Comparison of efficiency measures for Spanish first division football teams using data envelopment and stochastic frontier analyses

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Abstract

The aim of this study was to analyse the robustness of the efficiency ratios calculated by the original version of Data Envelopment Analysis (DEA), the modified version, as proposed by Andersen and Petersen (1993), and stochastic frontier analysis. The study was performed on the football teams that played in the Spanish football First Division between 1998 and 2010. The results show that consistency conditions are generally met between the two versions of the DEA, but the correlation between them and stochastic frontier analysis is less. The conclusions offer some possible interpretations of the discrepancies identified.

Resumen

El objetivo de este estudio fue analizar la robustez de los ratios de eficiencia calculada por la versión original del análisis de envolvente de datos (DEA), la versión modificada, como se ha propuesto por Andersen y Petersen (1993) y el análisis de fronteras estocásticas. El estudio fue realizado en los equipos de fútbol que jugaron en el primera división del fútbol español entre 1998 y 2010. Los resultados muestran que generalmente se cumplen las condiciones de consistencia entre las dos versiones de la DEA, pero la correlación entre ellas y el análisis de fronteras estocásticas es menor. Las conclusiones ofrecen algunas posibles interpretaciones de las discrepancias identificadas.

1. Introduction

The interest in establishing the efficiency of economic units is to provide a mechanism for comparing results to design more effective policies and improve decision-making processes. This is because establishing the efficiency of a unit can detect whether differences, between results and others considered comparable, are due to the effect of the production environment or, conversely, internal or management factors of the particular business.

It would be preferable to have a proper definition and a precise method for calculating the understanding of the efficiency of an economic unit, so that it had the utility that Lovell (1993) attributed to it. However, there is no agreement, especially regarding calculation methods. In addition, despite the lack of unanimity regarding the definition of the term efficiency, and its confusion with similar terms such as efficacy or productivity, it is always considered that efficiency is related to the allocation of resources. Leibenstein (1966) proposed a differentiation between allocative efficiency and X-efficiency. In reality this distinction reflects the fact that the term “efficiency” is applied to the resource allocative function by both markets and agents. According to that author, allocative inefficiency measures only the impact of price distortions and amounts due to, for example, monopolies, restrictions on international trade, the existence of subsidies or government enterprises. However, this is under the assumption that companies use their resources efficiently. This internal efficiency is what Leibenstein (1966) called X-efficiency. In turn, as noted by Lovell (1993), internal efficiency can be divided into technical efficiency and price efficiency. The first refers to the ability to prevent waste by producing as much output as possible with the productive resources used or using minimal amounts of resources to obtain the product, ie adopting a physical standpoint. The second relates to the ability to combine inputs and outputs in optimal proportions in light of prevailing market prices.

This study applies the concepts of technical efficiency to the Spanish First Division football teams in the seasons between 1998 and 2010. There are several reasons for choosing this sector. Firstly, among the criteria that should be pursued in any investigation, is a reflection on the current economic situation and an analysis of the existing reality. In this regard, the tertiary sector currently has the greatest influence on the GDP of developed economies. This sector includes leisure activities and, more specifically, activities related to sport. Secondly, despite the passions raised by competitive sport among fans, it is necessary for clubs, as units with limited resources, to achieve their goals, and to be managed with economic criteria using the tools provided by the economy. Therefore, it should not be surprising that it is increasingly common for studies within this discipline to take sporting institutions as business samples to be analysed.

Since the aim of this study is to calculate the technical efficiency of Spanish First Division football teams, economic issues often at the centre of comments in the media are left aside. For

example, the personnel policy reflected in the contracts, salaries and incentives should be considered if the intention is to study the pricing efficiency of football teams. Also not under consideration, are the sources of financing or investments made (information in the financial accounts), or income from advertising and other marketing activities. Moreover, public perception of the value or quality of the players is based on their previous track record or other characteristics. However, this study adopts the physical point of view of technical efficiency, and only evaluates the absence or not of waste in the achievement of the final product from the amounts of productive resources used. This is done by taking the unit of analysis to be the team and not individual players, as it is considered that the business of football teams is, as referred to in Organisational Theory, team technology, where it is very difficult to assess the individual contribution of each member in the final result.

Regarding the production process of sports teams, Schofield (1988), Carmichael and Thomas (1995) and Carmichael, Thomas and Ward (2000) consider a resourceful system one in which the team's success depends on the performance of players during a game and this, in turn, depends on their skills as well as the work of the manager. Therefore, if this is applied to the production function of football teams, it could be considered to consist of two phases, each with its inputs and outputs:

- The first phase could be considered as the players' individual skills (sporting talent, physical condition and form, experience, etc) together with the work of the manager (work in training, tactics put into practice, line-ups, etc) in building up team skills to maximise the joint effort during competitive matches.
- The second phase is the results achieved from the previous phase in competition with the other teams. Attacking and defensive moves (as a result of the first stage) are taken as inputs and are transformed into success during the encounters, which are considered as the output of this second phase.

This study only examines the technical efficiency of Spanish First Division football teams on the pitch, ie in the second of the two phases mentioned.

The frontier methods introduced by Farell (1957) were used as the method of calculating efficiency. These measure the efficiency of a unit from the best observations of a previously selected sample, and therefore correspond to optimisation processes and not estimates of averages. There are several different approaches among these frontier models, which are summarised in Førsund, Lovell and Schmidt (1980) and Lovell (1993), although the most used are the deterministic nonparametric and stochastic frontier models. Lovell (1993) states the advantages and disadvantages of these two methods. Stochastic frontiers try to distinguish between random effects and inefficiency; deterministic nonparametric frontiers confuse any error in measuring with inefficiency.

Moreover, as stochastic frontiers are parametric, ie they specify the functional form of the production frontier, they are more sensitive to the risk of incorrect specification and may

therefore lead to confusing errors in the specification with technical inefficiency, while deterministic nonparametric frontiers are less prone to this error. As neither method is superior to the other, De Borger and Kerstens (1996) highlight the usefulness of a comparative analysis of these efficiency calculating methods and, therefore, recommend using both and checking the robustness of the efficiency values obtained.

Accordingly, this study uses both the deterministic nonparametric and stochastic frontier methods as alternatives for measuring the efficiency of Spanish football teams. It then analyses the consistency of results reached, as it is important to make sure the values from which the measurements are intended to be made, for the management of the units analysed, should reflect reality and not depend on the method used to reach them. Although Dawson, Dobson and Gerrard (2000 a and b) also analyse the consistency of the efficiency values for sporting institutions by different calculation methods, there are several differences with the approach taken in this work. Firstly, the work of those authors analyses the consistency between different stochastic frontier estimation methods, while this study considers multiple stochastic frontiers and chooses the one that best fits the data in accordance with the comparison tests proposed by Coelli, Rao and Battesse (1998).

