucas model with leisure is very rich. It has been studied for example by Benhabib and Perli (1994) and Ladrón-de-Guevara et al. (1997 and 1999). Due to the aggregate increasing returns in the production technology and leisure in the utility function, there might exist a unique trajectory or arise a continuum of transitions towards a unique or multiple balanced growth paths. Gorostiaga et al. (2013) analyze the dynamics in the present model for the centralized economy.

In the present work we will pay attention to the balance growth path analysis and the transitional dynamics for a particular parametrization. We develop analytical results for Cobb-Douglass production function and logarithmic utility. In general, there might exist a parameter space where multiple balanced growth paths occur. Nevertheless, a part of our analysis is restricted to the case of total depreciation of both capitals, a model which exhibits unique equilibrium.

We evaluate welfare under the social planner solution and under different combinations of capital and labor income tax rates. Subsidy rate then depends on the obtained revenues, as the government always keeps its budget balanced. We compute the welfare loss of alternative policies with respect to the social planner solution measured in percentage of the preferred mix of consumption and leisure goods.

Our results suggest that it is optimal to use both available fiscal instruments: capital income taxation and labor income taxation. Positive tax on capital income distorts capital accumulation and may harm future consumption. However, welfare loss due to lower consumption can be compensated by higher leisure, given that in the second best environment time devoted to leisure is higher than the one in the centralized solution.[1] Our sensitivity analysis shows how the second

[1] In the social planner solution, working time is lower than in the decentralized case with no intervention and

best optimal policy tax ratio, labor income tax rate/capital income tax rate, varies over different sets of parameters.

The remainder of the paper is organized as follows. The model economy, competitive and social planner equilibria are outlined in section 2. Analytical results are developed in section 3. Second best policy and welfare is discussed in section 4. Final conclusions are stated in section 5.

2 The Model Economy

We consider a model economy that extends Uzawa (1965) and Lucas (1988) to include leisure-labor decisions. There are two production sectors: the final goods sector and the education sector that produces new human capital. The economy is populated by identical and infinitely lived agents. Total population is assumed to be constant and normalized to one. Households have initial endowments of physical and human capital, k_0 and h_0, respectively. Agents also have an endowment of one unit of time at each period t that they allocate to the production of the final good, u_t, leisure activities, l_t, or human capital accumulation, $1 - u_t - l_t$.

2.1 Final Goods Sector

The final goods sector produces a commodity that can be consumed or accumulated as physical capital. The technology in this sector combines physical capital, k_t, and efficiency units of labor, $u_t h_t$, and is described through the Cobb-Douglas production function

$$y_t = A k_t^{\alpha} h_t^{1-\alpha} u_t^{1-\alpha} h_{a_t}^{\gamma} \tag{1}$$

where A is the technology parameter, $0 < \alpha < 1$ is the share of physical capital in output, h_{a_t} is the average human capital stock and the term $h_{a_t}^{\gamma}$ captures the external effect of average human capital in the production of goods. Note that the parameter γ measures the degree of the externality and also the degree of increasing returns to scale at the social level.

Firms maximize profits taking prices and the average stock of human capital as given. Inputs' demands are such that

$$r_t = \alpha A k_t^{\alpha-1} h_t^{1-\alpha} u_t^{1-\alpha} h_{a_t}^{\gamma} = \alpha \frac{y_t}{k_t} \tag{2}$$

and

$$w_t = (1-\alpha) A k_t^{\alpha} h_t^{-\alpha} u_t^{-\alpha} h_{a_t}^{\gamma} = (1-\alpha) \frac{y_t}{h_t u_t} \tag{3}$$

where r_t is the return on capital and w_t is the wage per efficiency unit of labor.

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the studying time is higher.

2.2 Education Sector

The schooling sector produces human capital services. Human capital accumulation depends on the time spent studying $1 - u_t - l_t$ and on the level of human capital h_t according to

$$h_{t+1} = \phi (1 - u_t - l_t) h_t + (1 - \delta_h) h_t \tag{4}$$

where ϕ is a measure of productivity in the education sector and δ_h is the depreciation rate of human capital.

2.3 Households

Households derive utility from consumption, c_t, and leisure, l_t. Lifetime welfare is characterized by the utility function

$$\sum_{t=0}^{\infty} \beta^t (\ln c_t + b \ln l_t) \tag{5}$$

where β is the discount factor and b is the preference parameter on leisure.[2]

Households pay proportional labor income taxes at rate τ_{w_t} and capital income taxes at rate τ_{r_t}. The government subsidizes the investment in human capital funding a fraction s_t of wage income that is foregone while studying. The budget constraint that agents face at t can be written as

$$c_t + k_{t+1} - (1 - \delta_k) k_t \leq (1 - \tau_{r_t}) r_t k_t + (1 - \tau_{w_t}) w_t h_t u_t + s_t w_t (1 - u_t - l_t) h_t \tag{6}$$

where δ_k is the rate of depreciation of physical capital.

The problem of the representative agent is to maximize the lifetime utility (5), subject to the budget constraint (6), and the condition for the accumulation of human capital (4) and taking as given prices, policies and initial values for physical and human capital. Let η_t and ε_t be the non-negative Lagrange multipliers associated with the budget constraint (6) and the condition for the accumulation of human capital (4), respectively. The first order necessary conditions on

[2] As commented in Lucas (2003), the logarithmic utility is employed by many researchers, because the estimates of the elasticity of intertemporal substitution are close to unity. As shown in Benhabib and Perli (1994), the balanced growth path with positive growth and constant supply of labor hours exists for this utility function.

consumption, labor, leisure, physical and human capitals, respectively, are

$$\frac{1}{c_t} = \eta_t, \tag{7}$$

$$\eta_t \left(1 - \tau_{w_t} - s_t\right) w_t h_t = \phi \varepsilon_t h_t, \tag{8}$$

$$\frac{b}{l_t} = \eta_t s_t w_t h_t + \phi \varepsilon_t h_t, \tag{9}$$

$$\eta_t = \beta \eta_{t+1} \left[\left(1 - \tau_{r_{t+1}}\right) r_{t+1} + 1 - \delta_k\right], \tag{10}$$

$$\begin{aligned} \varepsilon_t = {} & \beta \eta_{t+1} \left[\left(1 - \tau_{w_{t+1}}\right) w_{t+1} u_{t+1} + s_{t+1} w_{t+1} \left(1 - u_{t+1} - l_{t+1}\right)\right] + \\ & + \beta \varepsilon_{t+1} \left[\phi \left(1 - u_{t+1} - l_{t+1}\right) + 1 - \delta_h\right] \end{aligned} \tag{11}$$

and transversality conditions are

$$\lim_{t \to \infty} \beta^t \eta_t k_{t+1} = 0, \tag{12}$$

$$\lim_{t \to \infty} \beta^t \varepsilon_t h_{t+1} = 0. \tag{13}$$

The Lagrange multipliers associated with the budget constraint and the constraint on human capital accumulation, η_t and ε_t, can be interpreted as the marginal utility of wealth and the shadow price of human capital, respectively. The first order condition on consumption (7) indicates the marginal utility of wealth. The first order conditions on working and leisure time, (8) and (9), determine the optimal allocation of time among the three activities, working, studying and leisure. The first order conditions on physical capital (10) and human capital (11) embody the costs and profits associated with investing one marginal unit of wealth in either capital.

