PARADOX AND LEARNING: IMPLICATIONS FROM PARADOXICAL PSYCHOTHERAPY AND ZEN BUDDHISM FOR MATHEMATICAL INQUIRY WITH PARADOXES

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Abstract:
This paper argues that paradox offers an ideal didactic context for open-ended group discussion, for the intensive practice of reasoning, acquiring dispositions critical for mathematical thinking, and higher order learning. In order to characterize the full pedagogical range of paradox, I offer a short overview of the effects of paradox, followed by a discussion of some parallels between the use of paradox in paradoxical psychotherapy and the use of the koan in Zen Buddhist spiritual training. Reasoning with paradoxes in a community of mathematical inquiry is interpreted as a comparative if not isomorphic pedagogical and cognitive phenomenon. Finally, some broad implications are drawn for mathematical inquiry with paradoxes.

Key-words: learning, paradox, community of mathematical inquiry, constructivist pedagogy

Paradoja y aprendizaje: Implicaciones de la Psicoterapia Paradójica y del Budismo Zen para la investigación matemática con paradojas

Resumen:
Este trabajo sostiene que la paradoja ofrece un contexto didáctico ideal para la discusión abierta en grupo, para la práctica intensa del razonamiento, adquiriendo disposiciones críticas para el pensamiento matemático, y un aprendizaje de más alto orden. Para caracterizar la gama pedagógica completa de paradoja, ofrezco una corta descripción de los efectos de la paradoja, seguida por una discusión de algunos paralelos entre el uso de la paradoja en sicoterapia paradójica y el uso del koan en el entrenamiento espiritual del zen budista. El razonar con paradojas en una comunidad de investigación matemática se interpreta como un fenómeno comparativo, si no isomorfo, de lo pedagógico y cognitivo. Finalmente, algunas implicaciones amplias se dibujan para la investigación matemática con paradojas.

Palabras clave: aprendizaje; paradoja; comunidad de investigación matemática; pedagogía constructivista.

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Paradoxo e aprendizagem: implicações da psicoterapia paradoxal e do zen budismo para a investigação matemática com paradoxos

Resumo:
Este trabalho sustenta que o paradoxo oferece um contexto didático ideal para a discussão aberta em grupo, para a prática intensa do raciocínio, adquirindo disposições críticas para o pensamento matemático e para uma aprendizagem de ordem mais alta. Para caracterizar a amplitude pedagógica do paradoxo, ofereço uma pequena descrição dos efeitos do paradoxo, seguinda por uma discussão de alguns paralelos entre o uso do paradoxo na psicoterapia paradoxal e o uso do koan nos treinamentos espirituais zen budista. O raciocinar com os paradoxos na comunidade de investigação matemática é interpretado como um fenômeno comparativo, se não isomórfico entre o pedagógico e o cognitivo. Finalmente, algumas implicações extensivas são desenhadas para a investigação matemática com paradoxos.

Palavra-chave: aprendizagem; paradoxo; comunidade de investigação matemática; pedagogia construtivista.

PARADOX AND LEARNING: IMPLICATIONS FROM PARADOXICAL PSYCHOTHERAPY AND ZEN BUDDHISM FOR MATHEMATICAL INQUIRY WITH PARADOXES

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If you have the staff, I will give it to you.
If you have no staff, I will take it away
from you!
Zen Koan[1]

Introduction

Historians trace the origins of paradox to the riddles of Greek folklore and the Delphic Oracle. This ancient epideictic[2] tradition was developed extensively in Greek tragedy and philosophy. An early example of its literary forms—a kind of "rhetorical" paradox—can be found in Sophocles' play *Oedipus the King*, in which wit, puzzlement, and ambiguity proliferate. According to the ancient myth, Oedipus was the first to encounter the Sphinx and answer her riddle, and for his accomplishment was rewarded with the throne of Thebes. Until then the Sphinx had positioned itself outside Thebes and strangled anybody who couldn't answer the following riddle: "What goes on four legs in the morning, two legs in the afternoon, and three legs in the evening?" (Seyffert, 1964) Oedipus resolved it by identifying a metaphor that unravelled its implicit contradictions.[3]