The efficiency values obtained from them are then compared with those obtained by deterministic nonparametric frontiers. Secondly, the unit of analysis taken by Dawson, Dobson and Gerrard (2000 a and b) is the managers, while this study considers football teams. Thirdly, those authors take the English Premiership as their study sample. Moreover, this work calculates the efficiency values according to the original DEA version and the modified one proposed by Andersen and Petersen (1993), since the latter can discriminate between efficient organisations. Finally, to assess the consistency between the efficiency ratios calculated by the different methods, compliance with the conditions proposed by Bauer et al (1998), Cummins and Zi (1998) and Luo and Donthu (2005) were analysed.

This study is structured as follows: the second section briefly reviews the theoretical and methodological issues of stochastic and deterministic non-parametric frontiers. The stochastic frontier methodology is then applied to the Spanish First Division football team data, and the models that best fit the data for calculating efficiency values are chosen according to hypothesis tests. The fourth section evaluates the consistency between the values obtained from the previously selected stochastic frontiers and those obtained from deterministic nonparametric frontiers using various correlation coefficients. The conclusions are at the end of the article.

2. Efficiency calculation methods[1]

2.1 Stochastic frontier analysis

Because stochastic frontier analysis is a parametric method, the functional form of the production frontier has to be specified, and the Cobb-Douglas function was chosen. According to Lovell (1993), this is the most common. Therefore, the function to be estimated is:

$$\ln(y_i) = x_i\beta + v_i - u_i \qquad (1)$$

where,

$\ln(y_i)$ is the natural logarithm of output obtained by the economic agent, i

x_i is a vector whose first element is unity and the remaining elements are the logarithms of the quantities of the n inputs used by company, i

β is a vector of unknown parameters to be estimated

v_i is a random term

u_i is a non-negative random variable associated with technical efficiency.

This study followed the proposals of Coelli, Rao and Battese (1998) regarding the model assumptions on the distribution of random terms, the method of econometric estimation and hypothesis testing.

The random terms v_i represent the influence of measurement errors or other random factors on the output obtained, such as the effects of unspecified input variables in the production function. It is assumed that these random terms follow a normal distribution with zero mean and variance σ^2_v, and are independently and identically distributed.

One of the problems attributed to stochastic frontiers is the lack of justification for the distribution chosen for u_i. In response to this, two estimates were made: one considering that u_i was a random variable of semi-normal distribution, and the other that it was a truncated normal distribution. It is assumed that the terms u_i are independently and identically distributed, according to a truncation at zero of a Normal distribution with zero mean and variance σ^2 in the first case, and a Normal of mean μ and variance σ^2 in the second. Furthermore, it is assumed that the distribution is independent of the v_i distribution.

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1 Technical efficiency, which is mentioned throughout the study, is the overall technical efficiency broken down into pure technical efficiency and efficiency of scale. For this work, constant returns to scale were assumed, so that the efficiency results obtained by deterministic nonparametric frontier and stochastic analysis were comparable. Therefore, it was decided to keep the simplest term of technical efficiency, so as not to cause confusion between overall and pure technical efficiency.

To estimate the production function, the maximum likelihood method was used which, according to Coelli, Rao and Battese (1998) has better properties than alternative methods, for example, the Corrected Ordinary Least Squares method.

According to the model considered in this study, the technical efficiency for the operator i with an input orientation (TE_i) is calculated using the expression:

$$TE_i = \exp(-u_i) \qquad (2)$$

However, this expression contains the value of u_i that cannot be observed. In the methodology of Coelli, Rao and Battese (1998) the technical efficiency of each agent in the sample considered is calculated as the arithmetic mean of the predictors of individual technical efficiency. These values are between zero and one.

When you have a sample of several companies over several periods, the following production function can be estimated considering panel data:

$$\ln(y_{it}) = x_{it}\beta + v_{it} - u_{it} \qquad (3)$$

where all variables have the same meaning as in equation (1), but relating to the company i during the period t.

Using stochastic frontier analysis means several hypothesis tests can be performed to choose the model that best fits the data, as indicated by Coelli, Rao and Battese (1998). Firstly, they proposed to check the absence of technical inefficiency in the model. To do this, they compared the null hypothesis $\gamma = 0$ with the alternative $\gamma > 0$ where $\gamma = \frac{\sigma^2}{\sigma_s^2}$ and $\sigma_s^2 = \sigma^2 + \sigma_v^2$.

To perform this test, the generalised likelihood ratio[2] is used, given that the values of the alternative hypothesis are bounded on one side. Secondly, once the production function is estimated if u_i is a truncated Normal distribution with mean μ and variance σ^2, it can be checked whether the most appropriate model is a semi-normal distribution by comparing with the null hypothesis $\mu = 0$ using the generalised likelihood ratio test. Finally, when the estimate is made based on panel data, it can be considered if the effects of technical inefficiency are invariant over time or, conversely, if they vary systematically over time. In reality, this is as a result of using the maximum likelihood method in the proposed estimates because, if traditional estimation methods were used in panel data, there would be no need to specify a distribution for the inefficiency effects. However, Coelli, Rao and Battese (1998) recognise the superiority of the estimation by the maximum likelihood method, unless there is reason to believe that the inefficiency effects and the function regressors are not independent. If there is any change in

[2].- The generalised likelihood ratio is calculated from the expression:

$$LR = -2\{\ln[L(H_0)] - \ln[L(H_1)]\}$$

where $\ln[L(H_0)]$ and $\ln[L(H_1)]$ are the natural logarithms of the values of the likelihood function under the null hypothesis (H_0) and alternative (H_1). This value is then compared with χ^2 or the Kodde and Palm table (1986) according to whether the alternative hypothesis is an inequality or an order relation, respectively.

efficiency values over time, the distribution of the inefficiency effects is assumed to follow the below expression:

$$u_{it} = \{\exp[-\eta(t-T)]\}u_i \qquad (4)$$

where T is the last period considered in the panel, η is a scalar to be estimated and u_i is a random variable which can be considered as the technical inefficiency of the company i in the last period of the panel. Depending on the sign of η, the evolution of the efficiency of each company will be increasing or decreasing. Moreover, given the functional form proposed, the efficiency value order of the management companies does not change over time. Therefore, it is interesting to verify the hypothesis of no efficiency changes over time in this model by comparing the null hypothesis $\eta=0$ using the generalised likelihood ratio.

