Optimal Public Policy in an Endogenous Growth Model when Social Discounting is Deemed Immoral

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Abstract

We use an overlapping generations model with physical and human capital to ascertain the consequences for optimality of a social planner adopting a social welfare criterion that treats all generations alike and is respectful of individual preferences. In particular, we consider a social planner who maximizes an non-discounted sum of individual utilities defined over consumption per unit of efficient labour. We show that, given some initial conditions, an economy commanded by such a social planner converges to the Golden Rule defined in this framework. Decentralizing the optimum requires taxing education along the entire growth path and that the intergenerational effects of the optimal policy are equivalent to those of a pay-as-you-go social security system.

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1 Introduction

The choice of an objective function with which to characterize social optimum has always been the subject of controversy. Ramsey (1928) assumed a social planner who maximizes an infinite, non-discounted sum of present and future utilities. The discounting of later enjoyments in comparison with earlier ones was for him an ethically indefensible practice. In his own words, (Ramsey, 1928, p. 543) discounting of future utility is "ethically indefensible and arises merely from the weakness of the imagination." Later, Cass (1965, 1966), Koopmans (1965, 1967), Malinvaud (1965) and Samuelson (1965) generalized Ramsey's approach to allow for the discounting of future utilities. In an explicit overlapping generations setting where individuals are pure life-cyclers, Diamond (1965) adhered to the maximization of the utility level of a representative individual under the constraint that any other achieves the same welfare level, giving rise to what the subsequent literature coined as the Two-Part Golden Rule, i.e., the combination of the Biological Interest Rate and the Golden Rule of (physical) capital accumulation [Samuelson (1968, 1975a, 1975b)]. Adding to this framework the dimension of intergenerational altruism à la Barro (1974), so that individuals behave as if they maximized dynastic utility, there is a new choice between considering the welfare level enjoyed by a representative child only [Carmichael (1982)] or that of all children [Burbidge (1983)].

Clearly, each of these views of social welfare leads to a different optimal resource allocation. But all of them share the feature that they assume economies without productivity growth, in which a steady state is a situation where consumption levels per unit of (natural) labour are kept constant. In contrast, in the presence of productivity growth that translates into consumption growth, these consumption levels will grow without limit. Under these circumstances, if a social planner adopted a social welfare function whose arguments were utility functions defined over individual consumptions per unit of natural labour, it is clear that, for plausible specifications, the utility index would grow without limit. Since utility will eventually be infinite along a balanced growth path, there would simply be no scope for utility maximization. A way to sidestep this is, of course,

to assume that the planner maximizes a discounted sum of utilities. This is the approach adopted by Docquier et al. (2007) to characterize the optimal balanced growth path in the endogenous growth model set up in the preceding section. An uncomfortable feature of this approach is that the precise cardinalization of the individual utility function is crucial for both the characterization of the social optimum and the policies that support it [Del Rey and Lopez-Garcia (2012)].

In light of these facts, it is tempting to ask whether it is possible for a social planner to continue to adhere to the Utilitarian criterion of maximizing a sum of individual utilities but: (i) without having to postulate something as elusive as a social discount rate; and (ii) without having to choose a specific functional form to cardinalize individual preferences. As it is argued below, the answer is affirmative provided that the social planner maximizes a non-discounted sum [thus solving (i)] of individual future utilities defined over consumption levels per unit of *efficient* rather than natural labour units [this allowing to sort (ii)]. It is important to stress that this "new" utility function is obtained by means of a monotonic transformation of the one guiding individual's behaviour, thus being fully respectul with individual *ordinal* preferences. Notice also that, assuming convergence, the limit of the time sequence of consumption levels expressed in terms of output per unit of efficient labour will be measured in such a dimension, as will be any variable which is constant along a balanced growth path.

This approach to the characterization of the optimal growth path can be related to the discussion in Del Rey and Lopez-Garcia (2013), who focus on the balanced growth path that maximizes the lifetime welfare of a representative individual subject to the constraint that everyone else's welfare is fixed at the same level, i.e., the counterpart in an endogenous growth framework of the so-called Two-Part Golden Rule [Samuelson (1968, 1975a, 1975b)]. As the analytics below will show, this is the balanced growth path towards which an economy converges when commanded by a social planner who, given some initial conditions, maximizes an non-discounted sum of individual utilities defined over consumption per unit of efficient labour.

The second purpose of this paper concerns the characterization of optimal policies,

i.e., those that once superimposed on private behaviour and interaction in the market place, allow the social planner to decentralize the optimal growth path. Del Rey and Lopez-Garcia (2013) show that along the optimal balanced growth path, a negative education subsidy and positive pensions to the elderly are called for. However, these results do not seem especially instructive when the focus turns from balanced growth paths to the discussion of the whole time trajectory leading to it. In particular, two questions become relevant: (i) the sign of tax parameters, particularly the one addressed to education decisions, along the transition periods starting from some arbitray initial conditions; and (ii) the intergenerational effects of the optimal tax instruments taken together. As far as (i) is concerned, it is shown below that education expenditures should not only be taxed along the optimal balanced-growth path but also along the *entire* optimal growth path. And referring to (ii), the directions of the intergenerational transfers associated with the whole tax system along the optimal balanced growth path are equivalent to those of a *pay-as-you-go* social security system where the middle-aged contribute and the old-aged retirees receive a pension benefit. Interestingly, the pay-as-you-go nature of the optimal policy is independent of any particular feature of the model. This is in sharp contrast with the results obtained from exogenous growth models, where the precise configuration of the optimal social security depends in a crucial way on the relationship between the interest rate and the growth rate along the laissez-faire balanced-growth path [Samuelson (1975b)].