Another rhetorical paradox comes to us via St. Paul who, in his Epistle to Titus, warns the Bishop of Crete that, "One of themselves, even a prophet of their own, said, 'The Cretans are always liars, evil beasts, slow bellies. This witness is true' " (Holy

[1] A koan is an ambiguous or paradoxical saying or a short story used in Zen Buddhist practice and thought of as material for meditation and training in reaching enlightenment or satori (Japanese) which literally means "to understand" (*The American Heritage Dictionary of the English Language,* 2004).
[2] *Epideixis* (Greek term) refers to a lavish and ornamental speech whose aim is persuasion (*Dictionary of Languages and Linguistics*, 2005).
[3] Oedipus' answer to the question is: At the beginning of one's life, as an infant she crawls on four legs, then she walks on two, and in her old age she uses a cane or three legs (Seyffert, 1964).

Bible, 1977, *Titus* 1:12-13). The prophet in question was Epimenides, who had a reputation among the Athenians as a philosopher and a seer. His statement "The Cretans always lie," has been quoted for centuries as an exemplary paradox, given that Epimenides himself was a Cretan. The statement didn't seem to bother the ancients a great deal, for history had shown that some Cretans did tell the truth sometimes, apart from the accounts of ancient historians who tended to confirm St. Paul's view of them.

The ancient world had different reactions to paradox. Pythagoreans, for example, who thought of order and perfection as attainable only through the elimination of conflict, resorted to denial of paradox. And when it invaded their idyllic mathematical *harmonia* through Hippasus's discovery of incommensurables (e.g. $\sqrt{2}$), which was in direct conflict with the Greek theory of rational numbers, the presented paradox was consigned to secrecy, and Hippasus expelled from the Pythagorean school. On the other hand, Heraclitus' idea of a cosmic order was grounded in ontological paradox, and his idea of the Logos encompassed notions both of unity and of a conflict of opposites. Similarly, the Eleatic school and its most notable philosopher Zeno, by emphasizing paradox developed a method of disarming his opponents by leading their arguments *ad absurdum*, and consequently became famous for his paradoxes, most of them concerned with dichotomy and the mathematical problem of infinity. Through his formulation of over forty paradoxes, Zeno touched upon fundamental issues connected with the nature of mathematical objects and fatally compromised the hitherto assumed-unshakable foundations of mathematics and logic which, centuries later, spurred discussions among mathematicians that further inspired the development of set theory and various trends in philosophy of mathematics.

After Zeno, the epideictic tradition—which up to then had been diffused throughout rhetoric, drama and nature philosophy—took a different direction. Plato's dialogues, the most famous documented example of the ancient Greek tendency to undertake rational discourse as a kind of play, initiates a form of dialectic with different philosophical purposes. Socrates–that iconic philosopher whose method of inquiry initiated the Western tradition of philosophical dialogue—promoted a form of dialectic

known as *elenchus*. In contrast to the eristic form, Socratic *elenchus* launched inquiry with a question, and then required that one "…submit nobly to the argument as you would to a doctor…." and follow it where it leads. It aimed at persuasion, not through lavish speech, but though the examination of one's opponent's assertions in order to draw out the inherent contradictions within the other's position. Socratic *elenchus* is believed to be a refined form of Zenonean reasoning, and to be highly influenced by Zeno's *reductio ad absurdum* approach (Edwards, 1967; Vlastos, 1983). This characteristic of Socrates' discursive style brings his interlocutor to a state of *aporia,*[4] whereby his system of propositions and beliefs collapse under the insurmountable pressure of the derived contradiction. Socratic dialectic, then, offers an idiosyncratic educational model which operates by exposing contradictions which act as catalysts for dialogue between opposites, and in which the facilitator functions maeutically[5] rather than didactically. Thus in the Socratic method paradox (contradiction) is assigned a central role, which is essential for the thinking and learning process.