Plugging (7) and (8) into (9) and (10), we can easily get the usual intertemporal and intratemporal first order conditions

$$\frac{c_{t+1}}{c_t} = \beta \left[\left(1 - \tau_{r_{t+1}}\right) r_{t+1} + 1 - \delta_k\right], \tag{14}$$

$$\frac{c_t}{l_t} = \frac{\left(1 - \tau_{w_t}\right)}{b} w_t h_t. \tag{15}$$

Note that the capital income tax distorts intertemporal consumption decisions and the labor income tax distorts consumption-leisure decisions.

2.4 Government

Fiscal policy targets human capital accumulation. The government taxes capital and labor incomes and subsidizes the investment into human capital. Wages lost while studying are subsidized by a flat rate. The government's budget constraint is

$$\tau_{r_t} r_t k_t + \tau_{w_t} w_t h_t u_t = s_t w_t \left(1 - u_t - l_t\right) h_t. \tag{16}$$

2.5 Competitive Equilibrium

Definition: *Given initial conditions k_0 and h_0 and a tax policy $\{\tau_{r_t}, \tau_{w_t}, s_t\}_{t=0}^{\infty}$, allocations $\left\{c_t^{CE}, k_{t+1}^{CE}, l_t^{CE}, u_t^{CE}, h_{t+1}^{CE}, h_{a_t}^{CE}, y_t^{CE}\right\}_{t=0}^{\infty}$ and prices $\left\{r_t^{CE}, w_t^{CE}\right\}_{t=0}^{\infty}$ constitute a competitive equilibrium if the following conditions are satisfied:*

(i) Given prices $\left\{r_t^{CE}, w_t^{CE}\right\}_{t=0}^{\infty}$ and policies $\{\tau_{r_t},\ \tau_{w_t},\ s_t\}_{t=0}^{\infty}$, allocations $\left\{c_t^{CE}, k_{t+1}^{CE}, l_t^{CE}, u_t^{CE}, h_{t+1}^{CE}, y_t^{CE}\right\}_{t=0}^{\infty}$ solve the household's problem.

(ii) Given prices $\left\{r_t^{CE}, w_t^{CE}\right\}_{t=0}^{\infty}$ and the average human capital $\left\{h_{a_t}^{CE}\right\}_{t=0}^{\infty}$, allocations $\left\{k_{t+1}^{CE}, u_t^{CE}, h_{t+1}^{CE}\right\}_{t=0}^{\infty}$ solve the firm's problem.

(iii) The average human capital $h_{a_t}^{CE}$ is equal to h_t^{CE} at each period t.

(iv) The government budget is balanced in every period.

(v) All markets clear.

Therefore, when we consider a decentralized economy, equilibrium allocations, prices and policies have to satisfy equation (4) and equations (6)-(13) from the household problem, equations (1)-(3) from the firm's problem, the government budget constraint (16) and an additional expression

$$h_{a_t} = h_t.$$

2.6 Social Planner Problem

In this section we present the centralized economy. We assume that a social planner who internalizes the externality of human capital allocates resources and time so as to maximize lifetime utility. There will be two constraints in the planner's problem: one characterizing the human capital production technology, equation (4), and the so called resource constraint,

$$c_t + k_{t+1} - (1 - \delta_k) k_t \leq A k_t^{\alpha} h_t^{1-\alpha+\gamma} u_t^{1-\alpha}. \tag{17}$$

The necessary conditions for the first best allocation $\{c_t, k_{t+1}, l_t, u_t, h_{t+1}\}$ are the transversality conditions, equations (12) and (13), and the following set of equations

$$\frac{1}{c_t} = \eta_t^{SP}, \tag{18}$$

$$\eta_t^{SP}(1-\alpha)\frac{y_t}{u_t} = \phi\varepsilon_t^{SP}h_t, \tag{19}$$

$$\frac{b}{l_t} = \phi\varepsilon_t^{SP}h_t, \tag{20}$$

$$\eta_t^{SP} = \beta\eta_{t+1}^{SP}\left(\alpha\frac{y_t}{k_t}+1-\delta_k\right), \tag{21}$$

$$\varepsilon_t^{SP} = \beta\eta_{t+1}^{SP}\left[(1-\alpha+\gamma)\frac{y_{t+1}}{h_{t+1}}\right] + +\beta\varepsilon_{t+1}^{SP}\left[\phi\left(1-u_{t+1}-l_{t+1}\right)+1-\delta_h\right]. \tag{22}$$

where ε_t^{SP} and η_t^{SP} are the Lagrange multipliers associated with constraints (4) and (17), respectively.[3]

Since the planner takes into account the impact of average human capital in the production technology, she will find it optimal to devote more time to schooling than in a competitive equilibrium with no public intervention.

As in the competitive equilibrium case, we can substitute first order conditions (18) and (19) into equations (20) and (21) to get the intertemporal and intratemporal first order conditions of the planner's problem

$$\frac{c_{t+1}}{c_t} = \beta\left(\alpha\frac{y_{t+1}}{k_{t+1}}+1-\delta_k\right), \tag{23}$$

$$\frac{c_t}{l_t} = \frac{1-\alpha+\gamma}{b}\frac{y_t}{u_t}. \tag{24}$$

The two equations above show that in the first best allocation, there is no wedge neither between the intertemporal marginal rate of substitution in consumption and the gross return on capital nor between the marginal rate of substitution between leisure and consumption and the marginal product of labor. Any policy designed to decentralize the first best should not lead to any wedge between marginal rates of substitution and marginal rates of transformation.

[3] We place SP index on the Lagrange multipliers to highlight that they do not have to be the same as the ones in the decentralized problem. The same holds for the other series as the centralized and decentralized solution do not necesarilly coincide. However we do not index other series to simplify the notation.

We can see that there exists no optimal policy when tax rates are to be positive to raise revenues for the subsidy. It only exists if the subsidy can be financed by lump sums taxes, an economy analyzed in detail in Gorostiaga et al. (2013).

3 Analytical analysis

This section will serve us to get highlights of the results. We first develop long run results for the model. Then we constraint values of some parameters to obtain analytical results also in the transition.[4] We begin our long run analysis by transforming the model so it exhibits a steady state. For this purpose, both tax rates will have to be set constant.

