THE FORMAL DISCIPLINE THEORY AND MENTAL LOGIC

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Abstract

The formal discipline theory claims that the study of mathematics develops individuals' logical skills. Attridge and Inglis carried out an experiment in order to check whether or not that theory holds. Their conclusion was that their results showed that, while mathematics can indeed improve logical abilities, it is necessary to assume the defective interpretation of conditional. However, López-Astorga countered that other interpretations of their results are possible, that they do not prove that the material interpretation of conditional is not valid, and that the mental models theory can also explain them. In this paper, I comment the problems linked to the other possible interpretations proposed by López-Astorga and try to argue that the best option in the interpretation of Attridge and Inglis' results is to assume the mental logic theory.

Keywords: *defective conditional; formal discipline theory; material conditional; mental logic; mental models.*

ISSN (I): 0120-4688 / ISSN (D): 2389-9387

Recibido: marzo 10 de 2015 - Aprobado: mayo 8 de 2015

La teoría de la disciplina formal y la lógica mental

Resumen

La teoría de la disciplina formal sostiene que el estudio de las matemáticas desarrolla las habilidades lógicas de los individuos. Attridge e Inglis realizaron un experimento con el fin de comprobar si esta teoría es correcta o no. Su conclusión fue que sus resultados mostraban que, si bien las matemáticas pueden, efectivamente, mejorar las capacidades lógicas, es necesario asumir la interpretación defectuosa del condicional. Sin embargo, López-Astorga contraargumentó que eran posibles otras interpretaciones de sus resultados, que tales resultados no prueban que la interpretación material del condicional no es válida y que la teoría de los modelos mentales también los puede explicar. En este trabajo, comento los problemas vinculados a las otras interpretaciones posibles propuestas por López-Astorga y trato de mostrar que la mejor opción para la interpretación de los resultados de Attridge e Inglis es asumir la teoría de la lógica mental.

Palabras clave: *condicional defectuoso; teoría de la disciplina formal; condicional material; lógica mental; modelos mentales.*

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Introduction

A theory attributed to the Greek philosopher Plato states that, if an individual studies mathematics, that individual will improve his (her) logical abilities. The theory is known as the formal discipline theory and it is obvious that it is an interesting and important theory to be considered by professionals and researchers in education field. In fact, several studies on this theory have been carried out. One of them, which is specially relevant, is that of Attridge and Inglis (2013). Their work offered suggesting results and their interpretation of them was, basically, that, although they showed that it is truth that mathematics learning develops logical abilities, they also demonstrated that the material interpretation of conditional does not hold and that the defective interpretation of it is a better alternative.

As it is well known, the material interpretation of conditional, which is attributed to another Greek philosopher, Philo of Megara, is the traditional interpretation that classical logic assumes. According to that, as it is also known, a conditional is only false when its antecedent is true and its consequent is false. Otherwise, it is true, even if, for example, the antecedent is false and the consequent is true. On the other hand, the defective

[1] This paper is a result of the Project N. I003011, "Algoritmos adaptativos e inferencias lógicas con enunciados condicionales", supported by the Directorate for Research of the University of Talca (Dirección de Investigación de la Universidad de Talca), Chile. The author, who is also the main researcher of that Project, would like to thank the mentioned institutions for their help in funding this paper.

interpretation comes from probability logic (e.g., Adams, 1998; Adams & Levine, 1975) and raises that, when the antecedent of a conditional is not true, the scenario described by that conditional is irrelevant. However, what is important here is that López-Astorga (2014a) responded to Attridge and Inglis' (2013) paper and argued that their results could be interpreted in different ways. In particular, he proposed that those results do be compatible with Philo's interpretation of conditional and that other theory, the mental models theory (from now on, MM) can account for them too.

Several points will be considered in this paper. On the one hand, I will comment the problems that the two alternative explanations indicated by López-Astorga (2014a) have. On the other hand, I will try to show that another cognitive theory, the mental logic theory (from now on, ML), can better explain Attridge and Inglis' (2013) results. In this way, I will also argue that such results are clearly consistent with the predictions of ML and that this last theory can offer a very fruitful framework that can lead further researches in fields such as those of cognitive science, linguistics, psychology, philosophy, or education.