2.2 Deterministic nonparametric frontier analysis

Deterministic nonparametric frontier analysis is also known as Data Envelopment Analysis (DEA). These types of models do not consider a specific functional form for the frontier, but are estimated by linear programming techniques such as the envelope of observed values. Efficient agents are considered to be those that belong to the determined frontier. The most notable aspect of the method, as noted by Farrell (1957), is the mathematical formulation by linear programming, solving the following problem:

$$\begin{aligned} \text{P1} \qquad & \text{Min } \lambda_i \\ \text{s.a.} \quad & y_i \leq z_i Y \\ & \lambda_i x \geq z_i X \\ & z_i \in R_+^k \end{aligned}$$

where λ_i is the technical efficiency index with input orientation, y_i is the vector representing the quantities of m products produced by the company, Y is the matrix of range k.m representing the quantities of m products for the k companies in the sample, x is the amount of the n productive factors used by the company whose efficiency is being measured, X is the matrix of range k.n for quantities of n productive factors used by companies in the sample and z_i is a vector of intensity parameters determining observed factor and product combinations. When λ_i =1, the company analysed belongs to the isoquant and its production vector with a radial reduction of all its resources cannot be obtained.

When the consistency between the efficiency ratios obtained by following different calculation methods is analysed, Andersen and Petersen (1993) proposed using a modified version of Data Envelopment Analysis to discriminate and differentiate between the different

efficient units. In the classical version of Data Envelopment Analysis, all efficient units have an efficiency ratio equal to unity. The Andersen and Petersen proposal (1993) is to re-calculate the following linear programming problem for all efficient units:

$$
\begin{aligned}
\text{P2} \qquad & \text{Min } \lambda_i^* \\
\text{s.a.} \quad & y_i \leq z_i Y^* \\
& \lambda_i^* x_i \geq z_i X^* \\
& z_i \in R_+^{k-1}
\end{aligned}
$$

where y_i is the vector representing the quantities of m products produced by the efficient unit analysed, Y^* is the matrix of range (k-1). m representing the quantities of m products for the sample units excluding that analysed, x_i is the quantity of the n productive factors used by the unit whose efficiency is being measured, X^* is the matrix of range (k-1). n for the quantities of the n productive factors used by the sample units excluding the one being analysed and z_i is a vector of intensity parameters determining factor and product combinations observed. As the efficient units are not included in the reference sample, the value of the ratio λ_i^* no longer has an upper limit of unity, but can have higher values, and the higher the value, the more efficient the unit and therefore the company. The interpretation of λ_i^* according to Andersen and Petersen (1993) is the proportion by which the amount of productive resources used could be increased to maintain the efficiency of the unit tested.

3.- Stochastic frontier selection to measure the efficiency of Spanish first division football teams

The analysis of the efficiency of Spanish First Division football teams was done following the Espitia-Escuer and Garcia-Cebrían proposals (2004, 2006 and 2008) for the variables of the resources used and output generated. The former consists of the number of players used throughout the season, the attacking moves made, the minutes of possession and shots made. Defensive moves were not included as they were regarded as measures to adapt to the environment created by the play of opposing teams, rather than as actions directly aimed at the achievement of sports results.

Also the influence of opponents can be considered as consistent for the entire sample, as the data for the entire season were used, during which all teams play each other twice. Output is taken as the number of points achieved during the season, as recommended by Dawson, Dobson and Gerrard (2000 a) who highlight the importance of drawn matches in football and,

therefore, conclude that a measure based solely on a team's victories would not include all the results. Descriptive statistics of the data used in this work, corresponding to the seasons between 1998 and 2010, are shown in Table 1.

Table 1. Descriptive statistics of the data used.

		Attacking moves	**Number of players**	**Minutes of possession**	**Shots**	**Points**
Season 98/99	Maximum	5227	31	1149	514	79
	Minimum	4468	19	851	342	27
	Average	4798.15	24.55	956.55	418.20	52.00
	Std Deviation	164.62	2.70	77.72	52.65	13.52
Season 99/00	Maximum	5162	32	1215	611	69
	Minimum	4697	22	852	462	28
	Average	4954.70	25.90	963.20	514.45	51.40
	Std Deviation	124.67	2.88	76.64	42.43	10.13
Season 00/01	Maximum	5080	30	1176	637	80
	Minimum	4288	23	914	399	39
	Average	4594.00	25.75	1002.55	518.00	52.05
	Std Deviation	179.63	2.20	63.81	62.56	12.50
Season 01/02	Maximum	4608	30	1243	611	75
	Minimum	4254	21	933	427	37
	Average	4410.85	26.15	1042.80	497.10	51.95
	Std Deviation	88.81	2.30	82.97	49.87	10.63
Season 02/03	Maximum	4272	32	1271	590	78
	Minimum	3765	22	971	407	32
	Average	3975.35	27.40	1113.10	492.65	51.75
	Std Deviation	149.31	2.82	69.84	53.55	12.75
Season 03/04	Maximum	4268	32	1105	621	77
	Minimum	3872	23	863	383	26
	Average	4123.70	27.05	958.10	470.85	52.25
	Std Deviation	97.63	3.17	64.54	58.73	12.50
Season	Maximum	4399	36	1174	621	84

04/05	Minimum	4021	23	829	392	28
	Average	4225.45	27.20	939.95	471.35	52.00
	Std Deviation	128.12	2.75	83.36	50.16	14.36
Season 05/06	Maximum	4675	31	1116	678	82
	Minimum	4177	23	758	382	24
	Average	4444.25	27.30	876.10	481.60	51.75
	Std Deviation	150.68	2.34	90.66	69.75	14.67
Season 06/07	Maximum	4865	30	1201	586	76
	Minimum	4385	22	758	391	28
	Average	4549.25	26.35	870.40	474.30	52.10
	Std Deviation	120.72	2.46	98.81	53.34	13.47
Season 07/08	Maximum	4701	31	1191	589	85
	Minimum	4171	23	783	394	26
	Average	4455.10	27.25	899.25	477.75	52.65
	Std Deviation	146.24	2.34	91.72	52.46	14.25
Season 08/09	Maximum	4695	30	1206	719	87
	Minimum	4222	22	768	417	33
	Average	4441.35	26.45	886.50	503.70	52.85
	Std Deviation	120.66	2.48	104.60	82.02	14.52
Season 09/10	Maximum	4669	33	1264	801	99
	Minimum	4246	22	771	359	34
	Average	4442.25	27.20	893.35	489.35	52.25
	Std Deviation	119.55	3.29	117.35	95.25	18.60

