Public debt monitoring and implicit debt thresholds: an application for the case of Spain*

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Abstract

We extend previous work that combines the Value at Risk approach with the estimation of the correlation pattern of the macroeconomic determinants of public debt dynamics by means of Vector Auto Regressions (VARs). These estimated VARs are then used to compute the probability that the public debt ratio exceeds a given threshold, by means of Montecarlo simulations. We apply this methodology to Spanish data and compute time-series probabilities to analyze the possible correlation with market risk assessment, measured by the spread with respect to the German bond. Taking into account the high correlation between the probability of passing a pre-specified debt threshold and the spread, we go a step further and ask what would be the threshold that maximizes the correlation between the two variables. The aim of this exercise is to gauge the implicit debt threshold or fiscal limit which is the most consistent with market expectations as measured by the sovereign yield spread. Finally, as a matter of illustration, we lay out a small, simplified model that helps rationalizing the empirical evidence. Within this theoretical framework we assess the effectiveness of alternative fiscal rules in the determination of public debt.

JEL Classification: H63; H68; E61; E62.

Keywords: Public debt; early warning indicators; forecasting; fiscal sustainability.

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1 Introduction

This paper presents an empirical exercise that aims at highlighting the dependence of the public debt level that a given country can afford or maintain in the medium-term on the macroeconomic and fiscal fundamentals of the country. Some countries are able to maintain a high level of public debt over long and sustained periods of time while keeping normal access to the international markets, while others have to keep a significant public debt buffer against adverse macroeconomic and fiscal developments. To illustrate this point, we choose the case of Spain, a country that has recorded a public debt level that has been well-below the euro area average since the 1980s, and in particular since the creation of the European Monetary Union, as exemplified by Figure 1. Indeed, in 2007 the Spanish public debt-to-GDP ratio amounted to some 36% of GDP, compared to 66%, 64%, 103% and 65% in the euro area average, France, Italy and Germany, respectively. Even after the impressive surge of debt witnessed in the period 2008-2011, the Spanish public debt-to-GDP ratio stood at 68.5% in 2011, as compared to 87% of the euro area aggregate. Despite this fact, Spanish public debt has been the subject of increased and distinctive market pressure since 2009 and is routinely grouped among "high-debt countries" by the international press. In the current policy environment and against this background, it is not surprising to acknowledge that Spain, among several countries in the euro area, has passed a reform of the Constitutional Law in order to incorporate explicit public debt limits. Following an urgent procedure the Parliament approved on 8 September 2011 a reform of the Constitution to include a public debt rule that sets the reference level of 60% specified in the Treaty on the European Union as an explicit limit, in order to anchor markets' expectations.

Our work is related to the literature on public debt sustainability. In particular, in this paper we extend the work of Garcia and Rigobon (2004) and Polito and Wickens (2011) that combines the Value at Risk approach with the estimation of the correlation pattern of the macroeconomic determinants of public debt dynamics by means of Vector Auto Regressions. These estimated VAR's are then used to compute the probability of public debt ratio being higher than a given threshold, by means of Montecarlo simulations. By doing so we study to what extent the computed time-varying probabilities are able to correctly predict the dynamics of the public debt over some quarters ahead, with the aim of checking whether they may act as an early warning indicator of the compliance of the public debt level with the reference level of 60% established in the Maastricht criteria and recently included in the Constitutional reform in Spain. We then move one step forward and analyze the correlation of the latter probability measure with market risk assessment, measured by the spread with respect to the German bonds, and then address the following issues. Taking into account the high correlation we estimate between the probability of passing a pre-specified debt threshold and the spread, we what would be the threshold that maximizes the correlation between the two variables. The aim of this exercise is to gauge the implicit debt threshold or fiscal limit which is "more" consistent with market expectations approximated by the spread. We perform this experiment for (i) different sample sizes in order to study the possible changes in the fiscal limit (measured by the debt threshold) across sample sub-periods and (ii) different time spans used to compute the time varying probabilities so as to study if the market perception is mainly driven by short term debt sustainability or, on the contrary, there is consistency between the time span of the spread (10 years) and the probabilities computed using the same temporal horizon.

The finding that our measure of risk is highly correlated with the sovereign spread raises the issue of the utility of the former in real-time monitoring, given that the latter is available on a daily basis. The advantage of complementing the standard analysis with the probability measures we compute is twofold. First, it is a measure based on fundamentals, and thus not subject to market volatility. Second, we show that our measures Granger-cause the information contained in the spread with a 1-quarter lead, and thus, in a sense, do contain advanced information that is only reflected in the market measures with some delay.

Some empirical and theoretical literature suggest that the "affordable" debt level of countries is inversely related to GDP volatility and, in particular, the volatility of public revenues, among other medium-term determinants (see Rostagno et al., 2008, and the references quoted therein). Rational markets should be able to assess the evolution of fundamentals and thus impose tighter debt limits on countries with weaker and/or more volatile fundamentals. With our exercise we try to gauge the implicit debt threshold or "fiscal limit" that would have been the most consistent with market expectations. This can be read as the debt level Spain could have afforded at any point in time. From a purely empirical point of view, thus, we try to put forward some of the issues that have been addressed by the theoretical literature dealing with the determinants of the optimal level of debt (see, e.g. Woodford, 1990, Aiyagari and McGrattan, 1998, or Floden, 2001, Debonnet and Kankamge, 2007). On related grounds, a typical result of the literature is that public debt should act as a shock absorber to help a smooth the response of other budgetary variables, in particular to avoid drastic increases in tax rates in downturns. Nevertheless, from a policy perspective, our empirical results underlie the fact that this result is obtained under perfect access to markets by the government; if a given government, on the contrary, were subject to market pressure and limited market access, then the policy advise could turnout to be different, including the possibility of debt reduction.

To illustrate this latter point, and as a way to enrich the discussion of our empirical results, we put forward a extremely simplified, standard DSGE macro model with public debt in which limited market access under certain circumstances makes it optimal to reduce debt in times of distress rather than increasing its level. Within this theoretical framework we assess the effectiveness of alternative fiscal rules in the determination of public debt.

The rest of the paper is organized as follows. In Section 2 we describe the evolution of public debt in Spain in the past few decades and provide some stylized facts. In Section 3 we discuss the data and the methodology used in the empirical analysis, while in Section 4 we discuss the main results. In Section 4.4, in turn, we pose the theoretical model and the discussion on alternative fiscal rules. Finally, in Section 5 we present some conclusions.

2 The Spanish public debt: hysteresis and fundamentals

The case of Spain is a good example to test our risk-monitoring approach. This is the case because, as mentioned in the Introduction, despite the fact that Spain has been over the past decades a relatively low debt country when one compares it with the average of its neighboring countries, it is currently at the center of the euro area public debt crisis.