Paradox and Its Effects

Plato held that paradox in philosophy acts to provoke reflection—to "compel the soul to be at a loss and to inquire"—and thus impels human reason toward those reconsiderations which in turn lead to a higher level of understanding. And in his *Republic*, Plato (1961) used contradiction (paradox) in a similar way in his preface to the Allegory of the Cave (vi. 509) to describe knowledge acquisition as a four-part process: beliefs are acquired, then examined—a process in fact of coming to know what those beliefs are—then tested to see whether they contradict each other (i.e. yield paradoxes), and, finally, justified, which legitimates them as "knowledge." So Greek *aporia* became so pervasive in human inquiry that it came to be seen by many philosophers as inherent in any cognitive enterprise.

Among later philosophers, we find the same idea that contradiction/conflict not only initiates thinking, but guides its course as well. Charles Pierce suggested a method

[4] *Aporia*—an encounter with an insoluble contradiction or paradox (from Greek *aporos* which means "impassable") (*The American Heritage Dictionary of the English Language*, 2004).
[5] From *maiusis* (Gk.), birthing assistance, or midwifery

of justifying belief which, like Plato's, involves the examination of beliefs that are contrary or contradictory—but he placed a new emphasis on the role of a community as opposed to the lone individual in the "fixation of belief" (Pierce, 1958, 1966). Quine and Ullian (1970) elaborate on this process of "the fixing of belief" by suggesting that when, in a dialogical process, a set of beliefs is found to be inconsistent, the necessity to assess the grounds for this set may lead to the rejection of the least firmly supported of the contradictory beliefs, and thereby restore the consistency of the "web of beliefs" (Quine & Ullian, 1970).

Similarly for Dewey (1910), thinking is launched by conflicts, which "often fuse into one conflict between conditions at hand and a desired and intended result. . . . The object of thinking is to introduce a congruity between the two" (Dewey, 1910). In his *Logic: The Theory of Inquiry* he says: "Nothing is more important in inquiry than the institution of contradictory propositions" (Dewey, 1939, p. 197). Dewey sees the introduction of negation as inevitably bringing contradictions, and thus forcing discrimination and producing differences. Although his idea seems similar to Plato's, Dewey's institution of contradictories is conceived as one step in an ongoing process of inquiry, which he understands as providing only provisional conclusions, which are always liable to modifications. A supposed contradiction may bring a revision of a generalization, concept or conclusion which in itself is a result of the preceding inquiry--which is to say that the contradiction and the respective modification may affect the current as well as the preceding inquiry. In short, philosophers seem to consider contradiction (paradox) to be the most pointed example of cognitive conflict and an essential force in thinking and the search for truth.

Dialecticians, however, understand contradiction to be at the very center of human development itself. The dialectical moment begins with "being conscious of it [the thing] as a unity of opposed determinations" (Lawler, 1975, p. 4) and of the unfeasibility of adequately comprehending the thing in its given form—a notion which in fact resembles Dewey's idea of the starting point of inquiry. Thus the dialectical approach seeks to grasp processes in the full complexity of their interrelationships, and

to understand them as comprising a dynamic, self-organizing unity of internal contradictions and configurations of oppositions which are in fact the internal forces for movement and change. Furthermore, dialectical analysis does not regard the process of development as a simple process of growth, but as a development which passes from insignificant and imperceptible quantitative changes to fundamental qualitative changes.

The two giants of 20th century psychological theory—Piaget and Vygotsky—pay special attention to the role of cognitive conflict in development, and both offer an explanation of the transformations in cognitive structures, which assumes the existence of irreducible contradictions and proceeds through overcoming them. In other words, both theorists understand cognitive development as a continuous process of reconstruction of cognitive structures, whether understood as a movement from disequilibrium to re-equilibration which in fact mirrors the mechanisms of dialectics (Piaget, 1977, although it has never been acknowledged by Piaget), or explicitly described as a dialectical process as in the case of Vygotsky (1981). In both thinkers, cognitive development is understood as a process of active adaptation to the external world which involves progressive changes, and therefore learning.