The rest of the paper is organized as follows. Section 2 presents the model and analyzes the market equilibrium in the presence of government. Section 3 discusses the optimal growth path chosen by a social planner who maximizes an non-discounted sum of individual utilities defined over consumption per unit of efficient labour. Section 4 characterizes the optimal tax policies that allow to decentralize the optimal growth path and clarifies the pay-as-you-go nature of the intergenerational effects induced by the optimal policy. Section 5 concludes.

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2   The model and the decentralized equilibrium in the presence of government

The framework of analysis is the overlapping generations model with both human and physical capital used in Boldrin and Montes (2005), Docquier et al. (2007) and Del Rey and Lopez-Garcia (2012,2013). At period t, L_{t+1} individuals are born, and coexist with L_t middle-aged and L_{t-1} old-aged. Population grows at the exogenous rate $n > -1$,so that $L_t = (1+n)L_{t-1}$. Agents are born with the level of human capital of their parents, h_{t-1}, measured in units of efficient labour per unit of natural labour. Human capital in period t results from the interaction of the amount of output that young individuals invest in education, d_{t-1}, and the inherited human capital h_{t-1} according to the production function $h_t = E(d_{t-1}, h_{t-1})$. Assuming constant returns to scale, the production of human capital can be written in intensive terms as $h_t/h_{t-1} = e(\tilde{d}_{t-1})$, where $e(.)$ satisfies the Inada conditions and $\tilde{d}_{t-1} = d_{t-1}/h_{t-1}$ is the amount of output devoted to education per unit of inherited human capital. Therefore, the growth rate of productivity from period $t-1$ to period t, g_t, satisfies $h_t/h_{t-1} = e(\tilde{d}_{t-1}) = (1+g_t)$.

There is a single good, Y_t , that is produced by means of physical capital, K_t, and human capital, H_t, according to a constant returns to scale production function $Y_t = F(K_t, H_t)$. Only the middle-aged work, supplying inelastically one unit of natural labour, so that $H_t = h_t L_t$. Physical capital is assumed to fully depreciate each period. Letting $k_t = K_t/L_t$ be the physical capital per unit of natural labour ratio and $\tilde{k}_t = K_t/H_t = k_t/h_t$ the physical capital per unit of efficient labour ratio, one can write $Y_t/H_t = f(\tilde{k}_t)$, where $f(.)$ also satisfies the Inada conditions.

Perfect competition prevails, so that, if $(1 + r_t)$ and w_t are respectively the interest factor and the wage rate per unit of efficient labour,

$$(1 + r_t) \;=\; f'(\tilde{k}_t) \tag{1}$$

$$w_t \;=\; f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t) \tag{2}$$

Two policy instruments are assumed to be available to the government: lump-sum

taxes, both on the middle-aged and the elderly, and education subsidies. The latter are related to the repayment, in the second period of life, of the loans taken in the first one to pay for education and the ensuing interests. Let $z_t^m > 0$ [resp. < 0] be the lump-sum tax [transfer] the middle-aged pay [receive], $z_t^o > 0$ [< 0] the lump-sum tax the old pay [the pension they receive], and let θ_t be the subsidy rate, all of them in period t. The government budget constraint can be written:

$$z_t^m L_t + z_t^o L_{t-1} = \theta_t(1 + r_t)d_{t-1}L_t \tag{3}$$

Notice that a subsidy to the repayment of loans is not the only way to model education subsidies. This approach, however, emphasizes the role of credit markets in financing human capital investments, and the interaction of this process with public policy.

Individuals are assumed to behave as pure life-cyclers and only consume in their second and third period. The lifetime utility function of an individual born at period $t-1$ is $U_t = U(c_t^m, c_{t+1}^o)$, where c_t^m and c_{t+1}^o denote her consumption levels as middle-aged and old-aged respectively. This function is strictly increasing in both arguments, strictly concave and homogeneous of degree $j < 1$. In their first period, individuals born at $t-1$ borrow in perfect credit markets the amount of output required to pay for the education level d_{t-1} that maximizes the present value of their lifetime resources. In their second period they work, pay taxes z_t^m, pay back the loan net of education subsidies $(1 + r_t)d_{t-1}(1 - \theta_t)$, consume and save to finance consumption in their third period. In this third period, individuals consume and pay taxes z_{t+1}^o. Letting s_t stand for savings of a middle-aged:

$$c_t^m = w_t h_t - (1 + r_t)d_{t-1}(1 - \theta_t) - z_t^m - s_t \tag{4}$$

$$c_{t+1}^o = (1 + r_{t+1})s_t - z_{t+1}^o \tag{5}$$

As a consequence, the lifetime budget constraint of an individual born at period $t-1$ is:

$$c_t^m + \frac{c_{t+1}^o}{(1 + r_{t+1})} = w_t h_t - (1 + r_t)d_{t-1}(1 - \theta_t) - z_t^m - \frac{z_{t+1}^o}{(1 + r_{t+1})} \tag{6}$$