Firstly, this study estimated a cross-sectional stochastic frontier for each season in the sample and, secondly, a single frontier for all data using the panel methodology. In the first case, the production function for each season is specified as:

$$\ln(\text{points})_i = \beta_0 + \beta_1 \ln(\text{attacks})_i + \beta_2 \ln(\text{players})_i + \beta_3 \ln(\text{possession})_i + \beta_4 \ln(\text{shots})_i + v_i - u_i \quad (5)$$

where the subscript i denotes each of the teams in the First Division for the appropriate season and varies between 1 and 20. The regression for the second case is estimated as follows:

$$\ln(points)_{it} = \beta_0 + \beta_1 \ln(attacks)_{it} + \beta_2 \ln(players)_{it} + \beta_3 \ln(possession)_{it} + \beta_4 \ln(shots)_{it} + v_{it} - u_{it} \quad (6)$$

where the subscript i has the same meaning as in (5) and t denotes the season. It should be noted that this is an unbalanced panel, as the 20 teams that made up the First Division in Spain were not the same during the twelve seasons analysed, due to promotion and relegation occurring at the end of each season.

The regressions (5) and (6) were estimated according to different assumptions and by ordinary least squares (OLS). The signs of the regressors agree in virtually all cases for the OLS and stochastic frontier methodologies.

To analyse the consistency between the efficiency values obtained by using deterministic and stochastic methods it was thought appropriate to check the hypothesis beforehand. This was done using statistical tests to determine which of the estimated stochastic frontiers gave the best fit to the data; this was then used as a reference for calculating the efficiency of Spanish First Division football teams by stochastic methods. This procedure was considered more rigorous than analysing correlation coefficients for the coherence between the efficiency values obtained by different stochastic frontier estimates and those obtained by deterministic nonparametric frontiers. The generalised likelihood ratio was used to carry out all the tests, as proposed by Coelli, Rao and Battese (1998).

Firstly, it was checked if the term u_i followed a semi-normal distribution by comparing with the null hypothesis, H_0: $\mu = 0$. This hypothesis was rejected only for the 1998/1999 season and for the pooled sample. Therefore, the efficiency of the teams was calculated using stochastic frontiers with the regression where u_i is a truncated normal distribution for the season 1998/1999 and for the pooled sample, and a semi-normal distribution for the rest of the seasons. However, in response to the verification of the absence of inefficiency for the seasons 2000/2001, 2002/2003, 2003/2004, 2005/2006, 2006/2007, 2008/2009 and 2009/2010, for both u_i as a truncated and a semi-normal distribution, the null hypothesis, H_0: $\gamma=0$, was accepted. Therefore, the best estimate is the one provided by the OLS method[3]. However, the efficiency levels were calculated using stochastic frontiers so that comparisons could be made in a later section. Finally, to check that the efficiency does not change over time in models taking data panels, the null hypothesis, H_0: $\eta=0$ was compared by taking the likelihood function value of the two estimated models, assuming that the term u_i has a truncated normal distribution, as was previously accepted. The result reached was the acceptance of the null hypothesis, therefore the regression used was that estimated under the assumption that efficiency is invariant over time when calculating the efficiency values of Spanish First Division football teams, giving a single value per unit analysed.

[3] This estimate is also better than that provided for the 1998/1999 season under the assumption of a semi-normal distribution for u_i and for the 2004/2005 season under the assumption of truncated normal distribution of u_i. However, these models were rejected when checking the hypothesis.

4.- Analysis of the consistency of efficiency values calculated by stochastic and deterministic nonparametric frontier analyses

The efficiency was calculated separately for each of the seasons in the sample and by taking all the data together using Data Envelopment Analysis (DEA) and the modified version of Andersen and Petersen (1993), and the same variables representing output and input were considered for stochastic frontiers. It was also checked that the units rated as efficient by both DEA and its amended version did not give an excess of input or a lack of output. This ensured that the efficiency values calculated by different methods were comparable. This was also the reason why the efficiency values were calculated using both the traditional version of DEA and the amendment proposed by Andersen and Petersen (1993): while the efficient units have values equal to unity in the traditional version, in the modified version they have values greater than or equal to unity. This allows them to be discriminated, providing the efficiency is not a bounded variable. This property of the calculated efficiency levels is more consistent with those obtained by stochastic frontiers, since usually all units have different values with this method and they can be ranked accordingly.

To check the consistency of the results obtained in this study we followed the proposals of Bauer et al (1998). These authors proposed a set of consistency conditions for the efficiency measurements obtained from different methods to make them useful in making decisions. These conditions were:

a) Efficiency results generated by different methods should have similar values for the mean, standard deviation and other similar statistical properties.

b) The different methods should rank the units under study in a similar way depending on their efficiency.

c) The different methods should reasonably identify the best and worst performing organisations with the same units.

d) The efficiency values obtained by all methods should be reasonably consistent with other outcome measurements.

e) The results from all methods should be stable over time and consistent with market conditions.

Cummins and Zi (1998) also consider the conditions a, b and d in verifying consistency between the results obtained from applying different efficiency calculating methods; while Luo and Donthu (2005) consider conditions a and b together in the calculation of the Pearson correlation coefficient.

Two fundamental reasons for studying the efficiency of football teams can be considered. Firstly, as in any other economic sector, to assess the absence of waste in the use of productive resources, and to propose measures leading to more rational use of the inputs of organisations identified as inefficient; and secondly, to assess whether the efficient use of resources is rewarded with higher positions in the classification of competitions. The latter is because assessment based solely on sports results, regardless of the amount of resources used to achieve them, dominates in sport in general and football in particular. Therefore, this study sets out to analyse the consistency of different methods for calculating the efficiency of Spanish First Division football teams in evaluating these two aspects. Thus, to analyse the consistency between methods of assessing efficient use of resources, the Pearson correlation coefficient is calculated together with verification of the conditions a (using Welch's t test for the means) and b (using the Kendall Tau-b index) in each of the years under study, both separately and for the pooled sample. Condition c was checked by calculating the percentage of teams ranked similarly, using the different methods, among those classified as having the best and worst practices in each of the different seasons. In addition, to verify condition d, an extra outcome measure was taken of the points obtained by each team at the end of the season, and the consistency between this result and efficiency was analysed, both in terms of the values obtained (by calculating the Pearson correlation coefficient) and compared to the classifications obtained with respect to both variables (via the Kendall Tau-b index). In checking this condition via the Pearson correlation coefficient, the results for each of the seasons analysed and for the pooled sample are again taken in this study. However, verification using the Kendall Tau-b index was only done for each of the seasons separately. This is because, if the pooled sample was taken, the points order considered by the index would not correspond with the classification actually obtained by the teams in each of the seasons[4]. The results of the correlation coefficients obtained are shown in Tables 2 to 10.