As can be seen in Figure 2 the maximum level of debt as a percent of GDP in the period 1980-2010 was reached in 1996 (67.4% del PIB). After the 1996 maximum public debt got reduced until reaching a minimum of 36% in 2007. In the period 2008-2011, nevertheless, debt increased substantially to reach 68.5%, the maximum of the time series in the period 1980-2011. In relative terms, though, the Spanish public debt-to-GDP ratio represented in 2011 close to 80% of the euro area ratio, still well below the 90% that it represented back in 1996.

From a backward looking perspective, it is apparent that public debt increased in times of economic recession, but showed a significant downward rigidity in post-crisis times, at least until the mid 1990s. Between 1980 and 1987 public debt increased by 27 percentage points and got stabilized at 1987's level till 1991. This was the starting point for the second big increase in the sample period analyzed, as the outburst of the 1993 crisis pushed upwards public debt again, in such a way that the stock of government debt increased by some 25 percentage points between 1992 and 1996. This hysteresis-like behavior witnessed over the decade and a half that started in 1980 was curbed in the subsequent 1997-2007 period. In the latter 10-year period public debt got reduced by 30 percentage points of GDP, only to increase again substantially in the last part of the sample.

It is worth looking at the prolonged episode of debt reduction that started in the late 1990 through the lens of the government budget constraint. Let Y_t be real GDP at t and let D_t be the real value of government debt. The government budget constraint accounts for how a nominal interest rate i_t, net inflation π_t, net growth in real GDP, gdp_t, the net-of-interest deficit as a percent of Y_t, def_t, and the deficit-debt adjustment, DDA_t combine to determine the evolution of the government debt-to-GDP-ratio,

$$\frac{D_t}{Y_t} = \frac{1 + i_t}{(1 + \pi_t)(gdp_t)} \frac{D_{t-1}}{Y_{t-1}} + def_t + \frac{DDA_t}{Y_t} \tag{1}$$

were the nominal yield i_t and the real stock of debt D_t are averages of pertinent objects across terms to maturity. Its log-linearized version, suitable for accounting decomposition of the fundamental determinants of debt, takes the form

$$\frac{D_t}{Y_t} = (i_t - \pi_t - gdp_t) \frac{D_{t-1}}{Y_{t-1}} + \frac{D_{t-1}}{Y_{t-1}} + def_t + \frac{DDA_t}{Y_t} \tag{2}$$

With this decomposition at hand it is possible to analyze the determinants of changes in the debt-to-GDP ratio. In Figure 3 we decompose these determinants for each year over the period 1997-2011. Focusing in a first stage in the period 1997-2007, the primary balance contributed to an average debt reduction of 2.3 percentage points per year, an amount similar in size to the average contribution of real GDP (2.1 percentage points per year on average) and inflation (1.9 points per year on average). These three factor were partly compensated by an average 0.5 points per year debt-increasing contribution stemming from deficit-debt adjustments, and the interest payments, that amounted to some 2.8% of GDP per year on average. As regards the 2008-2011 period, in the first 3 years the sizeable increase in debt occurred in a period of still benign interest rates dynamics, and was basically due to the worsened primary balance, while the year 2011 combined the latter with adverse interest rate contributions. Figure 4, in turn, shows the same information as before, but cumulated, i.e. calculated by means of equation:

$$\frac{D_t}{Y_t} = \sum_{s=0}^{\tau-1} \left[(i_{t-s} - \pi_{t-s} - gdp_{t-s}) \frac{D_{t-s-1}}{Y_{t-s-1}} + def_{t-s} + \frac{DDA_{t-s}}{Y_{t-s}} \right] + \frac{D_{t+\tau}}{Y_{t+\tau}} \tag{3}$$

Between 1997 and 2007, the 31 percentage points of public debt reduction can be break down as follows: (i) 25 percentage points of reduction due to the adjustment of the primary balance; (ii) 22.6 points of reduction due to favorable real GDP growth; (iii) 20.4 percentage points of reduction due to inflation; (iv) these three factors more than compensated the increase by 30.7 points due to the interest payments effected during the period, and the 5.2 percentage points due to the deficit-debt adjustments. The debt-increasing contribution of the interest burden veils a favorable evolution of the implicit interest rate, as it becomes clear in Figure 5. Interestingly, implicit interest rate dynamics, that averages interest rates of newly issued, including refinanced debt, and rates of non-maturing debt issued in the past, contributed to contain the increase in the public debt ratio in 2008, 2009 and 2010, only turning to a positive contribution in 2011, when rates at issuance increased substantially. Beyond this latter factor, in the course of the four years that span from 2008 to 2011 the abrupt reversal of all positive factors, most notably the significant primary deficits, undid the results of the 1997-2007 consolidation period.

In the subsequent Section we will continue with the analysis of the role of fundamental in public debt evolution, but from a forward looking, and model based, point of view rather than a backward-looking and descriptive one.

3 Empirical methodology and data

3.1 Outline of the methodology

The approach to assess the dynamics and sustainability of public debt is based on the sequential estimation of a Vector autoregression model, updated each period. As claimed recently by Polito and Wickens (2011) this is instrumental for the problem at hand in that it exploits the well-documented advantages of time-series models for forecasting in the short run (see e.g. Stock and Watson,2001) and also due to the fact that time-series models tend to give better forecasts than structural models, particularly in the presence of structural breaks (see e.g. Clements and Hendry, 2005). Even though we take an agnostic view and estimate unrestricted VARs (structural models can be written as restricted VARs), we use the theory to guide our selection of variables. In particular we use the government budget constraint, as in (1) to select the variables to be included in the analysis.

More in detail, the procedure consists of the following steps:

- Given an initial sample size, a VAR is estimated using the variables that enter into the public debt equation, which in its simplest form are: the ratio of primary deficit over GDP, the nominal interest rate, the inflation rate and the growth rate of GDP.

- Given the estimated parameters of the VAR and the corresponding variance-covariance matrix of the errors, a large number of Monte-Carlo simulations are performed to obtain a large number of realizations of the innovations and the associated realizations of the macroeconomic fundamentals of the debt equation. This step entails the computation of the implied debt-to-GDP ratio for different time horizons depending on the criteria of interest, where the period-by-period government budget constraint, expressed as a ratio to total GDP in its non-linear form as in (1)

The procedure can be applied sequentially and allows for the calculation of the probabilistic distribution of the debt ratio for each quarter of the projection. Notice that at each point in time t the procedure uses the macroeconomic information available up to $t-1$ which are the initial conditions of the relevant variables from which the simulated paths depart, consequently the exercise can be used to out-of-sample monitor the evolution of public debt both in the short and the long term.