This study takes as its theoretical framework the broad philosophical tradition briefly reviewed above, which understands cognitive conflict and contradiction in their extreme form as triggers for inquiry—a tradition which provided an epistemological basis for conflict-theory in psychology, and which assumes the inherent ability of humans to overcome or transcend constraints by creating new intellectual tools and developing new modes of thinking. Paradox could be said to represent the most pointed example of cognitive conflict, and thus merits inquiry as to its role as a mediator in cognitive development and learning.

The questions which naturally follow are: what form of pedagogy is both appropriate and effective in creating this form of cognitive conflict specifically in a mathematics classroom? Do we regard all learning as equally important? And what kinds of problematic situations can be engineered in order to support different types of

learning? In preparing to answer this question, it is necessary to examine some effects of paradox and aspects of learning and for this purpose I will turn to a brief description of two traditions that use paradox as a tool for triggering cognitive change and promoting learning in their specific contextual frames—paradoxical psychotherapy and Zen Buddhism.

Paradoxical Psychotherapy and Zen Buddhism

I will precede the description of paradoxical psychotherapy with an explanation of what is understood by paradoxical communication—which in fact is what usually brings people to paradoxical therapy. Paradoxical communication is defined as a medium in which various communicational modes are consciously or unconsciously mixed, and their differences are not recognized as such by some of the communicative partners. Bateson's Double Bind Theory was formulated as an attempt to explain paradoxical communication and its therapeutic treatment. Schizophrenic patients in particular are often unable to differentiate between message and context or message and metamessage. To illustrate this, Jay Haley (1955)—a co-author of *The Double Bind Theory*--describes a patient who always diligently knocked on the door of an office as he passed by it—literally following the request on the door which read "Please knock." And more generally, mixing different levels of abstraction in communication tends to lead to paradoxes, as in humorous discourses, which often take one message-mode for another—for example a literal message for a metaphorical one, or visa versa, or choosing messages whose ambiguity allows them to be read in different modes. In such cases, the moment of discovery or "punch line" comes with the realization of the existence of these alternative modes, which triggers a sudden reinterpretation of the message.

A double bind situation is created when: 1) at least two messages of different levels of abstraction are communicated; 2) one of them is a message about the other—that is, a metamessage; and 3) both messages are contradictory. If they contradict each other, they create a paradox similar to the Liar Paradox, or what Bateson and his colleagues referred to as a double bind situation. In the sphere of human relationships,

Bateson (1972) finds the majority of double bind situations in long-term relations in which one of the communicators is understood—consciously or not— as more powerful than the other. He provides a prime example in describing a relationship between a mother and a child or a teacher and a child in which the mother/teacher is communicating a primary injunction of the form "Do not do this or I will punish you," and a secondary injunction which communicates the message "Do not see this as punishment," or even "Do not submit to my prohibitions." The child finds herself in a situation in which the two messages produce a paradox. Such a communication system is characterized as homeostatic—that is, as a system that perpetuates its communicative patterns, responses, and roles—i.e. "a game without end" (Watzlawick, Beavin & Jackson, 1967). A similar case is produced by the injunction "Be spontaneous!" which one can only obey by disobeying, since complying with the order implies the rejection of externally imposed rules and following one's own internal motivation. The concept of "child-centered" or "democratic" education might also create paradoxes, since the requirement that students engage in studying what really interests them sounds more like "We require that you wish to study what we don't require you to study."