Table 2 shows the Pearson index has a greater than 95% correlation between the two versions of the DEA for all seasons analysed. The correlation between the efficiency ratios calculated via stochastic frontiers and either of the two DEA versions, however, is more varied. Thus, the 1999/2000 season shows a maximum value above 90% for both cases while the minimum values in the sample are seen in the 2008/2009 season: almost 55% with the original DEA and 46% with the amended version. Also, correlations over 75% were found between the efficiency gained by using stochastic frontiers and the two DEA versions during four seasons (1998/1999, 2002/2003, 2006/2007 and 2007/2008), with the same occurring between the stochastic frontiers and the original DEA version in the 2000/2001 season. In each of the seasons studied, is can also be seen that, although the correlation between the efficiency values obtained by the stochastic frontiers and both versions DEA are similar, there is always a higher correlation with the original version than with the modified version.

4. The consistency condition e was not verified in this work, because it would make little sense in a case such as this where competition conditions are different every season.

Table 2. Pearson correlation coefficient values.

Season	Calculation method	Original DEA	Modified DEA	Stochastic frontier
1998/1999	Original DEA	1	0.96641292	0.83745876
	Modified DEA		1	0.83618463
	Stochastic frontier			1
1999/2000	Original DEA	1	0.99450324	0.9169814
	Modified DEA		1	0.91092181
	Stochastic frontier			1
2000/2001	Original DEA	1	0.98205045	0.76332267
	Modified DEA		1	0.73274433
	Stochastic frontier			1
2001/2002	Original DEA	1	0.96682623	0.58423138
	Modified DEA		1	0.56587846
	Stochastic frontier			1
2002/2003	Original DEA	1	0.98889434	0.7923316
	Modified DEA		1	0.75833634
	Stochastic frontier			1
2003/2004	Original DEA	1	0.97790959	NA*
	Modified DEA		1	NA*
	Stochastic frontier			1
2004/2005	Original DEA	1	0.98232825	0.72865815
	Modified DEA		1	0.67829785
	Stochastic frontier			1
2005/2006	Original DEA	1	0.98750483	0.66236915
	Modified DEA		1	0.63573198
	Stochastic frontier			1
2006/2007	Original DEA	1	0.98550752	0.88319421
	Modified DEA		1	0.81433857
	Stochastic frontier			1
2007/2008	Original DEA	1	0.99006148	0.83042115
	Modified DEA		1	0.80671267
	Stochastic frontier			1
2008/2009	Original DEA	1	0.98074504	0.54932966
	Modified DEA		1	0.46476637
	Stochastic frontier			1

2009/2010	**Original DEA**	1	0.95411418	0.68227884
	Modified DEA		1	0.57697852
	Stochastic frontier			1

* These results could not be calculated because the efficiency ratios calculated using stochastic frontiers are very similar for all teams in the season.

The results for Welch's t test in Table 3 show that the differences between the means in the original and modified DEA versions are not significant for all seasons studied. Moreover, in the seasons 2000/2001, 2001/2002, 2002/2003, 2003/2004 and 2008/2009, the average efficiency values obtained by stochastic frontiers is significantly different from the average efficiency calculated with any of the two DEA versions. Also, the seasons 2005/2006 and 2009/2010 have significant differences between the average efficiency calculated using the original DEA version and stochastic frontiers.

Table 3. Welch's t test results.

Season	**Calculation methods**	**Modified DEA**	**Stochastic frontier**
1998/1999	**Original DEA**	-0.3154	-0.3723
	Modified DEA		0.0058
1999/2000	**Original DEA**	-0.1237	-0.8166
	Modified DEA		-0.6588
2000/2001	**Original DEA**	-0.1991	-7.3602*
	Modified DEA		-6.2027*
2001/2002	**Original DEA**	-0.2121	-1.6229
	Modified DEA		-1.2900
2002/2003	**Original DEA**	-0.1462	-2.2549*
	Modified DEA		-1.9600*
2003/2004	**Original DEA**	-0.2908	-7.7882*
	Modified DEA		-6.0339*
2004/2005	**Original DEA**	-0.2308	-1.0061
	Modified DEA		-0.6759
2005/2006	**Original DEA**	-0.1978	-1.9609*
	Modified DEA		-1.5558
2006/2007	**Original DEA**	-0.2282	-0.5205
	Modified DEA		-0.2202
2007/2008	**Original DEA**	-0.1907	-1.5126

	Modified DEA		-1.2191
2008/2009	**Original DEA**	-0.2846	-5.3267*
	Modified DEA		-4.278*
2009/2010	**Original DEA**	-0.404	-2.2762*
	Modified DEA		-1.4267

* The values marked with an asterisk are those where the differences in means are significant, at 90% or more.

The Kendall Tau-b correlation coefficient calculates the consistency between the rankings provided by different criteria. Table 4 shows the results for this index for efficiency measurements of Spanish First Division football teams calculated by different methods. These values show a correlation of over 98% between the rankings provided by efficiency ratios calculated via both DEA versions for all analysed periods, which can be explained by the fact that the inefficient teams have the same efficiency index in the two DEA versions. This could also explain the very similar values found for the Kendall Tau-b correlation coefficients for the ranking given by the efficiency calculated from stochastic frontiers and the two DEA versions for each of the seasons studied. However, these values show a wide variation between seasons, with the maximum in season 1999/2000 (over 80%) and minimum for 2005/2006 (42%). Moreover, since the modified DEA version assigns values higher than one for efficient teams, and consequently they can be placed in order, one would expect the Kendall Tau-b coefficient for the stochastic frontier method to be higher for the modified DEA version than for the original, however, the opposite happens in seasons 2000/2001, 2005/2006, 2006/2007 and 2008/2009.

Table 4. Kendall Tau-b correlation coefficient values.