3.2 The data

We use quarterly data from the first quarter of 1986 to the fourth quarter of 2011 of the following variables: primary deficit over GDP, inflation rate, the implicit interest rate (computed as the ratio between the interest payments in a quarter and the stock of public debt of the previous period), the growth rate of GDP in real terms and the stock of public debt. Concerning the spread, the data used covers the period from 1991Q4 to 2011Q4 and it is computed as the difference between the Spanish 10-year government bond yield and the German one.

3.3 VAR model

The first step of the approach entails the sequential estimation of a Vector Autorregression Model of the following form

$$Y_t = B(L)Y_{t-1} + U_t. \tag{4}$$

where

$$Y_t = \begin{pmatrix} gdp_t \\ \pi_t \\ i_t \\ def_t \end{pmatrix} \tag{5}$$

and U_t is the vector of reduced-form residuals which are assumed to be distributed according to a multinomial distribution with zero mean and covariance matrix Ω, i.e. $U_t \sim N(0, \Omega)$. Given the estimated parameters of the VAR and the corresponding variance-covariance matrix of the errors, a large number of Monte-Carlo simulations are performed to obtain a large number of realizations of the innovations and the associated dynamics of the macroeconomic fundamentals. This step entails the computation of the implied debt-to-GDP ratio for different time horizons depending on the criteria of interest, where the period-by-period government budget constraint, expressed as a ratio to total GDP is given by equation (1).

3.4 The empirical distribution function of debt levels

The sequential estimation of the VAR and the computation of the simulated paths for public debt generates, at each point in time, an empirical distribution function of debt levels. This distribution function, that also depends on the simulation horizon, can be used to monitor the evolution of public debt in both the short and the long term by computing the corresponding moments of the simulated data. For notational purposes, conditional on the information available up to period $t-1$, denoted as I_{t-1} (composed of past data and the estimated parameter coefficients of the VAR model) we denote as $F_t^T(d \mid I_{t-1})$ the empirical cumulative distribution function of debt realizations of length T.

For illustrative purposes Figure 6 displays the cumulative distribution of debt at different simulation spans (one quarter, one year and two years) when the associated VAR is estimated with data until 2011Q4. Notice that at short simulation's horizons, debt outcomes are distributed tightly around the mean and as the projection horizon lengthens, uncertainty increases in such way that the distribution becomes more fat-tailed and more extreme outcomes cannot be ruled out.

Given that the information used covers the period until 2011Q4, the distributions generated could be used to perform debt projections starting in the first quarter of 2012. However, in order to address the forecasting performance of the procedure in the very short run when compared to the data, Figure 7 displays the average 1-quarter out-of-sample forecast of the macro fundamentals and the public debt over GDP against the observed data of the corresponding variable. The visual inspection of the graph indicates that the model correctly approximates the evolution of the main macroeconomic fundamentals and the public debt in the very short term (1 quarter ahead). The out-of-sample performance ability of the model to forecast the dynamics of public debt is further illustrated in Table 1 in which we compare the observed evolution of this item in the data against the forecast implied by the model at different time horizons. In particular, using the information available up to the fourth quarter of 2010, we present the expected dynamics of public debt in each quarter of 2011 and the first one of 2012 implied by the model. As mentioned before, the standard deviation of the forecast increases at longer horizons.

4 Results

4.1 A risk measure of public debt

Apart from the calculation of forecasts, the time dependent empirical distribution function can also be used to monitor the expected evolution of public debt in probabilistic terms. In particular, given the distribution function, it is possible to compute the probability of public debt being higher than a particular threshold θ some quarters ahead T, given the information available at some point in time denoted as I_{t-1}. One particular example that is especially relevant in the context of the Stability and Growth Pact and the Spanish Constitutional limit is the 60% threshold. Notice that the sequence of probabilities will be time dependent. Formally, at each point in time t the following probability is computed

$$p_{t/t-1}^{T}(\theta) = P(d > \theta \mid I_{t-1}) = 1 - F_t^T(\theta \mid I_{t-1}) \tag{6}$$

and the sequence of such probabilities $\{p_{t/t-1}^{T}(\theta)\}$ which will be denoted as $P_{t/t-1}^{T}(\theta)$.As suggested before one possible way of summarizing the information provided by the time dependent distribution function of public debt level is to compute the probability of observing a debt level above some threshold of interest which is deemed to be critical for its sustainability. If this threshold is empirically relevant in terms of this criteria, the probability series computed taking into account this threshold should be link with some market based measure of risk in the market for public debt. For this reason it may be useful to compute the correlation between the probability and the spread, as illustrated in Figure 8.

For a given threshold (60%) we have computed several probability series for different time spans so as to study if the market perception is mainly driven by short term debt sustainability criteria or on the contrary, there is consistency between the time span of the spread (10 years) and the probabilities computed using the same temporal horizon. The next Table shows the results. In particular, we find that the correlation is higher when the forecasting horizon approaches that of the spread, as shown in Table 2.

4.2 The "affordable" debt level

Taking into account the high correlation between the probability of passing a pre-specified debt threshold and the spread, in this section we go a step further and ask what would be the threshold that maximizes the correlation between the two variables. The aim of this exercise is to gauge the implicit debt threshold or fiscal limit which is the most consistent with market expectations approximated by the spread.

Formally,

$$\theta^* = \arg\max_{\theta} corr(P^T_{t/t-1}(\theta), S_t). \tag{7}$$

The outcome of the implementation of this procedure to the whole sample until the last quarter of 2011 in the case of running the simulations with time span consistent with the spread (10 years) is displayed Figure 9. There it is easy to see that the correlation increases until it reaches a maximum when the debt threshold is 56%, displaying a coefficient of 0.7.

In this context, a natural question that arises is whether the debt threshold has changed over time, and in particular, to what extent the effects of the crisis has affected the debt level that is perceived as sustainable by the market. In order to answer this question, we apply the procedure sequentially starting with a minimum sample size and extending it until the whole sample is considered, allowing us to study the temporal convergence towards the 56% debt threshold obtained before. The results of this latter exercise are shown in Figure 10 and indicate that until the third quarter of 2008 the debt threshold was around 75%. However, with the arrival of the crisis the debt tolerance decreases until it settles down around the 56%. This behavior is mainly related to the deterioration of both GDP growth prospects and the primary deficit observed in the Spanish economy during the crisis period because under this environment it is more likely that debt to GDP ratios displays explosive paths dampening the sustainability of public finances.

4.3 The probability measure and the market risk indicator

The probability measure Granger-causes in crisis times the market risk indicator as it is clear from Table 3 (computed on probabilities to breach the 60% limit). Thus, the usefulness of our measure of market risk is twofold compared to S_t: (i) as a real-time leading indicator;

(ii) it is based on macro fundamentals, and thus free from volatility and/or irrational (i.e. non-fundamental-based) signals.