The first-order conditions associated with the individual decision variables, d_{t-1}, c_t^m and c_{t+1}^o, are:

$$w_t e'(d_{t-1}/h_{t-1}) = (1 + r_t)(1 - \theta_t) \tag{7}$$

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$$\frac{\partial U(c_t^m, c_{t+1}^o)/\partial c_t^m}{\partial U(c_t^m, c_{t+1}^o)/\partial c_{t+1}^o} = (1 + r_{t+1}) \tag{8}$$

where use has been made of the homogeneity of degree one of the E function, i.e., $h_t = e(d_{t-1}/h_{t-1})h_{t-1}$. Equation (7) shows that the individual will invest in education up to the point where the marginal benefit in terms of second period income equals the marginal cost of investing in human capital allowing for subsidies. Rewriting (7) as $e'(\tilde{d}_{t-1}) = (1 - \theta_t)(1 + r(\tilde{k}_t))/w(\tilde{k}_t)$, this expression implicitly characterizes the optimal ratio $\tilde{d}_{t-1}$ as a function of $\tilde{k}_t$ and θ_t, i.e., $\tilde{d}_{t-1} = \phi(\tilde{k}_t, \theta_t)$. Since $e'' < 0$ it can readily be shown that the greater $\tilde{k}_t$ and θ_t, the greater $\tilde{d}_{t-1}$.

Using (3), (6) becomes

$$c_t^m + \frac{c_{t+1}^o}{(1 + r_{t+1})} = \omega_t \tag{9}$$

where ω_t is the present value of the net lifetime income of an individual born at $t - 1$:

$$\omega_t = w_t h_t - (1 + r_t)d_{t-1}(1 - \theta_t) - z_t^m - \frac{(1 + n)}{(1 + r_{t+1})}[\theta_{t+1}(1 + r_{t+1})d_t - z_{t+1}^m] \tag{10}$$

The homogeneity assumption on preferences implies that the c_{t+1}^o/c_t^m ratio is a function of r_{t+1} only. This allows one to write consumption in the second period as $c_t^m = \pi(r_{t+1})\omega_t$, where the function $\pi(.)$ depends on the interest rate only. Equilibrium in the market for physical capital will be achieved when the physical capital stock available in $t + 1$, K_{t+1}, equals gross savings made by the middle-aged in t, $s_t L_t$, minus the amount of output devoted to human capital investment by the young in t, $(1 + n)d_t L_t$. That is, when $K_{t+1} = s_t L_t - (1 + n)d_t L_t$, or, equivalently, $\tilde{k}_{t+1} = \tilde{s}_t/e(\tilde{d}_t)(1 + n) - \tilde{d}_t/e(\tilde{d}_t)$, where $\tilde{s}_t = s_t/h$. This equilibrium condition can be rewritten as

$$\tilde{k}_{t+1} = \frac{(1 - \pi(r_{t+1}))\tilde{\omega}_t}{e(\phi(\tilde{k}_{t+1}, \theta_{t+1}))(1 + n)} - \frac{\tilde{z}_{t+1}^m}{(1 + r_{t+1})} - \frac{(1 - \theta_{t+1})\phi(\tilde{k}_{t+1}, \theta_{t+1})}{e(\phi(\tilde{k}_{t+1}, \theta_{t+1}))} \tag{11}$$

where $\tilde{z}_{t+1}^m = z_{t+1}^m/h_{t+1}$ and $\tilde{\omega}_t = \omega_t/h_t$ is the present value of lifetime resources expressed in terms of output per unit of efficient labour. Taking into account (1) and (2), this expression implicitly provides $\tilde{k}_{t+1}$ as a function of $\tilde{k}_t$, $\tilde{z}_t^m$, $\tilde{z}_{t+1}^m$, θ_t, and θ_{t+1}, i.e., $\Psi(\tilde{k}_t; \tilde{z}_t^m, \tilde{z}_{t+1}^m, \theta_t, \theta_{t+1})$.

Finally, using factor prices in (1) and (2), the government budget constraint (3), the individual budget constraints in middle and old-age, (4) and (5), and the equilibrium

condition (11), one can find the aggregate feasibility constraint expressed in terms of output per unit of efficient labour:

$$\tilde{c}_t^m + \frac{\tilde{c}_t^o}{e(\tilde{d}_{t-1})(1+n)} = f(\tilde{k}_t) - (1+n)\tilde{d}_t - e(\tilde{d}_t)(1+n)\tilde{k}_{t+1} \tag{12}$$

where $\tilde{c}_t^m = c_t^m/h_t$ and $\tilde{c}_t^o = c_t^o/h_{t-1}$ are, respectively, consumption of a middle-aged and of an old-aged in period t per unit of labour efficiency.[1]