To do so, firstly, I will briefly explain what Attridge and Inglis' (2013) research was exactly. Then I will describe the two alternative approaches proposed by López-Astorga (2014a), both that based on the material interpretation and that following MM, and indicate their difficulties. Finally, I will expose the general framework of ML and argue why Attridge and Inglis' (2013) results are coherent with it and what such results really mean if interpreted from ML.

Attridge and Inglis' (2013) experiment

Attridge and Inglis' (2013) experiment refers to two basic rules of classical logic related to conditional coming from Stoic logic (in particular, they are attributed to Chrysippus of Soli), the *Modus Ponens* rule (from now on, mP) and the *Modus Tollens* rule (from now on, mT), and to two well known schemata that are considered to be fallacies in that logic, the affirming the consequent fallacy (from now on, aC) and the denying the antecedent fallacy (from now on, dA). The formal structure of those rules and fallacies are also well known:

$$\text{mP: } x \rightarrow y,\ x \ /\ \text{Ergo } y$$
$$\text{mT: } x \rightarrow y,\ \neg y \ /\ \text{Ergo } \neg x$$
$$\text{aC: } x \rightarrow y,\ y \ /\ \text{Ergo } x$$
$$\text{dA: } x \rightarrow y,\ \neg x \ /\ \text{Ergo } \neg y$$

Where '→' denotes conditional relationship, '¬' stands for denial, and 'Ergo' means that the right formula follows from the left formulae.

The aspect of Attridge and Inglis' (2013) experiment relevant for the aims of this paper is that they used abstract tasks (with letters and numbers) corresponding to the previous four schemata. In those tasks, the left formulae appeared as premises and it was asked whether or not the conclusion, i.e., the right formula could be drawn form the premises. Participants had only the response options 'yes' and 'no'.

According to Attridge and Inglis (2013), the mentioned tasks are interesting because they allow checking how participants interpret conditional. Attridge and Inglis (2013) take four possible interpretations into account, biconditional, material, defective, and conjunction, and link these four possible interpretations to the schemata indicated above. In their view, if an individual interprets conditional as biconditional, that individual must accept the four schemata. This is because, in standard logic, the following deduction are all correct:

$$x \leftrightarrow y, x \ / \ \text{Ergo } y$$
$$x \leftrightarrow y, \neg y \ / \ \text{Ergo } \neg x$$
$$x \leftrightarrow y, y \ / \ \text{Ergo } x$$
$$x \leftrightarrow y, \neg x \ / \ \text{Ergo } \neg y$$

Where '↔' represents biconditional relationship [remember that, in classical logic, $x \leftrightarrow y$ is equivalent to $(x \rightarrow y) \cdot (y \rightarrow x)$, where '·' is conjunction].

However, if a material interpretation of conditional is made, the only valid schemata are mP and mT. As said, only those schemata are correct in classical logic, which assumes Philo's interpretation. According to that logic, aC and dA are, as also indicated, fallacies.

On the other hand, if the interpretation is defective, only mP can be accepted. As also mentioned, the defective interpretation is linked to probability logic, and, in this last logic, only the situations in which the antecedent, i.e., x, is true are relevant (note that only in mP the second premise ensures that x is true).

Finally, if conditional is interpreted as conjunction, i.e., as $x \cdot y$, only mP and aC can be admitted, since mP and aC are the only schemata in which both x and y are true (remember that a conjunction is true when its two conjuncts are true too).

Based on this, Attridge and Inglis (2013) presented tasks such as those indicated to English high school students. The part of their research that is relevant here is that students that completed a year of post-compulsory mathematics made up a group of their participants. Such students executed the tasks on two occasions: before and after completing that year. Obviously, this fact limits the scope of my conclusions. As López-Astorga (2014a) also mentions, Attridge and Inglis' (2013) results do not actually give information on Plato's formal discipline theory. Such results can only lead to approximate conclusions on that theory, since, strictly speaking, they only enable to draw conclusions on a possible relation between the logical abilities improvement and the mentioned advanced level in mathematics in England.