In the homeostatic communication system like the one described above, there is no way out—whatever the response is, it will contradict one of the initial messages, and will perpetuate the paradoxical communication pattern. The only way to escape the situation is to recognize the paradox (the psychotherapist usually helps the patient with such a recognition), which means to become aware of the impossibility of establishing the consistency of the series of communicational messages just by choosing an alternative response. Such a realization initiates a *reductio ad absurdum* line of reasoning (Haley, 1955), which facilitates a transgression of this communication level, and the arrival at an inference that, given the established rules, a non-contradictory response is impossible, and therefore the "rules" must be changed. Paul Watzlawick, John Weakland, and Richard Fish (1974)—a research group from The Mental Research Institute in Palo Alto concerned with invariance and change in human communicative systems--differentiate between first and second order change in communicative

systems. The former is understood as change *within* the communicative system; the latter concerns change *of the system itself*—i.e. a change of the rules on which the system is based. Second order change is associated with a psychological phenomenon identified by Arthur Köestler as "bisociation"—which he describes as "an abrupt transfer of the train of thought from one associative context to another" (1969, p.60). He understands such an event as characteristic of "acts of creation," sudden illuminations that result in conceptual, artistic, or experiential synthesis.

A key psychotherapeutic technique which is thought to facilitate "bisociation" and second-order change is termed *reframing* which, according to Watzlawick, Weakland, & Fish (1974), means "to change the conceptual and/or emotional setting or viewpoint in relation to which a situation is experienced and to place it in another frame which fits the 'facts' of the same concrete situation equally well or even better, and thereby changes its entire meaning" (p.95). Understood in this way, second-order change does not change a system directly, but rather alters the concept of the system, which, in turn, entails a system change. Reframing could also be described as the modification of a concept by assigning it to a different class of members. The change in class membership introduces a whole new set of conceptual relationships, a process somewhat similar to the "ontological shift" which, according to Michelene Chi (1997), occurs when a concept "changes the ontological tree to which [it] belongs" (p. 220).

Therapeutic interventions in pathological communication usually aim at second-order change, and because the interventions themselves employ paradox and are based on dialectical strategies, are often referred to as "paradoxical psychotherapy" (Mozdzierz, Macchitelli, & Lisiecki, 1976). Alfred Adler (1956) is known as the first Western psychotherapist to have utilized—influenced by Hegelian dialectical thinking—paradoxical strategies in the interest of behavioral change. He was preceded, however, by the centuries' old tradition of Zen Buddhist philosophy in Japan, whose goal was to achieve "enlightenment" (*satori*). Zen masters use paradox in the form of the *koan*—a brief statement or question designed to trap the mind in a contradiction in

order to "awaken" the disciple. The following is a *koan* is in the form of a dialogue between a monk and his spiritual master:

> Joshu asked the teacher Nansen, "What is the true Way?"
> Nansen answered, "Everyday way is the true Way."
> Joshu asked, "Can I study it?"
> Nansen answered, "The more you study, the further from the Way."
> Joshu asked, "If I don't study it, how can I know it?"
> Nansen answered, "The Way does not belong to things seen: nor to things unseen. It does not belong to things known: nor to things unknown. Do not seek it, study it, or name it. To find yourself on it, open yourself wide as the sky." (Reps, 1970, p. 105)

The *koan*, it would seem, initiates a form of deliberation in the seeker reminiscent of the Hegelian oscillation between thesis and antithesis, and a leap—described by Daisetz Suzuki (1973) as a result of having "exhausted everything belonging to his intellect or his conscious deliberation" (p. 222)—into synthesis. *Koans* force one into a double bind, and trigger transcendence toward deeper meanings beyond words, and beyond object-thinking.

> Shuzan held out his short staff and said: "If you call this a short staff, you oppose its reality. If you do not call it a short staff, you ignore the fact. Now what do you wish to call this?"
> Mumon' commentary[6]: If you call this a short staff, you oppose its reality. If you do not call it a short staff, you ignore the fact. It cannot be expressed with words and it cannot be expressed without words. (Reps, 1970, p. 124)

For Zen, truth is of a higher level of abstraction than words, and can only be achieved through transcending the dualism which thinking necessarily implies. Zen does not value abstraction or conceptualization, for any conceptualization implies a division into categories and compartmentalization. In fact, thinking and even perception itself are bound to dualism, for conceiving or even perceiving an object means to set boundaries between what the object is and what the object is not—that is