A balanced growth path will be a situation where all variables expressed in terms of output per unit of natural labour grow at a constant rate, and, as a consequence, all variables per unit of efficient labour will remain constant over time. One can then delete the time subscripts in (11) and write $\tilde{k} = \Psi(\tilde{k}; \tilde{z}^m, \theta)$. An equilibrium ratio of physical capital to labour in efficiency units along a balanced growth path in the presence of government intervention, $\tilde{k}_G$, will then be a fixed point of the Ψ function, i.e., $\tilde{k}_G = \Psi(\tilde{k}_G; \tilde{z}^m, \theta)$. Such an equilibrium will be locally stable provided that $0 < \partial\Psi(\tilde{k}_G; \tilde{z}^m, \theta)/\partial\tilde{k} < 1$. We will focus on situations where the equilibrium is unique and stable, so that the relationship between $\tilde{k}$ and the tax parameters can be written, with an obvious notation, as $\tilde{k}_G = \tilde{k}(\tilde{z}^m, \theta)$. As for the determination of $\tilde{d}_G$, the amount of output devoted to education per unit of inherited human capital along a balanced growth path will be governed by the relationship arising from the education decision (7),so that we can write $\tilde{d}_G = \phi(\tilde{k}(\tilde{z}^m, \theta), \theta)$ or sinply $\tilde{d}_G = \tilde{d}(\tilde{z}^m, \theta)$.

The fact that the utility function is homogeneous implies that the marginal rates of substitution in the (c_t^m, c_{t+1}^o) and $(\tilde{c}^m, \tilde{c}^o)$ spaces will be the same. Thus, individual behavior along a balanced growth path in the presence of government intervention can be summarized by:

$$\frac{\partial U(\tilde{c}_G^m, \tilde{c}_G^o)/\partial\tilde{c}^m}{\partial U(\tilde{c}_G^m, \tilde{c}_G^o)/\partial\tilde{c}^o} = (1 + r_G) \tag{13}$$

$$w_G e'(\tilde{d}_G) = (1 + r_G)(1 - \theta) \tag{14}$$

$$\tilde{c}_G^m + \frac{\tilde{c}_G^o}{(1 + r_G)} = \tilde{\omega}_G \tag{15}$$

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[1]Note that $c_t^m L_t$ and $c_t^o L_{t-1}$ are measured in units of output. Since middle-aged individuals supply one unit of natural labour, c_t^m and c_t^o are expressed in units of output per unit of *natural* labour. The interpretation of $\tilde{c}_t^m$ and $\tilde{c}_t^o$ in terms of units of output per unit of *efficient* labour follows naturally.

where

$$\tilde{\omega}_G = w_G - \frac{(1+r_G)\tilde{d}_G}{e(\tilde{d}_G)} - \frac{\left[\theta(1+r_G)\tilde{d}_G - e(\tilde{d}_G)\tilde{z}^m\right]}{(1+r_G)e(\tilde{d}_G)}\left[e(\tilde{d}_G)(1+n) - (1+r_G)\right]. \quad (16)$$

is the present value of the individual's lifetime resources along a balanced growth path, and $e(\tilde{d}_G) = (1+g_G)$ is the growth factor of productivity and of any variable expressed in terms of output per unit of natural labour.

3   The optimum: A non-discounted sum of utilities defined over consumption per unit of efficient labour

In this section, we argue that a social welfare function can be postulated that maximizes a non-discounted sum of individual utilities without being obliged to adopt a specific cardinalization of individual ordinal preferences. To this end, notice that since the utility function is homogeneous of degree j, we can take the monotonic transformation of $U_t = U(c_t^m, c_{t+1}^o)$ resulting from dividing by h_t and obtain a new utility $\tilde{U}_t = U(\tilde{c}_t^m, \tilde{c}_{t+1}^o)$ while ensuring that ordinal preferences are respected.

The social planner's objective can then be written as the maximization of a non-discounted sum of the divergences between individual utilities derived from consumption expressed per unit of efficient labour and some "bliss utility" à la Ramsey (1928). This bliss utility level, $U(\tilde{c}_*^m, \tilde{c}_*^o)$, is the one associated with those values $\tilde{c}_*^m$ and $\tilde{c}_*^o$ that maximize $\tilde{U}_t$ under the balanced-growth-path version of the feasibility constraint (12). Formally, this can be written [Samuelson (1968), De la Croix and Michel (2002)] as:

$$\tilde{W} = \sum_{t=0}^{\infty}\left[U(\tilde{c}_t^m, \tilde{c}_{t+1}^o) - U(\tilde{c}_*^m, \tilde{c}_*^o)\right]. \quad (17)$$

The planner's problem is then to maximize (17) subject to the sequence of aggregate feasibility constraints (12) for given values of $\tilde{k}_0, \tilde{c}_0^o$ and $\tilde{d}_{-1}$ as initial conditions.[2] The