3.1 Transformation of the variables

For the growth rate of the marginal utility of wealth to be constant, equation (21), output and capital must grow at the same rate, and it also means that the ratio, $\frac{k_t}{h_t^{\frac{1-\alpha+\gamma}{1-\alpha}}}$ must be constant as $t \to \infty$. Therefore, the growth rates of both capitals are related in the following way

$$\lim_{t\to\infty} \frac{k_{t+1}}{k_t} = \lim_{t\to\infty} \left(\frac{h_{t+1}}{h_t}\right)^{\frac{1-\alpha+\gamma}{1-\alpha}}. \tag{25}$$

Goods market equilibrium (17) implies that consumption must grow as output and physical capital

$$\lim_{t\to\infty} \frac{y_{t+1}}{y_t} = \lim_{t\to\infty} \frac{k_{t+1}}{k_t} = \lim_{t\to\infty} \frac{c_{t+1}}{c_t} = g^* \tag{26}$$

where g^* denotes the long run growth rate of the economy. The first order condition on consumption (7) implies that

$$\frac{c_{t+1}}{c_t} = \frac{\eta_t}{\eta_{t+1}} \text{ for all } t, \tag{27}$$

and the one on working time (9) implies that

$$\lim_{t\to\infty} \frac{\eta_t}{\eta_{t+1}} = \lim_{t\to\infty} \frac{\varepsilon_t}{\varepsilon_{t+1}} \left(\frac{h_{t+1}}{h_t}\right)^{\frac{\gamma}{1-\alpha}}. \tag{28}$$

Using (28) and the first order condition on human capital (11) we can obtain that the shadow price of human capital decreases at the same rate as the human capital increases

$$\lim_{t\to\infty} \frac{\varepsilon_t}{\varepsilon_{t+1}} = \lim_{t\to\infty} \frac{h_{t+1}}{h_t}. \tag{29}$$

We can define the ratio of both capitals that has a steady state as

$$x_t = \frac{k_t}{h_t^{\frac{1-\alpha+\gamma}{1-\alpha}}} \tag{30}$$

[4] Later on we will proceed by solving the Ramsey problem.

and the transformed consumption and output as

$$\hat{c}_t = \frac{c_t}{h_t^{\frac{1-\alpha+\gamma}{1-\alpha}}},\ \hat{y}_t = \frac{y_t}{h_t^{\frac{1-\alpha+\gamma}{1-\alpha}}}. \tag{31}$$

Transformation of other variables to stationary forms and the transformed equations that must hold in equilibrium are stated in the Appendix.

3.2 Decentralized Economy: Balanced Growth Path Behavior

In the following, we place asterisk to mark variables that remain constant in the long run.

Proposition 1 *On the balanced growth path, when the capital and labor income tax rates are constant,* $\lim_{t\to\infty} \tau_{r_t} = \tau_r^*$ *and* $\lim_{t\to\infty} \tau_{w_t} = \tau_w^*$*, human capital grows at the rate*

$$g_h^* = \beta \left[\phi \frac{1-\tau_w^*}{1-\tau_w^*-s^*} (1-l^*) + 1 - \delta_h \right]$$

where l^* *is the leisure time*

$$l^* = \frac{1}{\frac{\theta_\eta^*(1-\alpha)(1-\tau_w^*)}{b\left[1-\beta\frac{1-\tau_w^*}{1-\tau_w^*-s^*}+(1-\beta)\frac{1-\delta_h}{\phi}\right]} + \frac{1-\beta\frac{1-\tau_w^*}{1-\tau_w^*-s^*}}{1-\beta\frac{1-\tau_w^*}{1-\tau_w^*-s^*}+(1-\beta)\frac{1-\delta_h}{\phi}}}, \tag{32}$$

the working time is

$$u^* = \frac{1}{\frac{b\left(1-\beta\frac{1-\tau_w^*}{1-\tau_w^*-s^*}\right)}{\theta_\eta^*(1-\alpha)(1-\tau_w^*)\left[1-\beta\frac{1-\tau_w^*}{1-\tau_w^*-s^*}+(1-\beta)\frac{1-\delta_h}{\phi}\right]} + \frac{1}{1-\beta\frac{1-\tau_w^*}{1-\tau_w^*-s^*}+(1-\beta)\frac{1-\delta_h}{\phi}}}, \tag{33}$$

where

$$\theta_\eta^* = \lim_{t\to\infty} \eta_t y_t = \frac{1}{1-\alpha\beta\left(1-\tau_r^*\right)\frac{g^*-(1-\delta_k)}{g^*-\beta(1-\delta_k)}}, \tag{34}$$

and the long run growth rate of consumption, physical capital and output is

$$g^* = \left(g_h^*\right)^{\frac{1-\alpha+\gamma}{1-\alpha}}. \tag{35}$$

Ratio of next period physical capital to output becomes

$$\theta_k^* = \lim_{t\to\infty} \frac{k_{t+1}}{y_t} = \frac{\alpha\beta\left(1-\tau_r^*\right)g^*}{g^*-\beta\left(1-\delta_k\right)},$$

consumption to output is

$$\theta_c^* = \lim_{t\to\infty} \frac{c_t}{y_t} = 1-\alpha\beta\left(1-\tau_r^*\right)\frac{g^*-(1-\delta_k)}{g^*-\beta\left(1-\delta_k\right)},$$

the relationship between physical and human capital defined as x_t takes the value

$$x^* = \lim_{t\to\infty} \frac{k_t}{h_t^{\frac{1-\alpha+\gamma}{1-\alpha}}} = \left[\frac{\alpha\beta A\left(1-\tau_r^*\right)}{g^*-\beta\left(1-\delta_k\right)}\right]^{\frac{1}{1-\alpha}} u^* \tag{36}$$

and the subsidy rate is

$$s^* = \frac{\left[\tau_r^*\alpha + \tau_w^*\left(1-\alpha\right)\right]u^*}{\left(1-\alpha\right)\left(1-u^*-l^*\right)}.$$

Proof. See Appendix. ■

Thus we obtain the long run policy functions. Capital evolves as

$$k_{t+1} = \frac{\alpha\beta\left(1-\tau_r^*\right)g^*}{g^*-\beta\left(1-\delta_k\right)}y_t,$$

consumption as

$$c_t = \left[1-\alpha\beta\left(1-\tau_r^*\right)\frac{g^*-\left(1-\delta_k\right)}{g^*-\beta\left(1-\delta_k\right)}\right]y_t$$

and human capital according to (4) with leisure and working times given by (32) and (33), and the long run growth rate given by (35).

3.3 Social Planner Solution: Balanced Growth Path Behavior

Proposition 2 *On the balanced growth path, human capital grows at the rate*

$$g_h^* = \beta\left[\phi\left(1-l^*+\frac{\gamma}{1-\alpha}u^*\right)+1-\delta_h\right],$$

where l^* and u^* are the working and leisure times

$$l^* = \frac{1}{\frac{1}{\left(1+\frac{1-\delta_h}{\phi}\right)}+\frac{\theta_\eta^*\left(1-\alpha+\beta\gamma\right)}{b\left(1-\beta\right)\left(1+\frac{1-\delta_h}{\phi}\right)}}, \tag{37}$$

$$u^* = \frac{1}{\frac{b}{\theta_\eta^*\left(1-\alpha\right)\left(1+\frac{1-\delta_h}{\phi}\right)}+\frac{1-\alpha+\beta\gamma}{\left(1-\beta\right)\left(1-\alpha\right)\left(1+\frac{1-\delta_h}{\phi}\right)}} \tag{38}$$

where

$$\theta_\eta^* = \lim_{t\to\infty}\eta_t y_t = \frac{1}{1-\alpha\beta\frac{g^*-\left(1-\delta_k\right)}{g^*-\beta\left(1-\delta_k\right)}}, \tag{39}$$