[6] Mumon (which means "no-gate') was a monk who compiled a collection of forty-eight koans, each accompanied by a commentary and a verse, and published in 1228 as *Mumonkan.* It is knows as "The Gateless Gate."

to resort to the standard logic of the excluded middle[7]. Contrary to this kind of Western thinking, Zen preaches the transcendence of dualism through the attainment of an imageless, speechless, holistic state of being (Suzuki, 1959). Zen philosophy implies understanding the world as a whole, without breaking it into pieces, and without making of it an object of thought—a concept that might cause a great deal of anxiety for the Cartesian mind. Words are not trustworthy tools in the search for meaning and truth, and yet they are all we have, for as the koan says, "It cannot be expressed with words and it cannot be expressed without words." And here we arrive at a paradoxical statement, which leaves the Zen devotee in a double bind. The role of the koan is to set up paradoxical situations like the one above in an attempt to obviate the limits of conceptual thought and verbalization. It is only by entering a state of bewilderment that one's mind can break free and make a leap to enlightenment. The leap is made, not as a result of a choice of "yes" or "no," but through the suspension of choice and the contemplation of both sides of the paradox, which makes the transgression of its boundaries possible. Like Double Bind theory, it requires "stepping out" of the system, thereby changing the concept of the system and consequently the system itself.

Gregory Bateson's Theory of Learning

Bateson (1972) associates cognitive change with learning, and he recognizes ahierarchical ordering of learning. What he calls Learning I is associated with habituation. Rote learning can be attributed to this category: in this case, to learn is to link a certain response with a specific stimulus/question/problem. The classical example of Learning I is the Pavlovian type of conditioned response based on instrumental reward. An instrumental "trial-and-error" problem solving process, where a revision of choice is made within a set of possible answers, could also be attributed to the same category. Multiple choice tests often call for a simple trial and error check in order to find out which of the suggested answers works.

[7] In standard logic, the law of the excluded middle states that "P or not-P is true upon any interpretations of P" (*The Oxford companion to philosophy*, 1995).

Learning II, Bateson holds, requires contextual evaluation of the problematic situation whereby a response is chosen from an altered set of alternatives. This, according to Bateson's theory, represents higher-order learning than Learning I, and implies the ability to reinterpret the context. Such a reinterpretation also allows for a different response chosen from among a new set of alternatives--but although a reinterpretation of the context can cause a change within the system, it won't change the system itself. On the other hand, Learning III is connected with what has already been referred to as a second-order change, i.e. with the change in the fundamentals of the system itself. This even higher-order learning than Learning II Bateson calls "learning to learn," which he regards as the highest category of human learning, offering as it does the possibility of transcending double bind situations and reaching dialectical synthesis. Each type of learning is characterized by a certain degree of flexibility—that is, freedom from habitual reactions–and the higher the category of learning, the greater the flexibility.

As we have seen, Bateson suggests that the double bind is encountered in the face of a contradiction which by definition is unresolvable, and forces what he calls a process of "learning to learn," which is realized through a complete reorganization of the understanding of the context of the problem, and consequently a profound change in the system or theory used to interpret it. It appears to resemble a Hegelian synthesis or "sublation," although it is not interpreted by Bateson in this way. Learning III or "learning to learn" seems to be a rare phenomenon, which coincides with profound emotional realization, deep cognitive restructuring, and extensive meta-analysis. The conditions that make Learning III possible resemble the koans practiced in the spiritual tradition of Zen Buddhism and the "double bind" situations created in ~~a~~ homeostatic systems that are approached by Paradoxical psychotherapy.