Thus, taking into account that this limitation has an influence on Attridge and Inglis' (2013) conclusions, López-Astorga's (2014a) conclusions, and the conclusions that I will expose below, it can be said that the results obtained by Attridge and Inglis (2013) that are more interesting for this paper show that, before completing the mathematics post-compulsory level, participants tended to respond 'yes' in the four types of problems described above. However, after completing it, they considered only the conclusions of the mP tasks to be valid. Obviously, Attridge and Inglis (2013) interpreted this fact as evidence that, without learning mathematics, people understand conditional as biconditional and, after doing that, they adopt the defective interpretation.

Therefore, in their view, learning mathematics does improve logical abilities. Nevertheless, it does that in a way that is not compatible with the requirements of classical logic. If conditional is materially interpreted, it cannot be said that mathematics leads people to a better conditional reasoning. We can state that only if the defective interpretation is assumed.

Nonetheless, López-Astorga (2014a) thinks that Attridge and Inglis' (2013) conclusions are questionable, since their results can be interpreted at least in two more different ways. His idea is not that the problems used by them are not good tasks to study logical reasoning. He acknowledges that those problems proved to be very useful in papers such as that of Evans, Clibbens, and Rood (1995). López-Astorga's (2014a) criticism is only that Attridge and Inglis' (2013) results do not show that what they claim. The two alternative explanations of them that he proposes and the difficulties that such explanations have are commented in the two next sections.

Philo's interpretation and mathematical learning

One of the two alternative accounts raised by López-Astorga (2014a) is based on the idea that Attridge and Inglis' (2013) results are really consistent with the material interpretation of conditional. His two main arguments seem to be the following:

Firstly, the fact that participants answered 'no' in the mT problems after completing the post-compulsory level does not demonstrate that they considered all the scenarios in which appeared ¬p to be irrelevant. It is not possible to know what students actually thought. So, another possibility is that they did not accept the inferences with the form x → y, ¬y / Ergo ¬x because, in their view, cases of x could be possible.

Undoubtedly, this is a problem for the material interpretation, since, if this interpretation is assumed, the mT inferences must be accepted. However, López-Astorga's (2014a) second argument, which is based on the literature on cognitive science and, in particular, in papers such as that of Byrne and Johnson-Laird (2009) and that of López-Astorga (2013a), is that mT is a lot harder than mP. Thus, the thesis is that Attridge and Inglis' (2013) participants responded 'no' in the mT tasks because, given that mT is more difficult than mP, they did not apply mT and thought that, in a scenario in which both x → y and ¬y are true, x can be both true and false.

Indeed, in classical logic, which, as said, adopts the material interpretation of conditional, the reason why mT is harder to use that mP is obvious. As it is well known, in systems such as that of Gentzen (1934), mP is a basic rule, but mT is not. As also explained by López-Astorga (2014a) and, of course, by Byrne and Johnson-Laird (2009) and by López-Astorga (2013a), deriving ¬x from x → y and ¬y requires a number of steps. Firstly, x has to be supposed. Secondly, mP has to be applied in order to draw y from x → y and x. Thirdly, the conjunction introduction rule (x, y / Ergo x · y) has to be used in order to deduce y · ¬y from y and ¬y. Fourthly, the *Reductio ad Absurdum* rule [x (supposition), y · ¬y / Ergo ¬x] has also to be considered in order to obtain ¬x from the initial supposition and the conjunction obtained in the third step. Therefore, while mP needs only one step, mT needs four steps.

It is evident that these arguments can explain why Attridge and Inglis' (2013) participants that completed the advanced mathematics level did not accept the inferences with the mT form. Nevertheless, this explanation has an important difficulty. It appears to claim that human mind follows the rules and the requirements of standard logic and, at present, there are no theories that hold the idea that human beings reason in accordance with such rules and requirements. There are theories, such as ML, which state that human reasoning is syntactic and that is leaded by logical rules. However, in general, those theories do not accept the idea that human thought uses all the formal rules of classical logic. For example, people do not often admit the disjunction introduction rule (x / ergo x v y, where 'v' denotes disjunction) and the conditional introduction rule {[x (supposed) / Ergo y] / Ergo x →y} is also problematic (see, e.g., Orenes & Johnson-Laird, 2012). In fact, as commented below, ML does not admit the disjunction introduction rule as a 'Core Schema' and proposes restrictions for the conditional introduction rule. In this way, this is a point that this alternative explanation given by López-Astorga (2014a) needs to clarify.