The table presents:

- Granger Causality test between S_t and $P_{t/t-1}$, for different specifications (lags from 4 to 6) and sample sizes ("recursive estimation") for the sake of robustness.
- Nevertheless the measure $P_{t/t-1}$ is the probability computed for quarter t with data and parameters up to $t-1$ and thus, it is information content is lagged with respect to S_t. That is why we also show in the table $P_{t+1/t}$ which is in phase with S_t (conjecture: using macro leading indicators one could solve the lagged nature of quarterly data releases).

The table shows:

- The probability measure Granger-causes in crisis times the market risk indicator.
- Indeed, $P_{t/t-1}$ (and $P_{t/t-1}$ forwarded) Granger-Causes S_t for all specifications that incorporate the financial/debt crisis years (i.e. the null hypothesi of no causation gets rejected).
- In the pre-crisis times, it is natural that $P_{t/t-1}$ plays no role: it is a period of debt reduction and strong fundamentals.
- The exercise is done for the 60% threshold (it could be done for a continuum of thresholds): but even so a signal is sent by the model as of the inclusion of information for 2009, when the debt level was substantially below that threshold - then systematically the measure leads S_t.

4.4 Public debt policies, fiscal rules and the fiscal limit

This section:

- Describe a simplified model with public debt in which the standard shock-absorber role holds under the usual assumptions. Extent the latter model to incorporate "limited market access" (some type of rule linking interest rates to debt??).

- In the latter model, simulate the path of the debt-to-GDP ratio imposing alternative fiscal rules (budget balance rules, public debt rules, limits to public spending).
- We also revisit the implicit fiscal limit under alternative rules: can the economy support a different fiscal limit under tighter fiscal rules.

5 Conclusions

In this paper we extend previous work that combines the Value at Risk approach with the estimation of the correlation pattern of the macroeconomic determinants of public debt dynamics by means of Vector Auto Regressions. These estimated VAR's are then used to compute the probability that the public debt ratio exceeds than a given threshold, by means of Montecarlo simulations.

We apply this methodology to Spanish data and compute time-series probabilities to analyze the possible correlation with market risk assessment, measured by the spread with respect to the German bond. Taking into account the high correlation between the probability of passing a pre-specified debt threshold and the spread, we go a step further and ask what would be the threshold that maximizes the correlation between the two variables. Thus, by pursuing this approach we are able to gauge the implicit debt threshold or fiscal limit which is consistent with market expectations as measured by the sovereign yield spread. We also assess the effectiveness of alternative fiscal rules in this framework.

Finally, as a matter of illustration, we lay out a small, simplified model that helps rationalizing the empirical evidence. Within this theoretical framework we assess the effectiveness of alternative fiscal rules in the determination of public debt.

References

Aiyagari S. R. and E. R. McGrattan, 1998. The Optimum Quantity of Debt, *Journal of Monetary Economics* 42, 447-469.

Aiyagari S. R., A. Marcet, T. J. Sargent and J. Seppälä, 2002. Optimal Taxation without

State-Contingent Debt, *Journal of Political Economy* 110, 1220-1254.

Andrés, J. and R. Domenech, 2006. Automatic Stabilizers, Fiscal Rules and Macroeconomic Stability, *European Economic Review* 50, pp. 1487-1506.

Bertola G. and A. Drazen, 1993. Trigger Points and Budget Cuts: Explaining the Effects of Fiscal Austerity, *American Economic Review* 83, 11-26.

Chari V., L. Christiano and P. Kehoe, 1994. Optimal Fiscal Policy in a Business Cycle Model, *Journal of Political Economy* 102, 617-652.

European Commission, 2011. Public Finances Report (June).

Flodén M., 2001. The effectiveness of government debt and transfers as insurance, *Journal of Monetary Economics* 48, 81-108.

Gali, J. and R. Perotti, 2003. Fiscal policy and monetary integration in Europe, *Economic Policy* 37, 533-572.

Kleven H.J. and C.T. Kreiner, 2003. The role of taxes as automatic destabilizers in New Keynesian economics, *Journal of Public Economics* 87 (2003) 1123—1136.

Mélitz J., 2000. Some Cross-Country Evidence about Fiscal Policy Behavior and Consequences for EMU, European Economy 2, 3-21.

Mitchell, P. R., Sault, J. E., and Wallis, K. F., 2000. Fiscal policy rules in macroeconomic models: Principles and practice. *Economic Modelling* 17, pp. 171–193.

Polito, V.and M. Wichens, 2011. A model-based indicator of the fiscal stance, *European Economic Review* 56, pp. 526-551.

Woodford M., 1990. Public Debt as Private Liquidity, *American Economic Review* 80, 382-388.

Wyplotz C. (1999), "Economic Policy Coordination in EMU: Strategies and Institutions", *Mimeo* (March).

Table 1: An illustration: public debt forecasts (with origin 2010Q4) vs. actual data

	2011				2012
	Q1	Q2	Q3	Q4	Q1
Actual Data	63.96	65.31	66.03	68.33	72.45
Actual (DDA excluded)	62.33	65.69	67.28	68.38	
Forecasts (DDA excluded)					
Min	60.75	61.04	61.76	62.09	61.69
Mean	62.73	64.77	66.86	69.00	71.16
Max	64.94	68.38	72.65	76.54	80.57
Std.	0.62	1.04	1.59	2.20	2.88

Table 2: Correlations between the probability and the spread

	Time span		
	1 year	5 years	10 years
1993Q1-2010Q4	0.34	0.57	0.66
1999Q1-2010Q4	**0.66**	**0.81**	**0.82**

Figure 1: Evolution of the debt-to-GDP in selected euro area countries

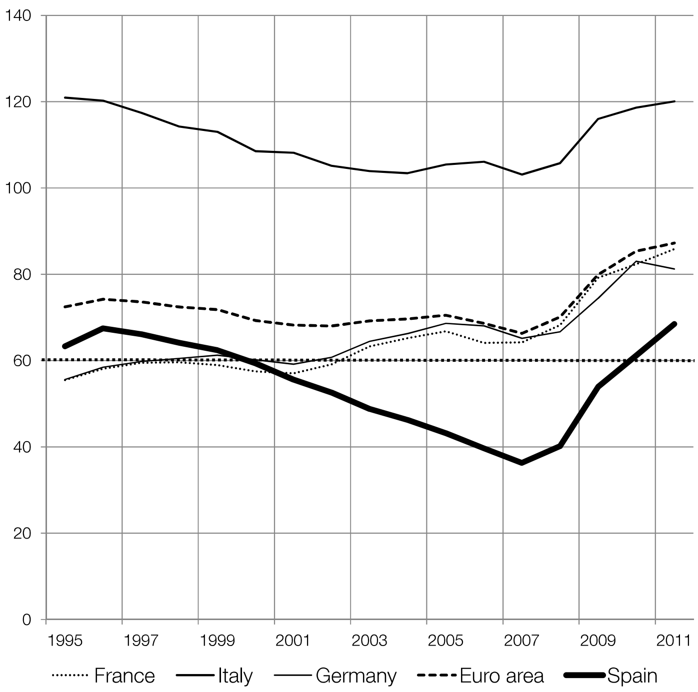

Table 3: The probability measure Granger-causes in crisis times the market risk indicator.