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[2]Strictly speaking, and to be fully coherent with the approach followed by Ramsey (17) should be written as $(-\tilde{W}) = \sum_{t=0}^{\infty}\left[U(\tilde{c}_*^m, \tilde{c}_*^o) - U(\tilde{c}_t^m, \tilde{c}_{t+1}^o)\right]$, so that the purpose of the social planner is to

socially optimum growth path can be characterized by means of the first-order conditions and the transversality conditions.[3] The Lagrangean function becomes:

$$\mathcal{L} = \sum_{t=0}^{\infty} \left[U(\tilde{c}_t^m, \tilde{c}_{t+1}^o) - U(\tilde{c}_*^m, \tilde{c}_*^o) \right] - \tag{18}$$

$$- \sum_{t=0}^{\infty} \mu_t \left(\tilde{c}_t^m + \frac{\tilde{c}_t^o}{e(\tilde{d}_{t-1})(1+n)} - f(\tilde{k}_t) + (1+n)e(\tilde{d}_t)\tilde{k}_{t+1} + (1+n)\tilde{d}_t \right) \tag{19}$$

where μ_t is the Lagrange multiplier associated with the resource constraint (12) at time t. From the first-order conditions corresponding to $\tilde{c}_t^m$, $\tilde{c}_{t+1}^o$, $\tilde{k}_{t+1}$, $\tilde{d}_t$, and μ_t, and adding the subscript $*$ to denote optimality, we obtain:

$$\frac{\partial U(\tilde{c}_{*t}^m, \tilde{c}_{*t+1}^o)/\partial \tilde{c}_t^m}{\partial U(\tilde{c}_{*t}^m, \tilde{c}_{*t+1}^o)/\partial \tilde{c}_{t+1}^o} = f'(\tilde{k}_{*t+1}) \tag{20}$$

$$\frac{\partial U(\tilde{c}_{*t}^m, \tilde{c}_{*t+1}^o)/\partial \tilde{c}_t^m}{\partial U(\tilde{c}_{*t-1}^m, \tilde{c}_{*t}^o)/\partial \tilde{c}_t^o} = e(\tilde{d}_{*t-1})(1+n) \tag{21}$$

$$e(\tilde{d}_{*t}) \left(\frac{\tilde{c}_{*t+1}^o}{f'(\tilde{k}_{*t+1})e(\tilde{d}_{*t})(1+n)} - \tilde{k}_{*t+1} \right) = 1 \tag{22}$$

as well as (12). The interpretation of (20) and (21) is straightforward. The former reflects the equality of the intertemporal rates of substitution in consumption (i.e., between second and third period consumptions) and of transformation in production (i.e., the marginal product of physical capital) between periods t and $t+1$. The latter captures the static conditions of optimal distribution of consumption available in period t between middle-aged and old-aged individuals, allowing for growth of both productivity and population. Although it may seem odd at first glance, expression (22) also has a natural interpretation. Indeed, it is an arbitrage condition between the returns from investing in physical capital and in education. The intuition can be grasped by making use of the fact that μ_t [resp. μ_{t+1}] is the shadow value (in terms of social welfare $\tilde{W}$) of a unit of output per efficient

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minimize the non-discounted divergence of the amount by which utility falls short of the bliss level. Obviously both are equivalent, as maximizing $\tilde{W}$ is tantamount to minimizing $(-\tilde{W})$.

[3]On the precise form of the transversality condition in Ramsey-like optmization problems, see Michel (1990) and De la Croix and Michel (2002).

labour in period t [resp. $t+1$]. Suppose that in period t the social planner slightly increases $\tilde{k}_{t+1}$. It is clear from the aggregate feasibility constraint (12) that this will affect its counterparts at periods t and $t+1$: a higher $\tilde{k}_{t+1}$ implies a reduction in the resources left for consumption in period t, given by $e(\tilde{d}_t)(1+n)$, and an increase of the resources available for consumption in period $t+1$, captured by $f'(\tilde{k}_{t+1})$. Thus, the marginal cost of investing in physical capital is $\mu_t(1+n)e(\tilde{d}_t)$ and the marginal benefit is $\mu_{t+1}f'(\tilde{k}_{t+1})$. The first order condition for $\tilde{k}_{t+1}$ imposes that this marginal cost and this marginal benefit should be equal along the optimum growth path. If, instead, the planner increases $\tilde{d}_t$, the feasibility constraints at periods t and $t+1$ will also be modified. The cost, incurred in period t, now has two components. On the one hand, there is a direct cost, $(1+n)$, that reduces consumption possibilities. There is, however, also an indirect cost, given by $e'(\tilde{d}_t)(1+n)\tilde{k}_{t+1}$: as a consequence of the effect of $\tilde{d}_t$ on the growth rate, the amount of output devoted to investment in physical capital must be increased if we are to achieve the optimal value of $\tilde{k}_{t+1}$. Using the shadow value μ_t, the marginal cost of an additional unit invested in education is thus $\mu_t[(1+n)+e'(\tilde{d}_t)(1+n)\tilde{k}_{t+1}]$. The benefits, however, do not take place until period $t+1$. Indeed, evaluating (12) at $t+1$, the increased growth rate lowers the marginal rate of transformation between third and second period consumption in the right hand side. This amounts to an expansion of consumption possibilities, so that the marginal benefit is $\mu_{t+1}e'(\tilde{d}_t)(1+n)\tilde{c}^o_{t+1}/[e(\tilde{d}_t)(1+n)]^2$. As before, the first order condition for $\tilde{d}_t$ imposes that these marginal costs and benefits should be equal along the optimum growth path. Both first order conditions involve the same ratio of shadow values, μ_t/μ_{t+1}, so that an arbitrage condition between the returns from investing in $\tilde{k}_{t+1}$ and $\tilde{d}_t$, measured in units of resources at $t+1$ per unit of resources at t, can be derived. This is precisely the way expression (22) is obtained.