consumption, real balances, physical capital and output grow at the rate

$$g^* = \left(g_h^*\right)^{\frac{1-\alpha+\gamma}{1-\alpha}}.$$

Ratio of next period physical capital to output becomes

$$\theta_k^* = \lim_{t\to\infty}\frac{k_{t+1}}{y_t} = \frac{\alpha\beta g^*}{g^*-\beta\left(1-\delta_k\right)},$$

consumption to output is

$$\theta_c^* = \lim_{t\to\infty} \frac{c_t}{y_t} = 1 - \alpha\beta \frac{g^* - (1-\delta_k)}{g^* - \beta(1-\delta_k)},$$

the ratio that contains the relationship between physical and human capitals x_t takes the value

$$x^* = \lim_{t\to\infty} \frac{k_t}{h_t^{\frac{1-\alpha+\gamma}{1-\alpha}}} = \left[\frac{\alpha\beta A}{g^* - \beta(1-\delta_k)}\right]^{\frac{1}{1-\alpha}} u^*. \tag{40}$$

Proof. We need to follow analogous steps as in the proof of the Proposition 1 setting all tax and subsidy rates to zero. ■

3.4 Transitional dynamics

Now let us assume that both capitals totally depreciate, $\delta_k = \delta_h = 1$. In that case the fraction of time allocated to each of the three activities -leisure, working and studying- remains constant over time. The previous long run results will hold also for the transition.

For the decentralized solution we keep the tax rates constant in all periods, $\tau_{r_t} = \tau_r$, $\tau_{w_t} = \tau_w$.

Proposition 3 *When the capital and labor income tax rates are kept constant, $\tau_{r_t} = \tau_r$, $\tau_{w_t} = \tau_w$ for all t, and $\delta_k = \delta_h = 1$, time allocation between leisure, working and studying is*

$$l_t = l = \frac{b[1-\alpha\beta(1-\tau_r)]\left(1-\beta\frac{1-\tau_w}{1-\tau_w-s}\right)}{b[1-\alpha\beta(1-\tau_r)]\left(1-\beta\frac{1-\tau_w}{1-\tau_w-s}\right)+(1-\alpha)(1-\tau_w)}, \tag{41}$$

$$u_t = u = \frac{(1-\alpha)(1-\tau_w)\left(1-\beta\frac{1-\tau_w}{1-\tau_w-s}\right)}{b[1-\alpha\beta(1-\tau_r)]\left(1-\beta\frac{1-\tau_w}{1-\tau_w-s}\right)+(1-\alpha)(1-\tau_w)}, \tag{42}$$

$$1 - u_t - l_t = 1 - u - l = \frac{(1-\alpha)(1-\tau_w)\beta\frac{1-\tau_w}{1-\tau_w-s}}{b[1-\alpha\beta(1-\tau_r)]\left(1-\beta\frac{1-\tau_w}{1-\tau_w-s}\right)+(1-\alpha)(1-\tau_w)}. \tag{43}$$

Policy functions for consumption and both capitals accumulation are

$$c_t = [1-\alpha\beta(1-\tau_r)]\, y_t = [1-\alpha\beta(1-\tau_r)]\, A k_t^\alpha h_t^{1-\alpha+\gamma} u^{1-\alpha}, \tag{44}$$

$$k_{t+1} = \alpha\beta(1-\tau_r)\, y_t = \alpha\beta(1-\tau_r)\, A k_t^\alpha h_t^{1-\alpha+\gamma} u^{1-\alpha}, \tag{45}$$

$$h_{t+1} = \phi(1-u-l)\, h_t, \tag{46}$$

and the subsidy rate is constant

$$s_t = s = \frac{[\tau_r \alpha + \tau_w(1-\alpha)]\, u}{(1-\alpha)(1-u-l)}.$$

Proof. Setting $\delta_k = \delta_h = 1$ and $\tau_{r_t} = \tau_r$, $\tau_{w_t} = \tau_w$ we can verify that the policy functions (44)-(46) and time allocations to leisure, working and studying (41)-(43) are compatible with equilibrium equations (1)-(4), (6), (7)-(11). Tax rate is obtained from the government budget constraint, (16). ■

For the centralized solution, we get the following results.

Proposition 4 *When $\delta_k = \delta_h = 1$, time allocation between leisure, working and studying does not change over time*

$$l_t = l = \frac{b\,(1-\alpha\beta)\,(1-\beta)}{b\,(1-\alpha\beta)\,(1-\beta) + (1-\alpha+\beta\gamma)}, \tag{47}$$

$$u_t = u = \frac{(1-\alpha)\,(1-\beta)}{b\,(1-\alpha\beta)\,(1-\beta) + (1-\alpha+\beta\gamma)}, \tag{48}$$

$$1 - u_t - l_t = 1 - u - l = \frac{(1-\alpha+\beta\gamma) - (1-\alpha)\,(1-\beta)}{b\,(1-\alpha\beta)\,(1-\beta) + (1-\alpha+\beta\gamma)}. \tag{49}$$

Policy functions for consumption and both capitals accumulation are

$$\begin{aligned} c_t &= (1-\alpha\beta)\,y_t = (1-\alpha\beta)\,Ak_t^{\alpha}h_t^{1-\alpha+\gamma}u^{1-\alpha}, && (50)\\ k_{t+1} &= \alpha\beta y_t = \alpha\beta Ak_t^{\alpha}h_t^{1-\alpha+\gamma}u^{1-\alpha}, && (51)\\ h_{t+1} &= \phi\,(1-u-l)\,h_t, && (52) \end{aligned}$$

Proof. As before we verify that the solutions are compatible with equilibrium equations. ■

4 Welfare maximizing policy

We look for the second best optimal policy. In this case it means looking for the combination of capital and income tax rates that lead to minimum welfare loss with respect to the optimal, social planner solution. The measure of welfare differences used is analogous to the one generally employed in the literature, as defined for example in Lucas (2003)

$$U\left[(1+\sigma)\,q^{A}\right] = U\left(q^{REF}\right)$$

where q contains variables that agents derive utility from. In our case it is a mixture of consumption and leisure given by the utility function. Number σ, in units of a percentage of the preferred mix of goods, gives us welfare gain or loss of following a policy A with respect to a reference case REF. Given that our reference policy is the social planner solution, $REF \equiv SP$, and using our utility specification we look for σ that satisfies the equality

$$\sum_{t=0}^{\infty}\beta^{t}\ln\left[(1+\sigma)\left(c_t l_t^{b}\right)^{A}\right] = \sum_{t=0}^{\infty}\beta^{t}\ln\left[\left(c_t l_t^{b}\right)^{SP}\right].$$

4.1 Second Best Policy for Total Depreciation of Both Capitals

4.1.1 Numerical Procedure

Note that working and leisure times are functions of the subsidy rate, $u = u(s)$ and $l = l(s)$, equations (41) and (42). Therefore, to find the time allocation between the three activities we have to proceed by a numerical solution. For a given combination of taxes, (τ_r, τ_w), we look for the subsidy rate, s, that fulfils

$$s = \frac{\alpha\tau_r + (1-\alpha)\tau_w}{(1-\alpha)} \frac{u(s)}{1 - u(s) - l(s)}.$$