Granger Causality test: S_t , $P_{t/t-1}$

4 lags	1999Q1-2012Q1	1999Q1-2011Q1	1999Q1-2010Q1	1999Q1-2009Q1	1999Q1-2008Q1	1999Q1-2007Q1	1999Q1-2006Q1
$P_{t/t-1}$ does not Cause S_t	0.071*	0.000***	0.000***	0.253	0.698	0.273	0.396
S_t does not Cause $P_{t/t-1}$	0.874	0.676	0.142	0.000***	0.796	0.817	0.847
$P_{t+1/t}$ does not Cause S_t	0.028**	0.000***	0.000***	0.005***	0.888	0.530	0.608
S_t does not Cause $P_{t+1/t}$	0.982	0.960	0.768	0.007**	0.420	0.423	0.523
5 lags	1999Q1-2012Q1	1999Q1-2011Q1	1999Q1-2010Q1	1999Q1-2009Q1	1999Q1-2008Q1	1999Q1-2007Q1	1999Q1-2006Q1
$P_{t/t-1}$ does not Cause S_t	0.013***	0.000***	0.000***	0.522	0.668	0.218	0.351
S_t does not Cause $P_{t/t-1}$	0.906	0.664	0.023***	0.000***	0.696	0.739	0.794
$P_{t+1/t}$ does not Cause S_t	0.000***	0.000***	0.000***	0.001***	0.578	0.182	0.277
S_t does not Cause $P_{t+1/t}$	0.990	0.957	0.016***	0.002***	0.033***	0.015***	0.038***
6 lags	1999Q1-2012Q1	1999Q1-2011Q1	1999Q1-2010Q1	1999Q1-2009Q1	1999Q1-2008Q1	1999Q1-2007Q1	1999Q1-2006Q1
$P_{t/t-1}$ does not Cause S_t	0.016***	0.000***	0.000***	0.550	0.705	0.365	0.536
S_t does not Cause $P_{t/t-1}$	0.717	0.490	0.006***	0.000***	0.460	0.520	0.650
$P_{t+1/t}$ does not Cause S_t	0.000***	0.000***	0.000***	0.000***	0.269	0.036	0.103
S_t does not Cause $P_{t+1/t}$	0.863	0.738	0.002***	0.001***	0.063*	0.014***	0.041***

Notes:

***: null hypothesis rejected at the 1% level, **: 5%, *: 10%.

Figure 2: Evolution of the debt-to-GDP in Spain: 1980-2011

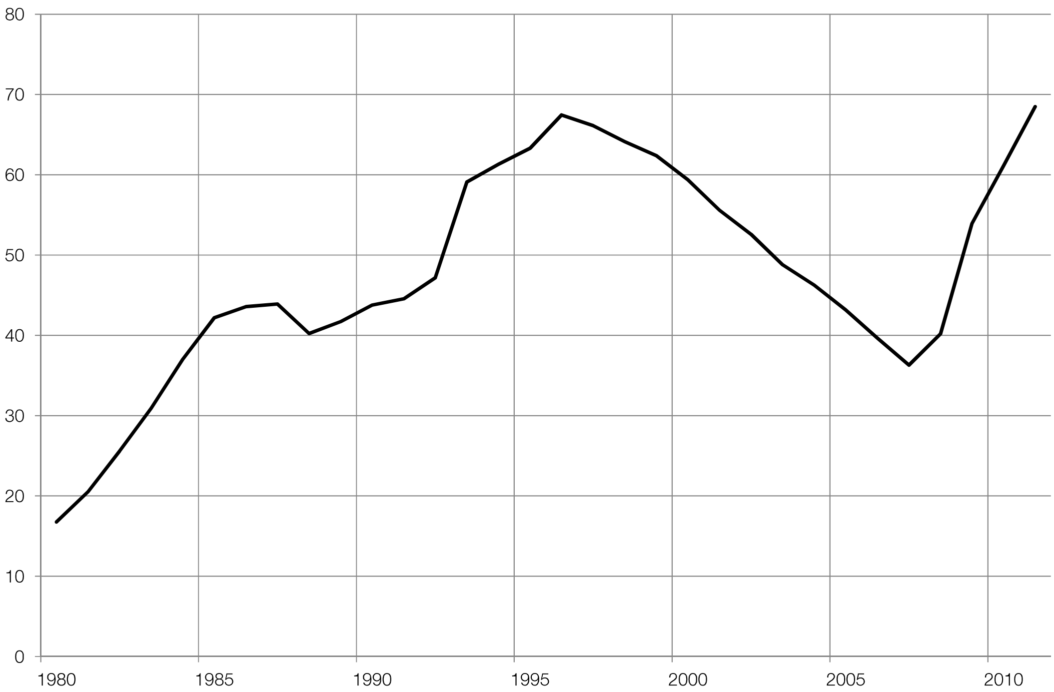

Figure 3: The determinants of the change in the debt to GDP ratio

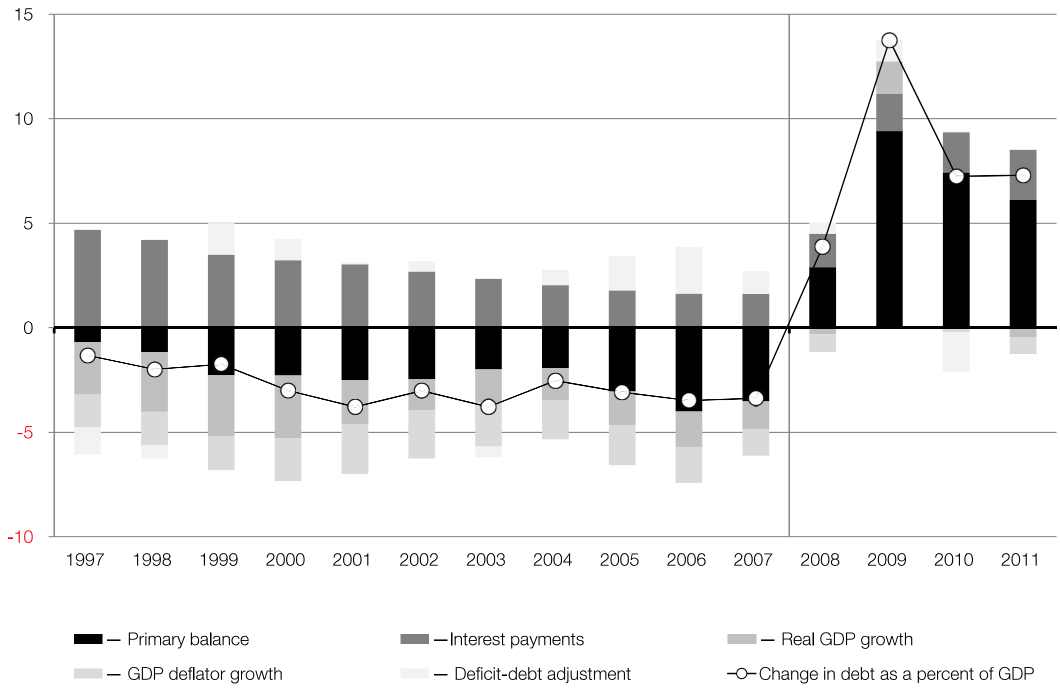

Figure 4: Cumulative sum of the components of the change in the debt to GDP ratio