We can advance the following definition:

Definition 1 *Given the initial conditions ($\tilde{k}_0, \tilde{c}^o_0$ and $\tilde{d}_{-1}$), the optimal path $\{c^m_{*t}, c^o_{*t+1}, k_{*t+1}, d_{*t}\}$ that provides the sequence $\{U_{*t}\}$ and maximizes the non-discounted sum (17) defined over consumption per unit of efficient labour from period t=0 to infinity, satisfies conditions (20), (21), (22) and (12).*

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Along a balanced growth path the variables measured in terms of output per unit of efficient labour will remain constant while the variables expressed per unit of natural labour will be growing at a constant common rate. Assuming convergence, and deleting the time subscripts in (20), (21) and (22), we obtain the following expressions characterizing the optimal balanced growth path:

$$\frac{\partial U(\tilde{c}_*^m, \tilde{c}_*^o)/\partial \tilde{c}^m}{\partial U(\tilde{c}_*^m, \tilde{c}_*^o)/\partial \tilde{c}^o} = e(\tilde{d}_*)(1+n) \tag{23}$$

$$f'(\tilde{k}_*) = e(\tilde{d}_*)(1+n) \tag{24}$$

$$e'(\tilde{d}_*) \left(\frac{\tilde{c}_*^o}{\left[e(\tilde{d}_*)(1+n) \right]^2} - \tilde{k}_* \right) = 1 \tag{25}$$

in addition to the balanced growth path version of (12). Together, these four equations provide the optimal-growth-path levels $\tilde{c}_*^m$, $\tilde{c}_*^o$, $\tilde{k}_*$ and $\tilde{d}_*$, and thus $\tilde{U}_* = U(\tilde{c}_*^m, \tilde{c}_*^o)$, termed by Del Rey and Lopez-Garcia (2013) as the "Golden Rule" in the current endogenous growth setting. Condition (23) is the equality of the marginal rate of substitution between second and third period consumptions and the economy's growth rate $e(\tilde{d}_*)(1+n)$. In turn, (24) is the equality between the marginal product of physical capital and the growth rate. Together, (23) and (24) are the counterpart of the so-called Two-Part Golden Rule [Samuelson (1968, 1975a, 1975b)], i.e., the (now endogenous) Biological Interest Rate [Samuelson (1958)] and the Golden Rule of (physical) capital accumulation [Phelps (1961)]. Notice that (24), and along with it the entire system of equations, is independent of the specific cardinalization of individual preferences that could have be chosen to describe individual behaviour. In other words, and in contrast to Docquier et al. (2007), the degree of homogeneity j of the utility function is now irrelevant.[4]

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[4]Indeed, using the double subscript $*$ to denote the optimal balanced growth path in Docquier et al. (2007), when the social planner maximizes a discounted (with a social factor γ) sum of utilities defined over consumption per unit of natural labour, the optimal balanced growth path is characterized by the Modified Golden Rule, $\gamma f'(\tilde{k}_{**}) = [e(\tilde{d}_{**})]^{1-j}(1+n)$, and the marginal product of physical capital will be greater than the economy's growth rate. The presence of the degree of homogeneity j in this

4 Optimal policy

We are now in a position to discuss the values, and not less important, the signs, of the optimal tax instruments $\left\{\theta_t, \tilde{z}_{*t}^m, \tilde{z}_{*t+1}^o\right\}$ that allow the social planner to decentralize the optimal time path characterized in the preceding section, $\left\{\tilde{c}_{*t}^m, \tilde{c}_{*t+1}^o, \tilde{k}_{*t+1}, \tilde{d}_{*t}\right\}$, as a market equilibrium. Of course, to do so, the set of tax parameters has to induce individuals to choose the optimal sequence of physical and human capital-labour ratios.

Using the feasibility constraint (12) evaluated at t, we can backward the arbitrage condition (22) one period and rewrite it in a way that resembles the first-order condition of the individual when she chooses the amount of resources invested in education (7):

$$
\left[f(\tilde{k}_{*t}) - \tilde{k}_{*t}f'(\tilde{k}_{*t})\right] e'(\tilde{d}_{*t-1}) = f'(\tilde{k}_{*t})\left(1 + \frac{e'(\tilde{d}_{*t-1})\Lambda(\tilde{k}_{*t+1}, \tilde{d}_{*t}, \tilde{c}_{*t}^m)}{f'(\tilde{k}_{*t})}\right), \qquad (26)
$$

where $\Lambda(\tilde{k}_{*t+1}, \tilde{d}_{*t}, \tilde{c}_{*t}^m) = (1+n)e(\tilde{d}_{*t})\tilde{k}_{*t+1} + (1+n)\tilde{d}_{*t} + \tilde{c}_{*t}^m > 0$. Mere comparison of (26) and (7) entails that the optimal education investment tax rate at period t, θ_{*t}, for all $t \geq 0$ will be:

$$
\theta_{*t} = -\frac{e'(\tilde{d}_{*t-1})\Lambda(\tilde{k}_{*t+1}, \tilde{d}_{*t}, \tilde{c}_{*t}^m)}{f'(\tilde{k}_{*t})} < 0. \qquad (27)
$$