Season	**Calculation method**	**Original DEA**	**Modified DEA**	**Stochastic frontier**
1998/1999	**Original DEA**	1	0.99736495	0.6174164
	Modified DEA		1	0.62105263
	Stochastic frontier			1
1999/2000	**Original DEA**	1	0.99736495	0.8179448
	Modified DEA		1	0.82105263
	Stochastic frontier			1
2000/2001	**Original DEA**	1	0.99207385	0.64225852
	Modified DEA		1	0.63716788
	Stochastic frontier			1
2001/2002	**Original DEA**	1	1	0.36842105
	Modified DEA		1	0.36842105
	Stochastic frontier			1

2002/2003	**Original DEA**	1	0.99736495	0.72295766
	Modified DEA		1	0.72631579
	Stochastic frontier			1
2003/2004	**Original DEA**	1	0.99736495	NA*
	Modified DEA		1	NA*
	Stochastic frontier			1
2004/2005	**Original DEA**	1	0.99736495	0.54353751
	Modified DEA		1	0.54736842
	Stochastic frontier			1
2005/2006	**Original DEA**	1	0.99736495	0.42744212
	Modified DEA		1	0.42105263
	Stochastic frontier			1
2006/2007	**Original DEA**	1	0.98408386	0.79154572
	Modified DEA		1	0.75789474
	Stochastic frontier			1
2007/2008	**Original DEA**	1	0.99736495	0.6174164
	Modified DEA		1	0.62105263
	Stochastic frontier			1
2008/2009	**Original DEA**	1	0.98408386	0.44690968
	Modified DEA		1	0.43281571
	Stochastic frontier			1
2009/2010	**Original DEA**	1	0.99736495	0.49076688
	Modified DEA		1	0.49473684
	Stochastic frontier			1

* These results could not be calculated because the efficiency ratios calculated using stochastic frontiers are very similar for all teams in the season.

Table 5 shows the percentage agreement between the teams that the modified DEA version[5] and the stochastic frontiers detected as having the best and worst practices. Following the proposals of Bauer et al (1998), they were ranked according to the season the football teams participated in the Spanish First Division based on their efficiency ratios calculated using these two methods. It was considered that the top 25% of the sample in each season represented the best practice, and the bottom 25%, the worst practice. The percentage of teams appearing in the most efficient group and those in the least efficient group were then calculated using the two classifications. The values in Table 5 show a degree of agreement greater than 50% in all but three cases (the most efficient teams in seasons 2001/2002 and 2009/2010, and the least efficient teams in season 2005/2006). It can also be seen that the most efficient teams are the same according to the two methods in the 2000/2001 season, while the least efficient also agree for seasons 2000/2001 and 2006/2007. Finally, both values were close in those seasons where the percentage of agreement between the most and least efficient teams was less than 50%.

Table 5. Percentage agreement between the most and least efficient teams according to the modified DEA version and stochastic frontiers.

Season 1998/1999		**Season 1999/2000**		**Season 2000/2001**		**Season 2001/2002**		**Season 2002/2003**		**Season 2003/2004**	
Most efficient teams	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**
0.8	0.6	0.6	0.8	1	1	0.4	0.6	0.8	0.8	0.8	0.8

Season 2004/2005		**Season 2005/2006**		**Season 2006/2007**		**Season 2007/2008**		**Season 2008/2009**		**Season 2009/2010**	
Most efficient teams	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**	**Most efficient teams**	**Least efficient teams**
0.6	0.8	0.6	0.4	0.8	1	0.6	0.6	0.8	0.6	0.4	0.6

The last consistency condition between efficiency calculation methods verified in this work is regarding the relationship between the different efficiency ratios and other performance

[5] The percentage agreement with the original DEA version was not analysed. This is because this method assigns an efficiency ratio equal to one for all efficient teams, and the representatives of best practice could not be determined when they exceeded 25% of the sample. Moreover, the percentage agreement of the worst practices between the two DEA versions would be 100% and each would have the same value compared with the stochastic frontiers.

measures. Having taken the Spanish First Division football teams as the sample under study, it was considered of interest to analyse the relationship between the efficiency ratios and each team's results as measured by points obtained in the season. In turn, the correlation between efficiency and points was analysed by calculating the Pearson correlation coefficient, and the correlation between the rankings provided by different efficiency calculation methods and the points gained (which tallies with the position in the table at the end of the season) via the Kendall Tau-b index. The results for both indices are shown in Table 6.

Table 6. Correlation between sporting results and team efficiency.

Season	Calculation method	Pearson coefficient	Kendall Tau-b index
1998/1999	**Original DEA**	0.94017303	0.8179448
	Modified DEA	0.94162386	0.81052632
	Stochastic frontier	0.81397776	0.6
1999/2000	**Original DEA**	0.97149833	0.85869415
	Modified DEA	0.96957822	0.86175089
	Stochastic frontier	0.87484465	0.70216739
2000/2001	**Original DEA**	0.92423494	0.77513891
	Modified DEA	0.94371206	0.77437264
	Stochastic frontier	0.70885432	0.63046242
2001/2002	**Original DEA**	0.93128558	0.86941062
	Modified DEA	0.93831725	0.86941062
	Stochastic frontier	0.35764838	0.27202418
2002/2003	**Original DEA**	0.982766	0.87533464
	Modified DEA	0.97638856	0.86773701
	Stochastic frontier	0.73274583	0.64551168
2003/2004	**Original DEA**	0.98094496	0.85869415
	Modified DEA	0.96331704	0.86175089
	Stochastic frontier	NA*	NA*
2004/2005	**Original DEA**	0.95497253	0.86635971
	Modified DEA	0.97499563	0.858743
	Stochastic frontier	0.60720513	0.44270602
2005/2006	**Original DEA**	0.96686755	0.81167394
	Modified DEA	0.96499169	0.81482622
	Stochastic frontier	0.57203783	0.33862908
2006/2007	**Original DEA**	0.95234348	0.88109395
	Modified DEA	0.97201083	0.89366759

	Stochastic frontier	0.73834451	0.68088959
2007/2008	**Original DEA**	0.96514614	0.86171432
	Modified DEA	0.96977055	0.85413845
	Stochastic frontier	0.74751461	0.51460515
2008/2009	**Original DEA**	0.88628989	0.87263193
	Modified DEA	0.94256543	0.88007823
	Stochastic frontier	0.30270529	0.38202872
2009/2010	**Original DEA**	0.92789618	0.84350429
	Modified DEA	0.96854508	0.8465727
	Stochastic frontier	0.45723358	0.40212203

* These results could not be calculated because the efficiency ratios calculated using stochastic frontiers are very similar for all teams in the season.