4.1.2 Parametrization

To look for the second best policy we have to assign values to the parameters of our model. We work with the case where $\delta_k = 1$ and $\delta_h = 1$. We set the long run growth rate of the economy to $g^* = 1.015$. Externality measure is set to $\gamma = 0.3$, preference parameter on leisure to $b = 1$. Technology parameter is chosen to be $A = 1$. Discount factor is $\beta = 0.9$, share of physical capital in output $\alpha = 0.4$ and the efficiency of learning is the one that delivers the desired long run growth rate in the centralized solution, $\phi = 1.184$. This parameter setting results in the following optimal time allocation: 6% of time is devoted to working, 7% to leisure and 87% to studying. We use as initial condition $k_0 = 1$ and $h_0 = 1$.[5]

4.1.3 Second best optimal policy

We calculate welfare loss for different combinations of capital and labor income tax rates with respect to the social planner solution, setting the number of periods to 9000. We obtain the welfare loss evaluating σ from the expression

$$1 + \sigma = \exp\left\{ \frac{\sum_{t=0}^{T} \beta^t \left[\ln c_t^{SP} + b \ln l_t^{SP}\right] - \sum_{t=0}^{T} \beta^t \left[\ln c_t^{A} + b \ln l_t^{A}\right]}{\sum_{t=0}^{T} \beta^t} \right\}$$

where $T = 9000$. The general results are illustrated in Figure 1 (in the vertical axe we plot the inverse of welfare loss, $\frac{1}{\sigma}$). The 'top of the hill' represents the second best optimal policy (welfare loss σ is the lowest, thus the ratio $\frac{1}{\sigma}$ is the highest).

[Figure 1 around here]

We can see that setting any value for the capital income tax rate, we will be able to identify the labor income tax rate that minimizes the welfare loss (points on the 'ridge' in Figure 1). We define such a labor income tax rate as τ_w^{2nd}. We plot the $\left(\tau_r, \tau_w^{2nd}\right)$ pairs in the bottom window of Figure 2. We can find the optimal capital income tax rate, τ_r^{2nd} (in the top window of Figure 2) as the one that minimizes the welfare loss, $\sigma^{\min} = \sigma(\tau_r^{2nd})$. We can then read the corresponding second best labor income tax rate, τ_w^{2nd}, see bottom window of Figure 2 (insets in both windows of Figure 2 show

[5] Results are not sensitive to changes in the initial condition.

the respective functions in the vicinity of the optimal solution). For the baseline set of parameter values, the second best optimal policy is $\left(\tau_r^{2nd}, \tau_w^{2nd}\right) = (0.04412, 0.29006)$. Corresponding welfare loss is $\sigma^{\min} = 0.0733$, i.e. under the second best optimal policy agents loose 7% of the stream of preferred consumption-leisure mix.[6]

[Figure 2 around here]

We state the optimal second best policy pairs, corresponding subsidy rate, welfare loss and time allocation to working, leisure and studying for various parameters in Table 1. The inability of the fiscal instruments to correct totally the effect of the externality implies that the allocation of time to the three activities is not optimal. In the second best, the working time is still lower than the optimal one −compare in the table the values u and u^{SP}, $\frac{u}{(u^{SP})}$, and the leisure is higher then optimal −compare l with l^{SP}, $\frac{l}{(l^{SP})}$.

We can see that as the externality increases, its best correction requires higher education subsidy which results in both higher capital and labor income tax rates. Given that capital income tax harms the capital accumulation, see equation (45), and consequently next period output and consumption, (1) and (44), it is always optimal to tax more labor than capital income.[7] For our set of parameters the optimal tax ratio $\frac{\tau_w^{2nd}}{\tau_r^{2nd}}$ decreases from 7.8 (for $\gamma = 0.1$) to 5.7 (for $\gamma = 0.5$).

[Table 1 around here]

In more capital intensive economies, when the participation of physical capital in output increases, it is harder to correct the effect of externality, as the gap between competitive equilibrium and the optimal leisure times increases. The ratio $\frac{\tau_w^{2nd}}{\tau_r^{2nd}}$ decreases with α.

When agents become more patient, higher β, optimal subsidy is lower and the optimal tax ratio $\frac{\tau_w^{2nd}}{\tau_r^{2nd}}$ increases. Nevertheless, as the discount factor increases, the second best equilibrium is further from the optimal one, i.e. the welfare loss increases with β.

In all cases exposed in the Table 1 (except of the case of $b = 0$, a model without labor-leisure choice), capital income tax is positive. We attribute the optimality of a positive capital income tax to the fact that higher leisure under the second best (compared to the social planner case) contributes to higher utility and can compensate the losses suffered in consumption due to distortions in physical capital accumulation.

We can observe in Table 1, that when the preference parameter on leisure $b \longrightarrow 0$, agents do not care for leisure, and we get the result of Gomez (2003), optimal tax on capital is zero, and the optimal labor income tax rate is

$$\lim_{b \longrightarrow 0} \tau_w = \frac{\gamma (1-u)}{1-\alpha+\gamma}$$

[6] Welfare loss under the *laisser-faire* is $\sigma|_{\tau_w=0}^{\tau_r=0} = 0.1646$, i.e. 16% of the stream of preferred consumption-leisure mix.

[7] Second best relative optimal tax, labor/capital tax is higher than unity.

and the subsidy rate is

$$\lim_{b \longrightarrow 0} s = \frac{\gamma u}{1 - \alpha + \gamma}.$$

In this case, the time allocated to work and study coincides with the social planner solution. It is thus no longer possible to compensate for the welfare loss caused by the distortion in the capital accumulation, and consequent decline in consumption, by higher fraction of time devoted to leisure. Therefore, the zero capital income tax becomes welfare maximizing policy. The optimal tax ratio $\frac{\tau_w^{2nd}}{\tau_r^{2nd}}$ decreases with b.

Changes in technology level, A, and social planner long run growth rate, g^{*SP}, do not affect the optimal policy results, they just scale the levels.

5 Conclusions

We show that in the Uzawa-Lucas model with externality in human capital and labor-leisure choice, in the absence of lump sum taxes, the second best optimal policy requires to tax both capital and labor incomes. Revenues to subsidize foregone earnings while studying can be raised imposing small positive tax rate on capital income, and a larger tax rate on labor income. This result differs from the optimal first best policy in the presence of lumps sum taxes, when no tax on neither capital nor labor should be used. The unavailability of lump sum taxation thus requires a tax schedule to be related to the structure of the economy.

Our results are shown on a particular case of an economy with total depreciation of physical and human capital. More general results are to be developed in the future.