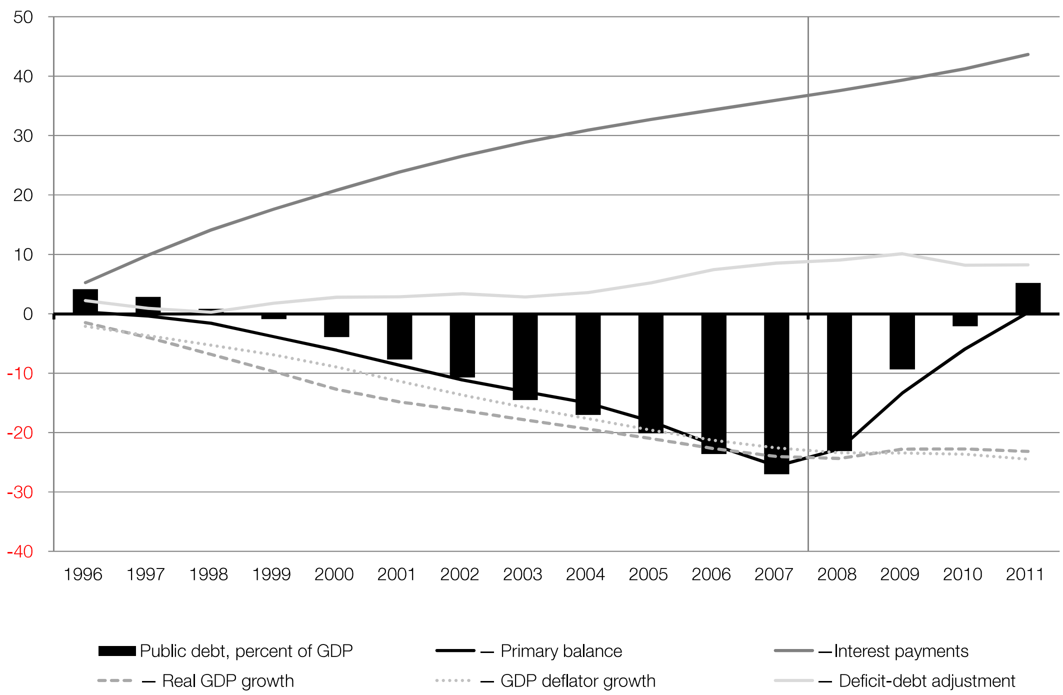

Figure 5: The determinants of the change in the interest payments to GDP ratio

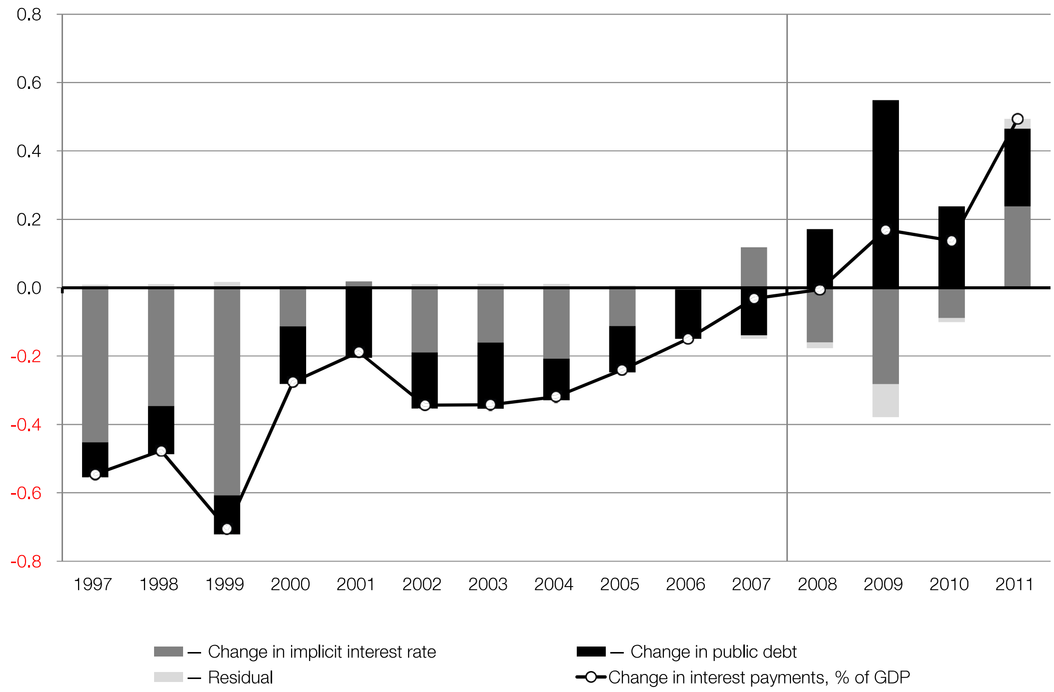

Figure 6: Empirical CDF with information up to 2011Q4

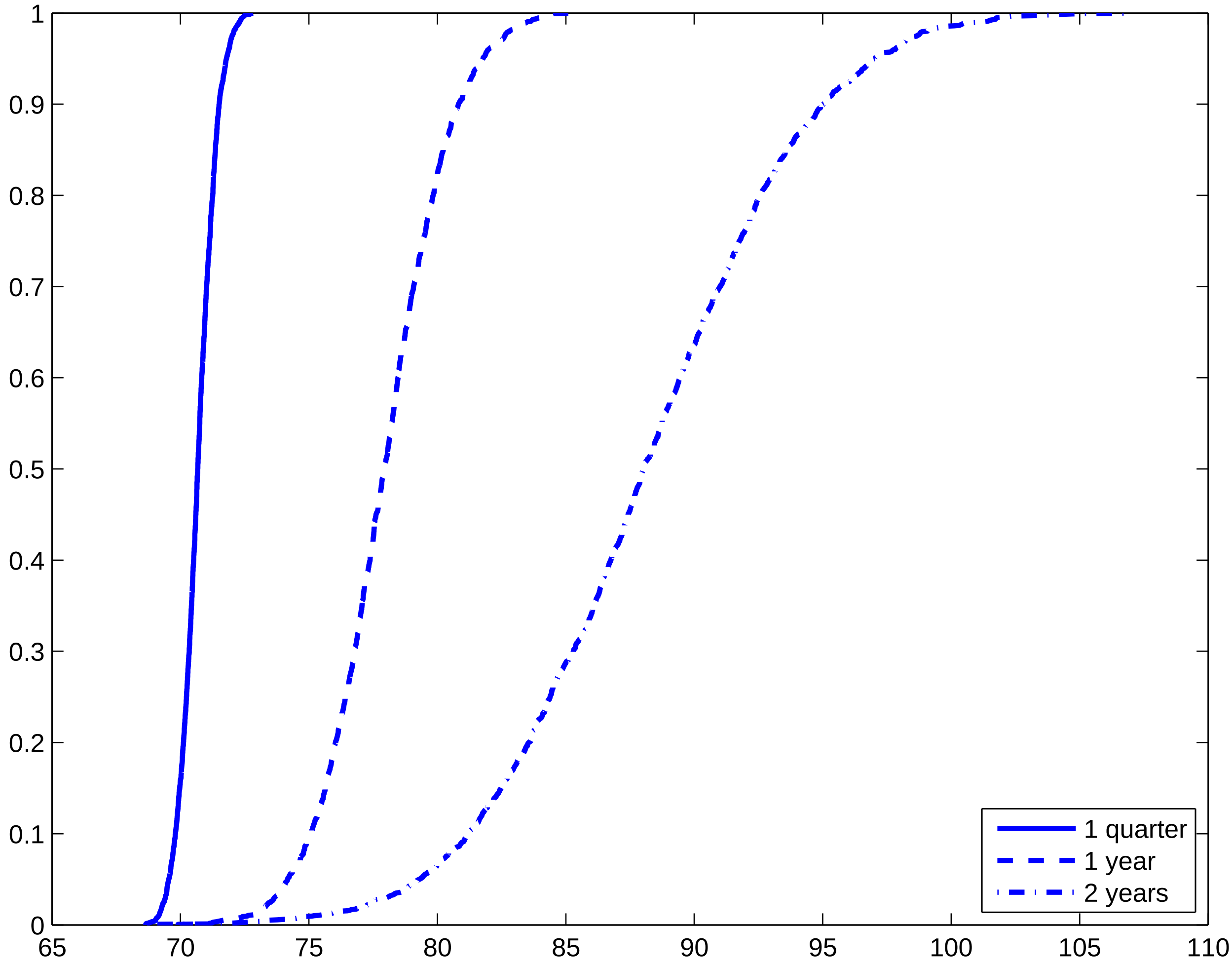

Figure 7: Data versus one-quarter forecasts