It can be seen that the correlation between the efficiency values calculated by both DEA versions and the points obtained by the teams is over 90% for all seasons except 2008/2009 for the original DEA version. The correlation between the efficiency ratios obtained by DEA and points does not show any generally higher values for one version, as there are seasons when the consistency is higher with the original and in others when the opposite occurs. Although the Pearson correlation coefficient between the efficiency calculated via stochastic frontiers and points is lower for all seasons with DEA, the values are all above 70% except in the seasons 2001/2002, 2004/2005, 2005/2006, 2008/2009 and 2009/2010. The correlation between the rankings provided by efficiency ratios and points show similar characteristics: the Kendall Tau-b index has similar values for the two DEA versions with neither being clearly superior and, again, the value of the index for efficiency values obtained using stochastic frontier is less in all cases. Also, in all seasons and for all three methods of calculation, the Kendall Tau-b index was lower than the Pearson coefficient. Thus, both DEA versions have values between 80% and 90% for all seasons except 2000/2001, which is slightly below 80%, and stochastic frontier calculations have smaller index values, reaching a minimum of 27% in the 2001/2002 season.

Calculations considering the sample as a whole required the most efficient teams to be identified on the basis of comparison with a larger reference set and, consequently, with respect to different technologies. This is why practically the same consistency analysis was done for efficiency ratios as with each season separately.[6]

6. The reason the correlation between the best and worst practice for the pooled sample was not analysed was because, having calculated the stochastic frontier efficiency with no change in time, it would have given a ranking with the same teams occupying the top positions as many times as they appeared in the sample, followed by the next, and so on, for the entire study sample. Also, the Kendall Tau-b index was not calculated for efficiency and sporting results because, having taken the entire sample as a whole, the ranking provided by the points variable would not necessarily coincide with the position in the table reached by the team in each season.

Table 7 shows that the correlation between the efficiency values calculated with the two DEA versions is very high, and is less between either of them and the results from applying stochastic frontiers, although in both cases they were above 60 %.

Table 7. Pearson correlation coefficient values for the sample taken as a whole.

		Original DEA	**Modified DEA**	**Stochastic frontiers**
PEARSON	**Original DEA**	1	0.995272	0.620985
	Modified DEA		1	0.6161516
	Stochastic frontiers			1

Welch's t test values for the efficiency ratios of the pooled sample shown in Table 8 show that differences in the means of the ratios calculated with the two DEA versions are not significant. However, the mean between any of them and those from the stochastic frontiers are different. One reason for this discrepancy may be that, while the application of DEA to the pooled sample gives different efficiency ratios for a team in each of the seasons, the stochastic frontier estimates used in this study did not permit any variation over time, so that a team was assigned the same efficiency value for all the seasons it appeared in the First Division between 1998 and 2010.

Table 8. Welch's t test values for the pooled sample.

Welch's t test	**Modified DEA**	**Stochastic frontiers**
Original DEA	-0.1309	-9.4171*
Modified DEA		-9.0484*

* The values marked with an asterisk are those where the differences in means are significant at 90% or more.

Table 9 shows the correlations between the rankings provided by the three efficiency calculation methods used in this work calculated using the Kendall Tau-b index. It can be seen that the correlation between the two DEA versions is very high, but when the stochastic frontier ranking is compared with either of the other two methods, the Kendall Tau-b index does not reach 50%.

Table 9. Kendall tau-b correlation coefficient values for the pooled sample.

KENDALL	**Original DEA**	1	0.998256	0.436978
	Modified DEA		1	0.467953
	Stochastic frontiers			1

Finally, the correlation between the efficiency values calculated for all the teams in the sample and the points obtained was calculated using the Pearson correlation index. The values are high for any of the three methods used, exceeding 85 % for the two DEA versions.

Table 10. Correlation between sports results and efficiency of the pooled sample.

Calculation method	**Pearson index**
Original DEA	0.8701505
Modified DEA	0.8742383
Stochastic frontiers	0.7096724

5.- Conclusions

The aim of this study was to evaluate the consistency of the efficiency ratios calculated by Data Envelopment Analysis (in its original and modified versions) and stochastic frontiers. The study sample taken was the Spanish First Division football teams that played in the seasons between 1998 and 2010. The consistency was assessed via correlation coefficients, mean tests and percentage agreement of the teams with the best and worst practice.

The results obtained for each of the seasons studied show a high correlation between the efficiency calculated by the two DEA versions in all the analyses performed (between the values obtained, in the ranking they provide and in their mean). Although the Pearson indices obtained for assessing the consistency of the efficiency ratios calculated by both DEA versions and stochastic frontiers gave high values in several of the seasons studied, they are not conclusive, as low values were also obtained. Welch's t test showed no significant differences between the efficiency calculated using the modified DEA and stochastic frontiers in five of the twelve seasons considered. Therefore, the efficiency average is more similar between these two methods than between the original DEA version and stochastic frontiers, whose differences are significant in two other seasons.

The Kendall Tau-b correlation coefficients calculated in the study do not show a general consistency between the rankings of efficiency ratios obtained by DEA or stochastic frontiers, as they take high or low values depending on the season in question. However, the modified DEA and stochastic frontiers show relatively high percentage agreement when identifying the most and least efficient teams in each season. Finally, the consistency between the points obtained by the teams and the efficiency is higher if we consider the values of those variables rather than looking at the rankings they provide, and in both cases the correlation is higher with either of the two DEA versions than with stochastic frontiers.

If efficiency is calculated by the three methods used in this work, but with a larger sample that includes all the seasons studied together, the consistency between different methods is no more evident. Also, even though the Pearson index between the efficiency ratios and the points obtained is high for all three methods, the correlation of the values and rankings between the stochastic frontiers and DEA is not high. In addition, the means test shows significant differences between the average efficiency calculated using stochastic frontiers and other methods.

If the results had shown a conclusive and generally very high consistency for the efficiency ratios between the DEA methods and stochastic frontiers, either could have been used to evaluate the use of resources made by football teams, and either analysis could have been chosen depending on its ease of calculation or familiarity. However, the lack of clear conclusions from this study could lead to a number of considerations to be taken into account.

Within the time frame of this study, all the calculations used to assess the robustness of the efficiency ratios obtained by different methods indicate a high consistency for a number of seasons, eg 1999/2000. Also, there are others (eg, 2008/2009) where the consistency analyses between methods provide very low values. In other words, if the same variables for inputs and outputs are used and the samples in each season are very similar in characteristics, such as the size and properties of the organisations, and the same evidence of consistency between the calculation methods used is not found, it may be useful to make recommendations for those cases where discrepancies are detected beyond those relating to the use of methods for calculating efficiency.