Appendix

Transformed equilibrium equations

The analysis of the long run growth rates in section 3.1 implies the definition of other variables that have steady state in the long run

$$\hat{\eta}_t = \eta_t h_t^{\frac{1-\alpha\gamma}{1-\alpha}}, \; \hat{\varepsilon}_t = \varepsilon_t h_t, \; g_{h_{t+1}} = \frac{h_{t+1}}{h_t}. \tag{53}$$

The equations that must hold in equilibrium, after the transformation, are the following: production function (1), accumulation of human capital (4), relationship between the growth rate of human capital and the one of the economy (25) and (28), goods market equilibrium (17), first order condition on consumption (7), first order condition on labor and leisure time, (10) and (11), growth rate of marginal utility of consumption (8) and growth rate of shadow price of human capital obtained by plugging (8) into (11)

$$\hat{y}_t = Ax_t^{\alpha} u_t^{1-\alpha}, \tag{54}$$

$$g_{h_t} = \phi\left(1 - u_t - l_t\right) + 1 - \delta_h, \tag{55}$$

$$g_t = g_{h_t}^{\frac{1-\alpha+\gamma}{1-\alpha}}, \tag{56}$$

$$\hat{y}_t = \hat{c}_t + x_{t+1} g_t - \left(1 - \delta_k\right) x_t, \tag{57}$$

$$\hat{\eta}_t = \frac{1}{\hat{c}_t}, \tag{58}$$

$$\hat{\varepsilon}_t \phi = \hat{\eta}_t \left(1 - \tau_{w_t} - s_t\right)\left(1 - \alpha\right) Ax_t^{\alpha} u_t^{-\alpha}, \tag{59}$$

$$\frac{b}{l_t} = \hat{\varepsilon}_t \phi + \hat{\eta}_t s_t \left(1 - \alpha\right) Ax_t^{\alpha} u_t^{-\alpha}, \tag{60}$$

$$\hat{\eta}_t = \beta \frac{\hat{\eta}_{t+1}}{g_{h_t}^{\frac{1-\alpha+\gamma}{1-\alpha}}} \left[\left(1 - \tau_{r_{t+1}}\right) \alpha A x_{t+1}^{\alpha-1} u_{t+1}^{1-\alpha} + 1 - \delta_k\right], \tag{61}$$

$$\hat{\varepsilon}_t = \beta \frac{\hat{\varepsilon}_{t+1}}{g_{h_t}} \left\{\phi\left[\left(\frac{\mathcal{H}_h}{1-\alpha} - 1\right) u_{t+1} + \frac{1 - \tau_{w_{t+1}}}{1 - \tau_{w_{t+1}} - s_{t+1}}\left(1 - l_{t+1}\right)\right] + 1 - \delta_h\right\} \tag{62}$$

$$\text{where } \mathcal{H}_h = \begin{cases} 1 - \alpha \text{ for the decentralized solution, and} \\ 1 - \alpha + \gamma \text{ for the centralized solution.} \end{cases}$$

In the case of centralized solution all tax rates should be set to zero for all t, i.e. $\tau_{r_t} = 0, \tau_{w_t} = 0$

and $s_t = 0$.

Transformed equilibrium equations: steady state

Denoting the long run values with an asterisk, we can write the system (54)-(62) in the steady state as

$$\hat{y}^* = A(x^*)^{\alpha}(u^*)^{1-\alpha}, \tag{63}$$

$$g_h^* = \phi(1 - u^* - l^*) + 1 - \delta_h, \tag{64}$$

$$g^* = (g_h^*)^{\frac{1-\alpha+\gamma}{1-\alpha}} \tag{65}$$

$$\hat{y}^* = \hat{c}^* + x^* g^* - (1-\delta_k)x^*, \tag{66}$$

$$\hat{c}^* = \frac{1}{\hat{\eta}^*}, \tag{67}$$

$$\hat{\varepsilon}^* = \hat{\eta}^* \frac{[(1-\tau_w^* - s^*)(1-\alpha)]}{\phi}\frac{\hat{y}^*}{u^*}, \tag{68}$$

$$\hat{\varepsilon}^*\phi = \frac{b}{l^*} + \hat{\eta}^* s^*(1-\alpha)\frac{\hat{y}^*}{u^*}, \tag{69}$$

$$1 = \frac{\beta}{g^*}\left[(1-\tau_r^*)\alpha A(x^*)^{\alpha-1}(u^*)^{1-\alpha} + 1 - \delta_k\right], \tag{70}$$

$$1 = \frac{\beta}{g_h^*}\left\{\phi\left[\left(\frac{\mathcal{H}_h}{1-\alpha} - 1\right)u^* + \frac{1-\tau_w^*}{1-\tau_w^*-s^*}(1-l^*)\right] + 1 - \delta_h\right\} \tag{71}$$

$$\text{where } \mathcal{H}_h = \begin{cases} 1-\alpha \text{ for the decentralized solution, and} \\ 1-\alpha+\gamma \text{ for the centralized solution.} \end{cases}$$

In the case of centralized solution all tax rates should be set to zero, i.e. $\tau_r^* = 0, \tau_w^* = 0$ and $s^* = 0$.

Proposition 1

Proof. Balanced growth path behavior, equations (26), (27) and (29), implies that some ratios become constant.[8] Define

$$\lim_{t\to\infty}\frac{c_t}{y_t} = \theta_c^*, \quad \lim_{t\to\infty}\eta_t y_t = \theta_\eta^*, \quad \lim_{t\to\infty}\varepsilon_t h_t = \theta_\varepsilon^* \tag{72}$$

where θ_i^*, $i = c, \eta$ and ε are constants. Let us use the transformed goods market equilibrium (57). If the the ratios $\frac{\hat{c}_t}{\hat{y}_t} = \frac{c_t}{y_t}$, $\frac{x_t}{\hat{y}_t} = \frac{k_t}{y_t}$ are constant, the ratio $\frac{x_{t+1}g_t}{\hat{y}_t} = \frac{k_{t+1}}{y_t}$ will be also constant. We define that constant as

$$\theta_k^* = \lim_{t\to\infty}\frac{k_{t+1}}{y_t}. \tag{73}$$

[8] Consumption and output grow at the same rate, Lagrange multiplier η_t (ε_t) decreases at the same rate as output (human capital) increases.

Using (63), (66) and (70) we can obtain

$$\lim_{t\to\infty}\frac{x_t}{\hat{y}_t}=\lim_{t\to\infty}\frac{k_t}{y_t}=\frac{x^*}{\hat{y}^*}=\frac{1}{A\left(x^*\right)^{\alpha-1}\left(u^*\right)^{1-\alpha}}=\frac{\alpha\beta\left(1-\tau_r^*\right)}{g^*-\beta\left(1-\delta_k\right)},$$

$$\lim_{t\to\infty}\frac{x_{t+1}g_t}{\hat{y}_t}=\lim_{t\to\infty}\frac{k_{t+1}}{y_t}=\frac{x^*g^*}{\hat{y}^*}=\frac{g^*}{A\left(x^*\right)^{\alpha-1}\left(u^*\right)^{1-\alpha}}=\frac{\alpha\beta\left(1-\tau_r^*\right)g^*}{g^*-\beta\left(1-\delta_k\right)}$$

and

$$\lim_{t\to\infty}\frac{\hat{c}_t}{\hat{y}_t}=\lim_{t\to\infty}\frac{c_t}{y_t}=\frac{\hat{c}^*}{\hat{y}^*}=\lim_{t\to\infty}\left(1-\frac{x_{t+1}g_t}{\hat{y}_t}+\left(1-\delta_k\right)\frac{x_t}{\hat{y}_t}\right)=1-\alpha\beta\left(1-\tau_r^*\right)\frac{g^*-\left(1-\delta_k\right)}{g^*-\beta\left(1-\delta_k\right)}.$$