This result can be stated as the following

Proposition 1 *When the social planner maximizes the non-discounted sum of utilities (17) and decentralizes the allocation of resources through the market mechanism, investment on education should be taxed along the entire growth path according to the tax rate given in (27).*

As far as the physical capital-labour ratio is concerned, the equilibrium condition was discussed above to be $\tilde{k}_{t+1} = \tilde{s}_t/e(\tilde{d}_t)(1+n) + \tilde{d}_t/e(\tilde{d}_t)$. Denoting $\tilde{s}_{*t}$ the amount of saving per unit of efficient labour made by each middle-aged to support $\tilde{k}_{*t+1}$, one can use (4)

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expression makes it apparent that different cardinalizations of the same ordinal preferences will entail different balanced growth paths (and, consequently, different optimal configurations of the tax parameters designed to decentralize them).

and (5), with the latter backwarded one period, to obtain the required lump-sum taxes and transfers for any period $t \geq 1$:[5]

$$\tilde{z}_{*t}^m = w_{*t} - (1 + r_{*t})\tilde{d}_{*t-1}(1 - \theta_{*t})/e(\tilde{d}_{*t-1}) - \tilde{s}_{*t} - \tilde{c}_{*t}^m \qquad (28)$$

$$\tilde{z}_{*t}^o = (1 + r_{*t})\tilde{s}_{*t-1} - \tilde{c}_{*t}^o \qquad (29)$$

Proposition 2 *When the social planner maximizes the non-discounted sum of utilities (17) and decentralizes the allocation of resources through the market mechanism, the sequence of lump-sum taxes or transfers on middle-aged and old-aged along the optimal growth path are given by (28) and (29).*

Admittedly, Proposition 2 is not very instructive in the characterization of the signs of $\tilde{z}_t^m$ and $\tilde{z}_t^o$, and in particular on whether lump-sum taxes in old-age are positive or negative. Del Rey and Lopez-Garcia (2013, Prop. 3) provide a result related the sign of $\tilde{z}_*^o$. In particular, they show that decentralizing the "Golden Rule" entails positive pensions to the elderly, i.e., $\tilde{z}_*^o < 0$, a result that obviously applies to the optimal balanced growth path when the social planner maximizes the non-discounted sum of utilities (17).

Although we are unable to characterize the sign of the lump-sum taxes along the entire growth path, we can at this point take ask a related question and inquire about the *intergenerational effects* of the optimal tax policy, i.e., the direction of the net intergenerational transfers implied by the tax system that allow the social planner to decentralize the optimal balanced growth path. To advance in the analysis, notice that along the

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[5]As for the optimal lump-sum taxes and education tax in period 0, we have that $\tilde{k}_0$ (and thus $\tilde{s}_{-1}$, w_0 and r_0), $\tilde{c}_0$ and $\tilde{d}_{-1}$ are given as initial conditions, so that (27), (28) and (29) become:

$$\theta_{*0} = -\frac{e'(\tilde{d}_{-1})\Lambda(\tilde{k}_{*1}, \tilde{d}_{*0}, \tilde{c}_{*0}^m)}{f'(\tilde{k}_0)}$$

$$\tilde{z}_{*0}^m = w_0 - (1 + r_0)\tilde{d}_{-1}(1 - \theta_{*0})/e(\tilde{d}_{-1}) - \tilde{s}_{*0} - \tilde{c}_{*0}^m$$

$$\tilde{z}_{*0}^o = (1 + r_0)\tilde{s}_{-1} - \tilde{c}_0^o$$

Together, they provide three equations to solved in θ_{*0}, $\tilde{z}_{*0}^m$ and $\tilde{z}_{*0}^o$.

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optimal balanced growth path, the individual's present value of lifetime resources in (16), $\tilde{\omega}_*$, is:

$$\tilde{\omega}_* = w_* - \frac{(1+r_*)\tilde{d}_*}{e(\tilde{d}_*)(1+n)} - \frac{\left[\theta_*(1+r_*)\tilde{d}_* - e(\tilde{d}_*)\tilde{z}_*^m\right]}{(1+r_*)e(\tilde{d}_*)}\left[e(\tilde{d}_*)(1+n) - (1+r_*)\right] \quad (30)$$

where $(1+r_*)$, w_* and $e(\tilde{d}_*)(1+n)$ are, respectively, the interest factor, the wage rate and the economy's growth rate along the "Golden Rule". Clearly, condition (24) assures that the third term on the right-hand side of (30) vanishes regardless of the sign of $\left[\theta_*(1+r_*)\tilde{d}_* - e(\tilde{d}_*)\tilde{z}_*^m\right]$, and thus we end up with $\tilde{\omega}_* = w_* - (1+n)\tilde{d}_*$, which is precisely the right-hand side of the feasibility constraint (12) evaluated along the "Golden Rule". Nevertheless, this should not obscure the crucial fact that from the government budget constraint (3) one gets $\left[\theta_*(1+r_*)\tilde{d}_* - e(\tilde{d}_*)\tilde{z}_*^m\right] = \tilde{z}_*^o/(1+n)$. Since, as stated above, optimal pensions in old age are positive, i.e., $\tilde{z}_*^o < 0$, (30) can be rewritten as:

$$\tilde{\omega}_* = w_* - (1+n)\tilde{d}_* + \frac{(-\tilde{z}_*^o)}{\left[e(\tilde{d}_*)(1+n)\right]^2}\left[e(\tilde{d}_*)(1+n) - (1+r_*)\right] = w_* - (1+n)\tilde{d}_* \quad (31)$$