There appear to be parallels between overcoming the barrier posed by Zen koans and the transcendence of the double bind, or what, according to Bateson (1972), makes Learning III attainable. Although the metalevel of Learning III-- to our Western minds—appears to be "thinking about thinking," and *satori*, according to the Zen masters, is a

state of not thinking and merging with the universe, both experiences are end products of a chain which begins with paradoxes of some kind that trigger a double bind situation, lead one to recognize the double bind, and, if successful, trigger a transcendental leap out of it. Analogously, practitioners of the paradigm of paradoxical therapy hold that only after the realization of a "double bind" situation that confronts them with an untenable absurdity are patients able to break out of a self-perpetuating, pathological and homeostatic system by changing the system and its rules (Weeks & L'abate, 1982; Watzlawick, Beavin, & Jackson, 1967). Similarly, the psychotherapist Hans Sachs, a colleague of Freud's, stated that as a general rule an analysis ends when the patient realizes that it could continue endlessly (Watzlawick et al., 1967). What seems compelling about paradoxical psychotherapy is that what is available as an immediate choice is the habitual response that maintains the homeostasis, and what breaks it is a completely new possibility born from the awakened mind.

The way out of a double-bind is not within the set of possibilities offered within the system, but beyond it. It comes with an awakening, which is an act of undoing, reversing, discontinuation, or unlearning. It would appear that this undoing mode is only a precursor of enlightenment, which is a continuous experience of the unity of subject and object, and this indeed is a new form of knowledge. The process at least evokes the dialectical spiral characteristic of Hegelian notions of development. Awakening is the undoing, or the negation of the products of previous learning, whereby—in the unity of both the previous learning and its negation, the synthesis, a new form of learning, is born. The new form is qualitatively different, in that it both retains aspects of the previous form and negates it at the same time.

Implications for Teaching and Learning with Paradox in a Community of Mathematical Inquiry

What implications for mathematics education can we draw from Bateson's theory of learning and the two paradigmatic modes of learning experiences encountered in Paradoxical therapy and Zen Buddhist spiritual training? Bateson's

conception of types of learning has obvious implications for teaching and learning understood as problem-solving. Mathematics education is still notorious for perpetuating Learning I—rote learning accompanied by a simplistic, instrumental approach to problem-solving. Learning II and III are closely connected, as we have already seen, with the concept of change—whether systemic and/or personal –and therefore with the Hegelian ideal of mental development—or *bildung*. Both types of learning are associated with an encounter of inadequacies, and conceptual change and transformation as the outcome of overcoming these inadequacies. The latter are, if we assume a dialectical perspective, inherent in every dialectical system, whether communicative, cognitive, social, or some combination of the three. Learning, then, is a characteristic and a dimension of these systems in all their combinations, acting both as an adaptive mechanism and as an agent of system change. I want to suggest that the distinctive problematic of the paradox, which represents inadequacies (contradiction in its "purest" form), when approached in a dialogical and therefore dialectical pedagogical form—that is "Community of inquiry"—may act in a powerful way to support Learning II and III.

Community of Inquiry

Community of inquiry as understood in this study may be broadly and simply described as the collective execution of a dialogical, language-based activity whose goal is to reach communal agreement through argumentation. The model of community of mathematical inquiry (henceforth CMI) so conceived is adapted from the model of community of philosophical inquiry developed by Mathew Lipman and Ann Sharp in the 1970's at the Institute for the Advancement of Philosophy for Children at Montclair State University in New Jersey, which is the pedagogical basis for the Philosophy for Children curriculum (Lipman, Sharp & Oscanyan, 1980; Lipman, 1991). One main objective of both is an emphasis on the construction of meaning and the formation of concepts, not through transmission, solitary reflection or debate, but through what is referred to as "building on each other's ideas"— that is through distributed thinking in a dialogical context (Kennedy, 1999). Communal inquiry is understood to advance

through reasoning, or "the giving of reasons," and each reasoning move ideally represents a reconstruction of the inquiring system in the direction of a more sophisticated conception. The ideal inquiry proceeds through a form of argumentation which is inherently dialogical and thus by implication a dialectical process, which is to say a process which moves forward through encountering and attempting to resolve tensions/inadequacies/contradictions. In the inquiring system as such, any given argument is built on or as a counter argument to a previous one.