That means that the constant in the policy function for capital is

$$\theta_k^*=\frac{\alpha\beta\left(1-\tau_r^*\right)g^*}{g^*-\beta\left(1-\delta_k\right)},$$

and the one in the policy function for consumption

$$\theta_c^*=1-\alpha\beta\left(1-\tau_r^*\right)\frac{g^*-\left(1-\delta_k\right)}{g^*-\beta\left(1-\delta_k\right)}. \tag{74}$$

The first order condition on consumption (58) together with (72) imply that

$$\theta_\eta=\frac{1}{\theta_c} \tag{75}$$

and the first order condition on working time (68) together with (72) lead to

$$\theta_\varepsilon^*=\frac{\theta_\eta^*\left[\left(1-\tau_w^*-s^*\right)\left(1-\alpha\right)\right]}{\phi u^*}. \tag{76}$$

Let us look for the balanced growth path working and labor times, $\lim_{t\to\infty}u_t=u^*$, $\lim_{t\to\infty}l_t=l^*$. Joining (68) and (69) we get one relationship between u^* and l^*

$$l^*=\frac{b}{\theta_\eta^*\left(1-\alpha\right)\left(1-\tau_w^*\right)}u^*. \tag{77}$$

Using (64) and (71) we get another relationship between u^* and l^*

$$u^*=\left(1-\beta\frac{1-\tau_w^*}{1-\tau_w^*-s^*}\right)\left(1-l^*\right)+\left(1-\beta\right)\frac{1-\delta_h}{\phi}\text{ for the decentralized case.} \tag{78}$$

■

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γ	τ_r^{2nd}	τ_w^{2nd}	s	σ	u (u^{SP})	l (l^{SP})	$1-u-l$ $(1-u^{SP}-l^{SP})$
0.10	0.0155	0.1215	0.0126	0.0092	0.0785 (0.0795)	0.0962 (0.0848)	0.8251 (0.8355)
0.20	0.0302	0.2153	0.0196	0.0344	0.0695 (0.0711)	0.0961 (0.0758)	0.8343 (0.8530)
0.30	0.0441	0.2901	0.0237	0.0733	0.0623 (0.0642)	0.0960 (0.0685)	0.8416 (0.8672)
0.40	0.0574	0.3511	0.0260	0.1246	0.0565 (0.0586)	0.0959 (0.0625)	0.8476 (0.8789)
0.50	0.0702	0.4018	0.0272	0.1879	0.0517 (0.0539)	0.0958 (0.0574)	0.8525 (0.8887)
α	τ_r^{2nd}	τ_w^{2nd}	s	σ	u (u^{SP})	l (l^{SP})	$1-u-l$ $(1-u^{SP}-l^{SP})$
0.30	0.0380	0.2666	0.0220	0.0555	0.0653 (0.0671)	0.0942 (0.0700)	0.8403 (0.8629)
0.40	0.0441	0.2900	0.0237	0.0732	0.0623 (0.0642)	0.0960 (0.0685)	0.8416 (0.8672)
0.60	0.0632	0.3465	0.0280	0.1489	0.0536 (0.0557)	0.1014 (0.0642)	0.8449 (0.8799)
β	τ_r^{2nd}	τ_w^{2nd}	s	σ	u (u^{SP})	l (l^{SP})	$1-u-l$ $(1-u^{SP}-l^{SP})$
0.85	0.0619	0.2693	0.0365	0.0618	0.0904 (0.0943)	0.1405 (0.1037)	0.7690 (0.8019)
0.90	0.0441	0.2900	0.0237	0.0732	0.0623 (0.0642)	0.0960 (0.0685)	0.8416 (0.8672)
0.99	0.0049	0.3286	0.0022	0.0962	0.0066 (0.0066)	0.0100 (0.0067)	0.9834 (0.9867)
b	τ_r^{2nd}	τ_w^{2nd}	s	σ	u (u^{SP})	l (l^{SP})	$1-u-l$ $(1-u^{SP}-l^{SP})$
0.00	0.0000	0.3103	0.0230	0.0000	0.0690 (0.0690)	0.0000 (0.0000)	0.9310 (0.9310)
0.10	0.0052	0.3080	0.0231	0.0078	0.0682 (0.0685)	0.0105 (0.0073)	0.9212 (0.9242)
0.80	0.0366	0.2935	0.0235	0.0594	0.0636 (0.0651)	0.0784 (0.0556)	0.8581 (0.8793)
1.00	0.0441	0.2901	0.0237	0.0733	0.0623 (0.0642)	0.0960 (0.0685)	0.8416 (0.8672)
1.50	0.0611	0.2823	0.0239	0.1064	0.0595 (0.0621)	0.1372 (0.0994)	0.8033 (0.8385)
4.00	0.1182	0.2560	0.0248	0.2456	0.0485 (0.0533)	0.2966 (0.2274)	0.6548 (0.7194)

Table 1: Second best optimal policy, τ_r^{2nd}, τ_w^{2nd} and s, and time allocation across period, u, l, $1-u-l$, over parameters' space. Baseline values: $\gamma = 0.3$, $b = 1$, $\alpha = 0.4$, $\beta = 0.9$, $A = 1$, $g^* = 1.015$, $\delta_k = 1$, $\delta_h = 1$. Optimal values of working, leisure and studying in parenthesis, u^{SP}, l^{SP}, $1 - u^{SP} - l^{SP}$.