The important point is that (31) is exactly the individual's present value of lifetime income in the presence of a pure *pay-as-you-go social security* that forces individuals to contribute the (positive) amount $(-\tilde{z}_*^o)/e(\tilde{d}_*)(1+n)$ in their middle age, entitling them to receive the (positive) pension benefit $(-\tilde{z}_*^o)$ in their old age. As individuals are actually obtaining the (counterpart in the present model of the) Biological Interest Rate $e(\tilde{d}_*)(1+n)$ through their "investment in shares of future generations" and the opportunity cost is exactly the market interest rate $(1+r_*)$, both terms cancel in present value along the optimal balanced growth path, as shown in (31). This point can also be easily illustrated by means of the physical capital market equilibrium condition (11), that along the optimal balanced growth path becomes:

$$e(\tilde{d}_*)(1+n)\tilde{k}_* = w_* - (1+n)\tilde{d}_* - \frac{(-\tilde{z}_*^o)}{e(\tilde{d}_*)(1+n)} - \tilde{c}_*^m - (1+n)\tilde{d}_* \quad (32)$$

Form (3) the optimal pay-as-you-go contribution will be given by:

$$\frac{-\tilde{z}_*^o}{e(\tilde{d}_*)(1+n)} = \frac{-\left[\theta_*(1+r_*)\tilde{d}_* - e(\tilde{d}_*)\tilde{z}_*^m\right]}{e(\tilde{d}_*)} > 0 \quad (33)$$

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so that we can state the following

Proposition 3 *The intergenerational effects of the optimal tax policy $(\tilde{z}_*^m, \tilde{z}_*^o, \theta_*)$ are equivalent to those of a pay-as-you-go social security system where middle-aged individuals contribute the amount in (33) when middle-aged and receive a pension benefit $(-\tilde{z}_*^o)$ when they retire.*

It should be stressed that it is the combination of the orthopaedics provided by the optimal values of $\tilde{z}_*^m$, $\tilde{z}_*^o$ and θ_*, the one that allows the decentralized behaviour of individuals to replicate the optimal balanced growth path. In other words, the *intergenerational income effects* associated with the scheme described in Proposition 3 have to be supplemented by the *price effects* in terms of investment in human capital induced by the tax on education reported in Proposition 1. If individuals faced the optimal social security scheme but not the optimal education tax, they would fail to achieve the "Golden Rule", and the same reasoning applies to the situation where the optimal education tax is in force but the intergenerational transfers are not optimally set. Notice also that the optimal lump-sum tax on middle-aged, $\tilde{z}_*^m$, can either be positive or negative but, together with the revenue obtained by taxing the repayment of education loans, will generate a net contribution in middle age to finance optimal pensions to the elderly.

It is important to emphasize that the result stated in Proposition 3 is in sharp contrast with the one that arises in life-cycle models with physical capital only and exogenous growth. Indeed, in such a setting, the nature of the optimal social security that supports the Two-Part Golden Rule depends in a crucial way on whether the laissez-faire capital-labour ratio is higher or lower than the optimum one. In other words, it entails a transfer from [resp. to] the younger generation to [resp. from] older one through a pay-as-you-go social security system [resp. a "reverse" pay-as-you-go system or a more-than-fully-funded one] when the interest rate is less [resp. greater] than the economy's growth rate in the laissez-faire steady state. In contrast, in the presence of human capital that drives endogenous growth, the intergenerational effects of the optimal policy mix are *always* equivalent to those of a pay-as-you-go social security system.

5 Concluding remarks

In this paper, we have used an overlapping generations model with physical and human capital to ascertain the consequences for optimality of a social planner adopting a social welfare criterion that treats all generations alike and is respectful of individual preferences. In particular, we have considered a social planner who maximizes an non-discounted sum of individual utilities defined over consumption per unit of efficient labour. We show that, given some initial conditions, an economy commanded by such a social planner converges to the Golden Rule defined in Del Rey and Lopez-Garcia (2013). Decentralizing the optimum requires taxing education along the entire growth path. With respect to lump-sum taxes on the middle-aged and old-aged individuals, no general result can be derived for the entire growth path. We know, however, from previous work that, along the optimal balanced growth path, pensions are positive. In this paper, we have moved a step forward and showed that the intergenerational effects of the optimal policy are equivalent to those of a pay-as-you-go social security system.

Although our social welfare function my seem ad hoc at first sight, it is worth emphasizing that, unlike what happens when we adopt alternative objectives, our optimal policy is independent of the degree of cardinalization of the utility function as well as the choice of an arbitrary discount rate. Other assumptions underlying the model are quite unrealistic, e.g. the assumption that individuals have access to perfect credit markets. It may well be the case that the insights emerging from the analysis differ when individuals face constraints when trying to borrow to finance their educational investments. We leave these issues for further research.

Acknowledgements

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