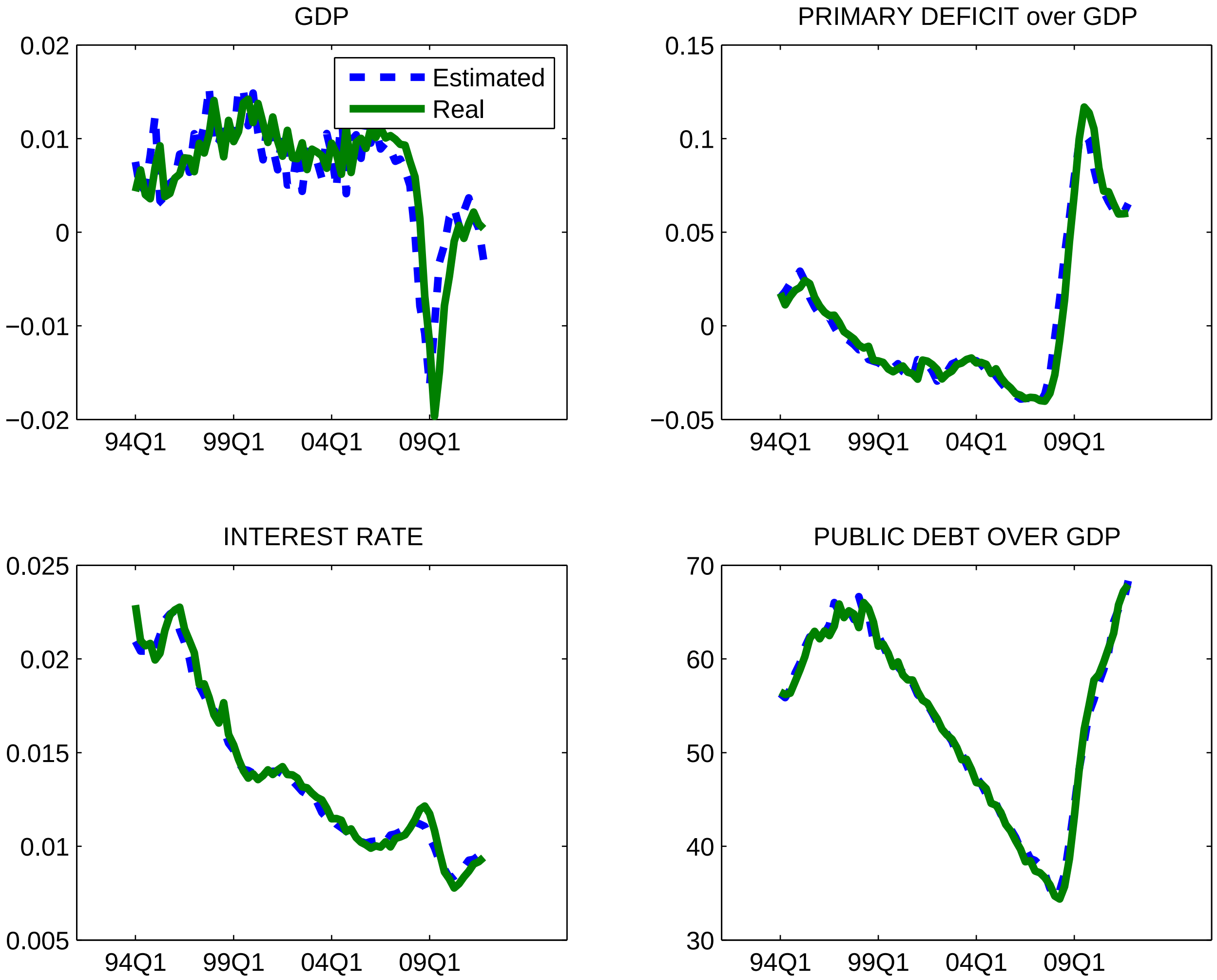

Figure 8: Probability $P^{10y}_{t/t-1}(60)$ vs. Spread

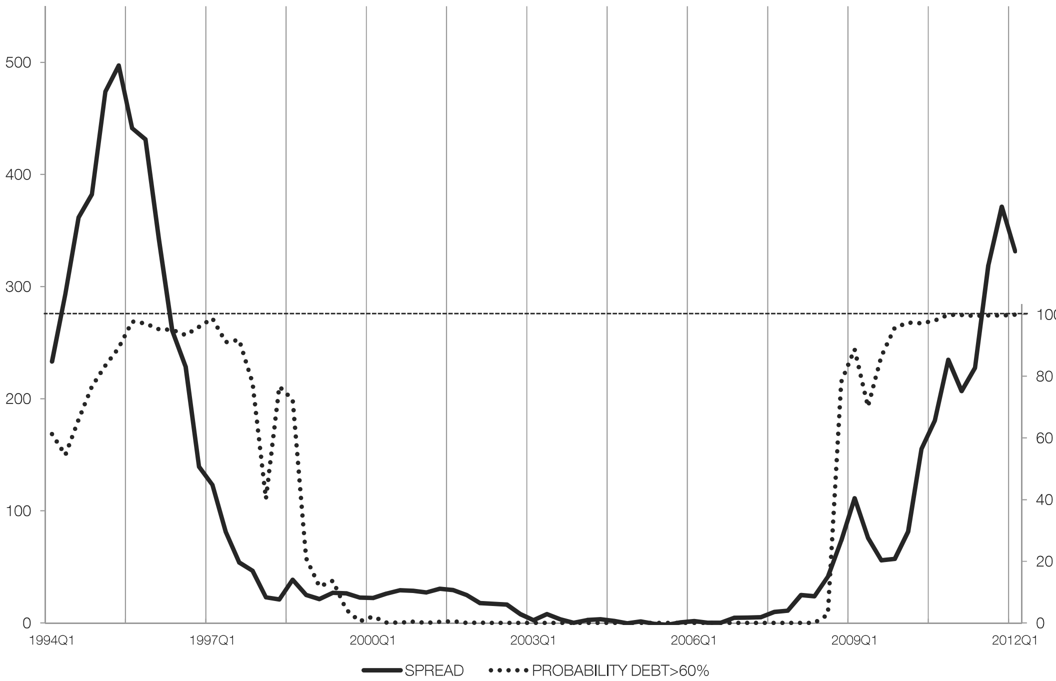

Figure 9: Correlation between the spread and the public debt thresholds

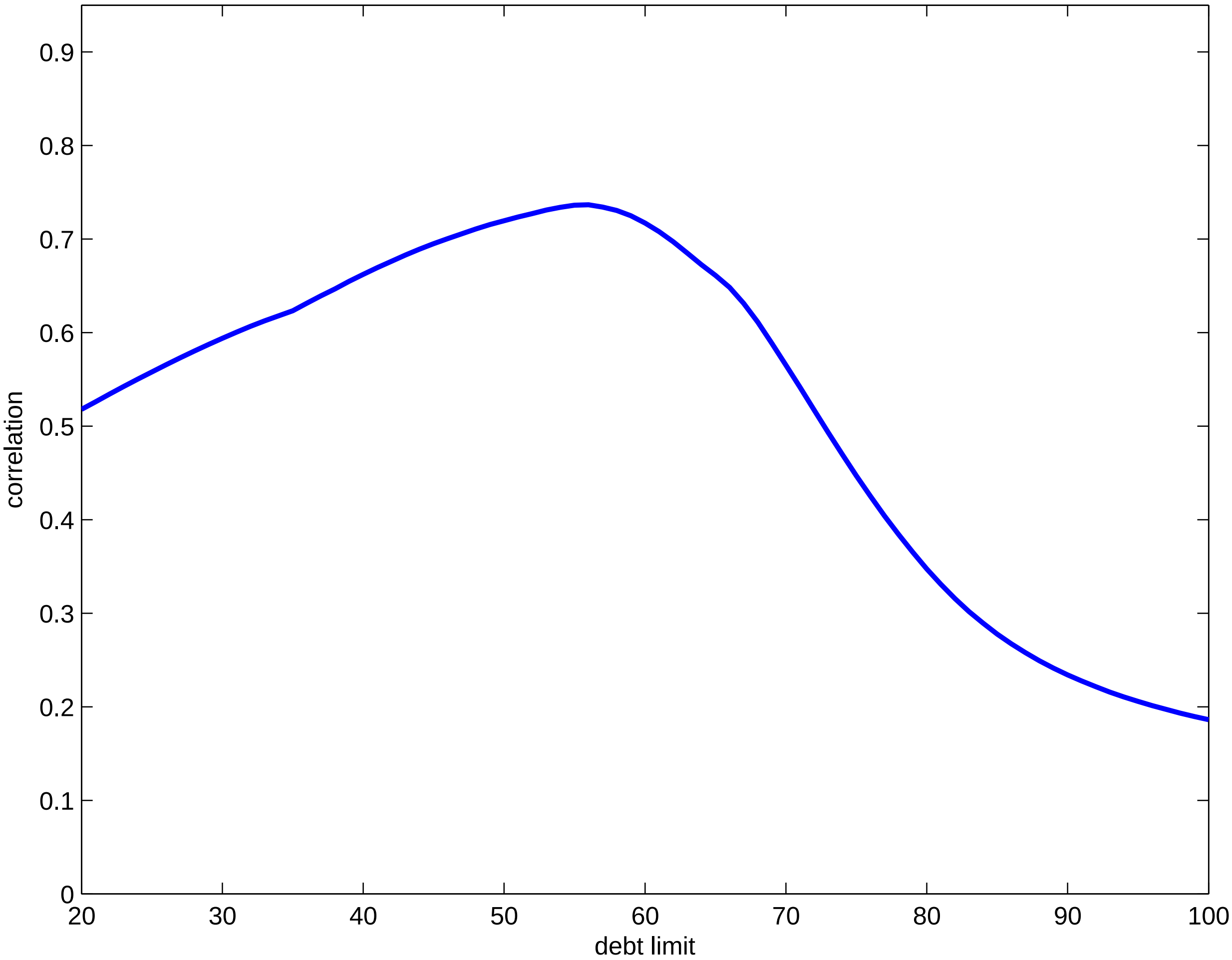

Figure 10: Time variation of the affordable debt limit