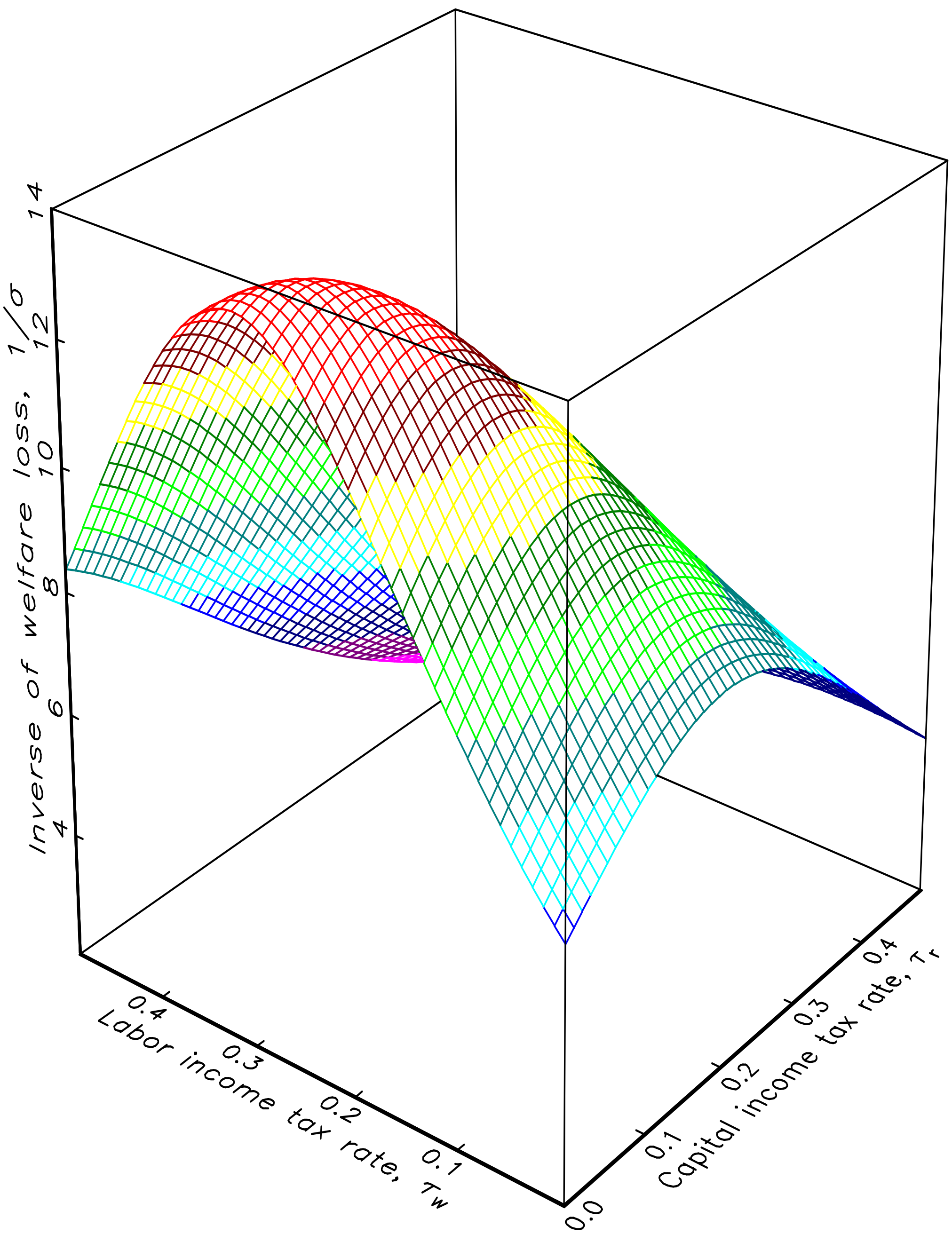

Figure 1: Inverse of welfare loss, $\dfrac{1}{\sigma}$, over the capital and labor income tax space, different combinations of τ_r and τ_w.

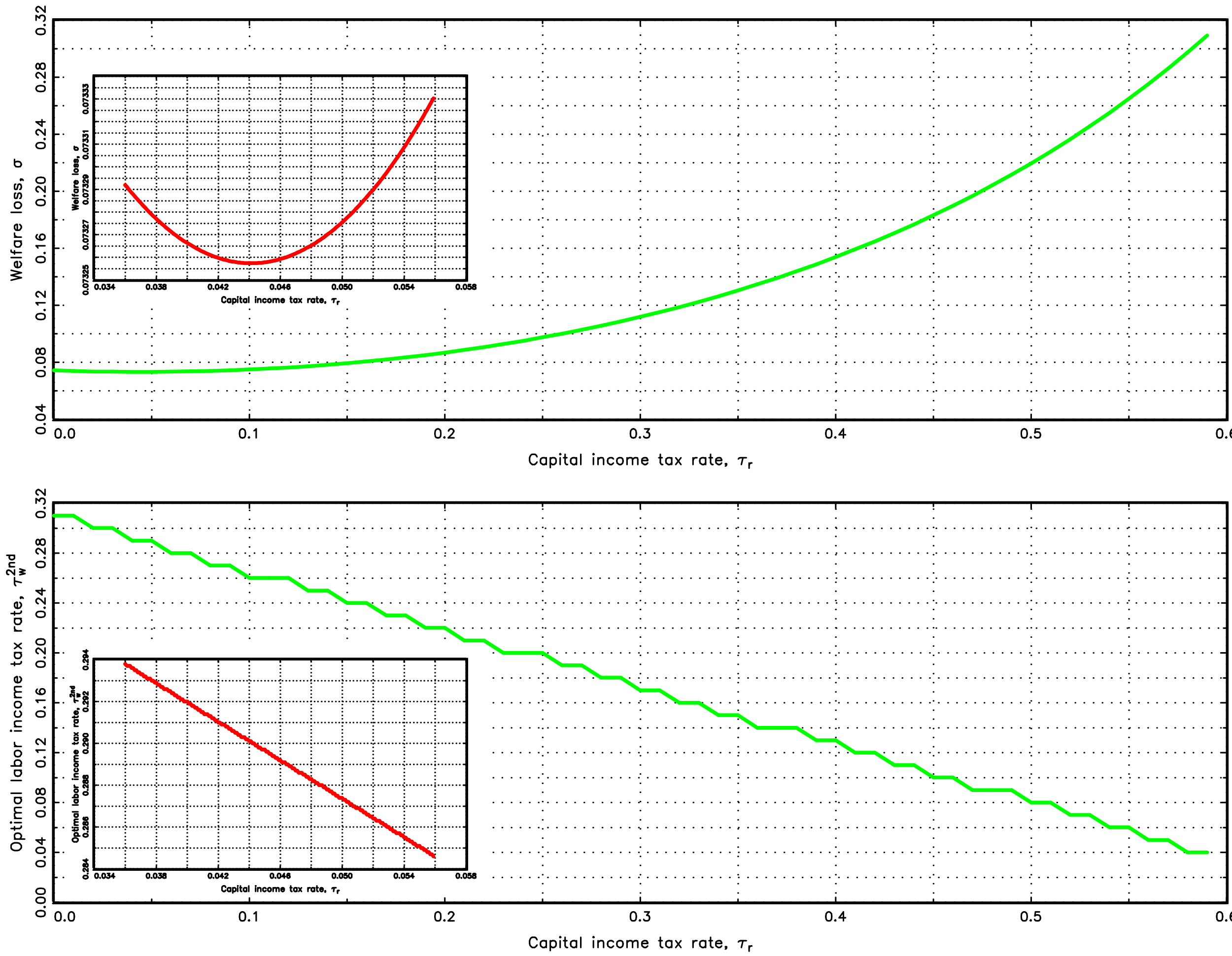

Figure 2: Relationship between the welfare loss and capital and labor income tax rates: for a given capital income tax rate τ_r (horizontal axe) we plot the labor income tax rate which leads to the highest welfare - minimum welfare loss, τ_w^{2nd} (vertical axe, bottom figure) and show the corresponding welfare loss σ (vertical axe, top figure). In the inset of the top figure we present the detail to identify the capital income tax rate that minimizes the welfare loss, τ_r^{2nd}. In the inset in the bottom figure one can read the corresponding labor income tax rate τ_w^{2nd}.