MM and its view of conditional

Many papers and works describe and explain the general theses of MM (e.g., Byrne & Johnson-Laird, 2009; Johnson-Laird, 2010, 2012; Khemlani, Orenes, & Johnson-Laird, 2012, 2014; Orenes & Johnson-Laird, 2012). This is a semantic theory that claims that reasoning is made by taking possibilities into account. Propositions refer to possibilities and, when they are faced with inferences, people only consider the possibilities that are consistent with each other in order to check or draw a conclusion. MM indicates the possibilities corresponding to each of classical logical operators (conjunction, disjunction, conditional, and biconditional). Nonetheless, for obvious reasons, only the possibilities related to conditional and biconditional are relevant here.

To describe which those possibilities are, firstly, it is necessary to mention that all the possibilities are not easy to be identified. Thus, MM distinguishes between 'mental models', i.e., possibilities that individuals detect without difficulties, and 'fully explicit models', i.e., possibilities that need certain cognitive effort to be identified. As far as conditional is concerned, a proposition such as $x \rightarrow y$ only has a mental model:

$$x \qquad y$$

This model represents a scenario in which both x and y happen. However, the fully explicit models of conditional are three:

$$\begin{array}{cc} x & y \\ \neg x & y \\ \neg x & \neg y \end{array}$$

So, the only scenario that conditional does not enable is that in which x is true and y is false.

This distinction of MM allows explaining many cognitive phenomena, including the fact that mP is easier than mT. mP can be applied by considering only the mental model of the conditional premise. Indeed, that premise, which is the first premise, presents, as said, a scenario in which both x and y are true. The second premise states that x is true, but, if the first premise is taken into account, if x is true, it is only possible that y is true as well.

The case of mT, on the other hand, is different. To use mT it is necessary to identify the fully explicit models, since only the third one reveals us that the only possibility when ¬y is true is that ¬x is also true. Furthermore, another important datum is that, while the mental model of biconditional is the same as that of conditional, the fully explicit models of biconditional are only these ones:

x	y
¬x	¬y

Based on this, MM can easily account for most participants' response before taking the post-compulsory year of mathematics in Attridge and Inglis' (2013) experiment. They interpreted the first premise, i.e., the conditional premise, as a biconditional. Thus, mP was accepted because x is only true in the first fully explicit model of biconditional and, in that model, y is also true. Secondly, aC was admitted because y is only true in that same model and, in it, x is true too. On the other hand, mT was accepted because y is only false in the second fully explicit model of biconditional and, in that model, x is false as well. Finally, dA was admitted because x is also only false in this last fully explicit model and, in it, y is false too.

The problem for MM is to explain what happens after students complete the post-compulsory level. As said, after that, they only consider mP to be correct, but, as López-Astorga (2014a) indicates, Attridge and Inglis (2013) seem to argue that MM cannot account for that result. From the framework of MM, the only possibility could be to assume that participants, in that case, only detected the mental model, whether or not the first premise was interpreted as a biconditional. However, this possibility does not enable to explain the fact that students only accepted mP, since, if they took the mental model into account, aC should also have been accepted (remember that the mental model refers to a scenario in which both the antecedent and the consequent are true). Strictly speaking, an individual would accept only mP if he (or she) identified both the first and the second fully explicit models and did not detect the third fully explicit model. In this way, the second fully explicit model (i.e., the scenario in which x is false and y is true) would reveal that the fact that y is true does not involve that x is true too (so aC could not be accepted), and that the fact that x is false does involve that y is false as well (so dA could not be accepted). At the same time, given that the third fully explicit model is not detected, mT could not be considered to be valid, since the individual would not know what happens when y is false.

This is a problem for MM because the theory, in principle, only allows situations in which the mental model or the fully explicit models are identified, but not intermediate situations. To solve this problem, López-Astorga (2014a) proposes an interpretation of MM claiming that individuals have several options about models and that to detect the mental model or the fully explicit models of conditionals (or biconditionals) are not the only possibilities. In my view, this idea is consistent with the general theses of MM, since this theory has another important concept: the concept of modulation. According to it, sometimes meanings, contexts, and pragmatic factors can block certain models. Thus, it could be said that López-Astorga's (2014a) interpretation leads one to think that, while mathematical learning caused students to improve their logical reasoning in a sense (they did not interpret conditionals as biconditionals anymore), that improvement was limited, because they could not identify all the fully explicit models of conditionals included in Attridge and Inglis' (2013) experiment.