If, in a given sample, the means tests show a significant difference between values obtained with different methods, it would not be advisable to make judgments of the management of organisations based on the average efficiency ratios. Also, if the calculation of efficiency through frontiers is intended to detect which are the organisations that do not squander their resources to guide the inefficient ones to take action to try to improve their efficiency, detecting consistency among the best and worst practices by the different methods would be more relevant than consistency between the rankings of the sample they provide, as found in this study. Finally, the lack of robustness between efficiency and other result variables would have to be interpreted in each case. In this study, with the sample being Spanish First Division football teams and their scores used as a performance variable, the high correlation between

the efficiency calculated by DEA and the points obtained would lead to the conclusion that efficient teams are those which occupy the top positions in the league table. Moreover, the lowest correlation with the efficiency calculated with stochastic frontiers could be interpreted as a lack of a relationship between efficiency and sports scores, or even as evidence that competitions cannot be won without wasting resources. Clearly, the question of whether efficient teams are rewarded by better sports results would not be answered by results obtained from this study. It would be appropriate to state that, in the case of stochastic frontiers, because a ratio of one is rarely assigned to an organisation in the sample, the range of variation of its efficiency values is lower than in DEA, and especially the modified version. This may be why the efficiency calculated with DEA shows the highest correlation with the points earned by the teams.

A possible solution, therefore, would be to use another results variable, or to transform the points variable so that its range of variation is lower. On the other hand, perhaps the inclusion of more variables which are representative of the production function of soccer teams in their games, or the consideration of the previous preparation stage for encounters as a farther step in the production process, would provide more conclusive results about the influence of efficiency on sports results.

The values of efficiency that provide frontier methods are not absolute values, but the result of the comparison among the units composing the sample under study. The findings of this study show that these values are not only sensitive to the sample used, but that different measurement methods can provide different efficiency ratios. Usefulness of these tools for organizations in general and for football clubs in particular, depends on the robustness in their results. This robustness guarantees that proposals made in order to improve efficiency are convenient regardless of the method of calculation used.

The differences in the results showed in this work don’t invalidate the use of frontier methods, but show the need to make additional recommendations, which in turn can be interpreted as future lines of research.

First, depending on the purpose for the calculation of efficiency, only one of the five types of consistency suggested by Bauer et al. (1998) could be determined and will be necessary and, in consequence, the other four could be considered as irrelevant.

Second, both DEA and stochastic frontiers estimate the production function for organizations. Therefore, the more representative of inputs and outputs are the variables chosen, efficiency ratios will conform better to reality, regardless of the method used for calculation. Besides, a criterion to evaluate the consistency of efficiency ratios is their comparison with other variables of results; so the need for a correct choice of the variables must be applied also to these variables of results, which in the case of football teams would be the sporting success, but it can be measured in different ways (goals, matches won, points...).

Neither DEA nor stochastic frontiers provide mechanisms to assess the relationship of causality between the variables chosen as representative of productive resources and products. In this regard Golany and Roll (1989) propose the use of regressions or the opinion of expert managers in the sector under study. These authors also recommend the inclusion as inputs of only factors that determine the efficiency and they suggest to left out variables that would explain the differences observed in the efficiency ratios.

References

ANDERSEN, P. y PETERSEN, N.C. (1993): "A Procedure for Ranking Efficient Units in Date Envelopment Analysis", Management Science, vol. 39, nº10.

BAUER, P.W., BERGER, A.N., FERRIER, G.D. and HUMPHREY, D.B. (1998): "Consistency Conditions for Regulatory Analysis of Financial Institutions: A comparison of Frontier Efficiency Methods", Journal of Economics and Business, vol. 56, nº2.

CARMICHAEL, F. and THOMAS, C. (1995): "Production and efficiency in team sports: an investigation of rugby league football", Applied Economics, vol. 27, nº 9.

CARMICHAEL, F., THOMAS, D. and WARD, R. (2000): "Team Performance: The Case of English Premiership Football", Managerial and Decision Economics, vol. 21, nº 1.

COELLI, T, RAO, D.S.P. and BATTESE, G.E. (1998): An Introduction to Efficiency and Productivity Analysis, Kluwer Academic Publishers.

CUMMINS, J.D. and ZI, H.C. (1998): "Comparison of Frontier Efficiency Methods: An Application to the U.S. Life Insurance Industry", Journal of Productivity Analysis, vol. 10, nº2.

DAWSON, P., DOBSON, S. and GERRARD, B. (2000, a): "Estimating Coaching Efficiency in Professional Team Sports: Evidence from English Association Football", Scottish Journal of Political Economy, vol. 47, nº 4.

DAWSON, P., DOBSON, S. and GERRARD, B. (2000, b): "Stochastic Frontiers and the Temporal Structure of Managerial Efficiency in English Soccer", Journal of Sports Economics, vol. 1, nº 4.

DE BORGER, B. and KERSTENS, K. (1996): "Cost efficiency of Belgian local governments: A comparative analysis of FDH, DEA and econometric approaches", Regional Science and Urban Economics, vol. 26, nº2.

GOLANY, B. and ROLL, Y. (1989): "An Application Procedure for DEA", OMEGA. International Journal of Management Science, vol. 17, nº3.

ESPITIA-ESCUER, M. and GARCIA-CEBRIAN, L.I. (2004): "Measuring the Efficiency of Spanish First-Division Soccer Teams", Journal of Sports Economics, vol. 5, nº4.

ESPITIA-ESCUER, M. and GARCIA-CEBRIAN, L.I. (2006): "Performance in sports teams: results and potential in the professional soccer league in Spain", Management Decision, vol. 44, nº8.

ESPITIA-ESCUER, M. and GARCIA-CEBRIAN, L.I. (2008): "Measuring the Productivity of Spanish First Division Soccer Teams", European Sport Management Quarterly, vol. 8, nº3.

FARRELL, M.J. (1957): "The Measurement of Productive Efficiency", Journal of the Royal Statistical Society, Serie A, vol. 120, Part III.

FØRSUND, F.R., LOVELL, C.A.K. and SCHMIDT, P. (1980): "A survey of frontier production functions and their relationship to efficiency measurement", Journal of Econometrics, vol. 13, nº1.

KODDE, D.A. and PALM, F.C. (1986): "Wald Criteria for Jointly Testing Equality and Inequality Restrictions", Econometrica, vol. 54, nº5.

LEIBENSTEIN, H. (1966): "Allocative efficiency vs. «X-efficiency»", American Economic Review, vol. LVI, nº3.

LOVELL, C.A.K. (1993): "Production Frontiers and Productive Efficiency", in "The Measurement of Productive Efficiency", Fried, H.O., Lovell, C.A.K. and Schmidt, S.S. (ed). Oxford University Press.

LUO, X. and DONTHU, N. (2005): "Assessing advertising media spending inefficiencies in generating sales", Journal of Business Research, vol. 58, nº1.

SCHOFIELD, J.A. (1988): "Production functions in the sports industry: an empirical analysis of professional cricket", Applied Economics, vol. 20, nº 2.