It is obvious that this interpretation is interesting, but it has some difficulties too. Firstly, in my view, it is necessary to pay attention to a general problem that MM needs to solve. It is unclear when individuals identify mental models and when they detect more models. It is important that MM clarifies under what circumstances it is only possible consider mental models and under what circumstances it is also possible to take other possibilities into account. Maybe it would be an exaggeration to require MM to describe an algorithm or a mental mechanism revealing the situations in which more cognitive effort is usually made and other models (and not only mental models) are available. Nevertheless, further explanations in this way are needed, since the exact reasons why a same group of participants can detect different numbers of models in different experimental conditions are in a sense unknown. On the other hand, there is also a difficulty specially related to Attridge and Inglis' (2013) results. The tasks used by them included abstract propositions with letters and numbers. In this way, it is hard to understand how modulation could lead the participants that completed the advanced level to accept the two first fully explicit models of conditional and to reject the third one. One might think that pragmatics, contexts, and meanings cannot play a relevant role in situations in which the propositions are very abstract. Therefore, this is another point that MM must account for.

So, both of the alternative explanations offered by López-Astorga (2014a) have problems. Nonetheless, ML can give an account without difficulties and, for this reason, it can be thought that it is the alternative approach that is more consistent with Attridge and Inglis' (2013) results. This idea is argued in the next section.

ML and the sophistication levels

As said, ML (e.g., Braine & O'Brien, 1998a; O'Brien, 2009, 2014; O'Brien & Li, 2013; O'Brien & Manfrinati, 2010) is a syntactic theory and assumes the idea that there is a logic that is followed by human mind. However, that logic is not standard propositional calculus. According to ML, human reasoning works by means of formal rules and those rules are valid in classical logic. Nevertheless, the theory does not admit all the rules of standard calculus or systems such as that of Gentzen (1934). As also mentioned, ML claims that the basic rules of reasoning are a number of schemata in which problematic rules such as that of disjunction introduction are not included and rules such as that of conditional introduction have certain restrictions. In particular, it distinguishes different kinds of rules. There are 'Core Schemata', which are rules with a greatest use, and 'Feeder Schemata', which are rules that are applied only if their use in turn allows using Core Schemata. There are also 'Incompatibility Schemata', which refer to contradictions, and other schemata. Nonetheless, what is important for this paper is that the theory also includes a 'Direct Reasoning Routine' that indicates the order and the way the schemata are used and that mT is not a basic schema in it.

Thus, ML can explain why mP is easier than mT as well. The first one is a Core Schema (in particular, it is Schema 7 in Braine & O'Brien, 1998b), but the second one, as said, is not a basic rule. This does not mean that mT is impossible for human beings. According to ML, mT requires certain reasoning strategies, including *Reductio ad Absurdum*, which, while they are possible for sophisticated individuals, they are not absolutely and easily available for all human beings.

This distinction between individuals that are more and less sophisticated made by ML is very relevant for the aims of this paper, since it is linked to the use of mT. As explained by Braine and O'Brien (1998c), a curious phenomenon regarding mT can be observed. Children often seem to respond better than adults to tasks in which mT is involved (to support this fact, Braine & O'Brien quote works such as, for example, O'Brien & Overton, 1982; O'Brien & Shapiro, 1968; or Rumain, Connell, & Braine, 1983). However, this is only an apparent better performance. What actually happens is that both children and unsophisticated adults tend to accept invited inferences (Geis & Zwicky, 1971), which in turn lead to give the correct answers in the mT problems. Indeed, the invited inferences cause conditional to be understood as biconditional and, while it is very hard to draw $\neg x$ from $x \rightarrow y$ and $\neg y$, it is very easy to do so from $x \leftrightarrow y$ and $\neg y$. In this way, the idea is that unsophisticated individuals, including children, tend to interpret conditional as biconditional, and that, when individuals become more sophisticated, that trend disappears.

The problem is that the fact that the trend disappears does not involve that individuals acquire the logical abilities needed to use reasoning strategies such as that related to *Reductio ad Absurdum*. So, there is an intermediate sophistication level in which individuals do not interpret conditional as biconditional and, therefore, do not make fallacies such as aC and Da. Nevertheless, given that their logical skills are not developed enough, they cannot use the *Reductio ad Absurdum* strategy and hence solve problems involving mT. They only execute correctly tasks linked to mP. Thus, only the most sophisticated individuals –that is, those that do not consider conditionals to be biconditionals and, in addition, can make inferences that require the *Reductio ad Absurdum* strategy- offer the logically correct answers in tasks referring to mP, mT, aC, and Da.

As it can be easily noted, this framework enables to interpret Attridge and Inglis' (2013) results without difficulties and it is absolutely coherent with them. It can be stated that, before completing the mathematics advanced level, most students participating in Attridge and Inglis' (2013) experiment are not sophisticated in logic. For this reason, they understand the conditionals included in the reasoning tasks as biconditonals and, therefore, consider mP, mT, aC, and Da to be valid inferences. After taking mathematics post-compulsory level, they reach a logical reasoning level that can be considered to be intermediate. In this level, they note that conditionals are not biconditionals. Although, undoubtedly, this is an important improvement, the level achieved is not the best possible level. Students do not have the necessary strategies to solve problems in which *Reductio ad Absurdum* is involved yet. Such strategies correspond to the most sophisticated level, and it seems that mathematics post-compulsory level in England does not help achieve this last logical level.

In any case, what is most relevant here is that the results obtained by Attridge and Inglis (2013) not only are consistent with the general theses of ML. It can also be said that ML predicts such results and can hence offer a clear account of what happens to students' logical abilities after completing the mathematics advanced level in England. In addition, the explanation of ML does not have the problems that can be found in the account based on Philo's interpretation of conditional or in that of MM.

Conclusions

A general problem in cognitive science is that the same experimental results can be explained from different approaches, systems, or frameworks (see, e.g., besides López-Astorga, 2014a, López-Astorga, 2013a, 2013b, 2014b, 2014c). However, the previous arguments show that, in the case of the particular problem analyzed in this paper, ML is the theory that can give the best explanation.

As commented, an account based on the material interpretation of conditional must solve the problems that the idea that human reasoning is leaded by classical logic causes, since it is obvious that people not always use all the formal rules of standard propositional calculus. On the other hand, an explanation from MM has also to address certain problems, because this theory needs to clarify when individuals can only identify mental models and when they can detect other semantic possibilities. It is true that MM proposes the concept of modulation, but it is absolutely necessary to explain how modulation works in abstract scenarios such as those used by Attridge and Inglis (2013).

As far as the defective interpretation assumed by Attridge and Inglis (2013) is concerned, it is clear that, in principle, it seems coherent with their results. Maybe this is not surprising, since the defective interpretation and ML share relevant theses, including the rejection of Philo's interpretation and the idea that it is very hard to suppose that people, when they reason about conditionals, consider situations in which the antecedent is false and the consequent is true. However, I think that the ML framework provides a much more detailed explanation. As shown, it can account for and predict all the results commented in this paper (both those obtained before completing the mathematics advanced level and those obtained after that). So, in my view, the explanation given by ML appears to be more finished because, in addition to describing the exact characteristics regarding logical reasoning that can be attributed to participants according to their responses in the tasks, it also predicts, based on the literature, not only that the intermediate individuals will only accept the mP inferences, but also that the unsophisticated individuals will interpret conditionals as biconditionals. This last fact is a result that Attridge and Inglis (2013) found as well, and, as I understand it, it demonstrates a posteriori that the general approach of ML is correct.

Obviously, the previous arguments are relevant in the cognitive science field, since ML has been often misunderstood and misinterpreted. In particular, the most frequent mistake has been to think that it is equivalent or very akin to classical logic or standard propositional calculus. Nevertheless, form my point of view, the arguments are also very important for other scientific fields. ML opens many explicative possibilities for topics such as that of information processing or that of propositions understanding. Likewise, it can be a useful instrument in fields such as philosophy or linguistics, and, of course, education. The ML approach can help detect the contents or the methodologies that can develop logical reasoning abilities, to what extent certain courses provide that development, and the particular subject areas that should be studied in the schools in order to achieve logical sophistication in students.

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