In an open, communicative system such as CMI, it is deemed important that students fully orient to the problem from the very beginning, so that they understood themselves as primary agents and "owners" of the process (Gal'perin, 1980). Further, it is considered essential that the goals toward which the activity is directed be negotiated collectively, and thereby at least putatively accepted by each individual member (Davydov & Markova, 1983). Such a negotiation represents one dimension of an implicit contract between facilitator and students, which is based on the mutual understanding that each individual is viewed as self-regulating subject who is responsible for her own learning process very much in the way patients in paradoxical therapy and Zen students come to understand themselves. Another dimension of the same contract is the implicit agreement that the facilitator's role is to support the students in advancing with the activity only if they need help—that is, the facilitator is expected to operate within the students' zone of proximal development, but not within their zone of actual development (Vygotsky, 1962). Such support proceeds without providing direct answers or authoritative perspectives, but more through a form of the Socratic *elenchus*—that is, through provocative questioning, reformulation, and the offering of counter-examples and counter-perspectives. In fact the constructive process can be influenced by any single element of the system—for example by any single participant—as well as by any element in the cognitive medium, for example the initial problem under consideration, specific examples and counterexamples, or by the presence of conscious or unconscious assumptions. The chief pedagogical significance of this process of CMI is that it operates in the collective zone of proximal development,

which acts to "scaffold" concepts, skills and dispositions for each individual. The scaffolding process functions through subprocesses such as clarification, reformulation, summarization, and explanation, as well as through challenge and disagreement.

When learning situations in CMI are structured around problems which are broadly described as paradoxes, they offer a strong element of cognitive surprise and, given their succinctness, a well-defined starting point for discussion. Here the term paradox is loosely employed to denote the general class of problems which hide either a real or a fictitious contradiction, or "paradoxical problems" which offer a surprise that runs contrary to the assumptions and inferences that we spontaneously subscribe to on first reading. The latter we would denote as Type I paradoxes, and the former would be called Type II paradoxes. An example of such a problem is: A clock strikes 6 times in 5 seconds. How long would it take to strike 12 times?

The idea that the clock would strike 12 times in 10 seconds is highly plausible, but not correct. Another competitor for a solution is the supposition that the clock would strike 12 times in 11 seconds. This situation is in fact characteristic of the entire group of Type I paradoxes—they provoke at least two competing contrary inferences[8] that immediately present a cognitive conflict, which provides a natural context for argumentation.

Even a problem as simple as this one--A bottle and its cork cost $ 1.10. The bottle costs $1 more than the cork. How much does the bottle cost?—usually provokes several contrary propositions which presents the group with a "forked-road" situation that calls for a review of the relevant (to the conflicting propositions) set of beliefs, assumptions or premises, for reflecting on the unwarranted ones, and for "weeding out" the incorrect ones (Quine & Ullian, 1978, p. 18). Thus, the problem situation has the potential of triggering a modification of the whole web of beliefs, assumptions, or premises into a more adequate one.

The Type I paradox is expected to structure argumentation so as to allow for the emergence and juxtaposition of contrary propositions/interpretations, which in turn

[8] Two propositions are called contrary if they cannot both be true, but *can* both be false. Each proposition entails the negation of the other, but is not entailed by it (see Geach, 1972).

create an urgent felt need for re-evaluation. The exigency of this re-evaluation has been already explained as one result of the experience of cognitive dissonance, and the felt frustration which results in a disruption of an expected consistency (Jecker, 1964; Festinger, 1964). As has already been pointed out, the phenomenon of juxtaposition of contrary interpretive frames has been studied by Köestler in his work on creativity (1969), where he introduced the concept of *bisociation,* which he defines as " …the perceiving of a situation or idea, L, in two self-consistent but habitually incompatible frames of reference. . ." (Köestler, 1969, p. 35). Thus any changes associated with Type I paradoxes can be attributed to a change in the subjectively used interpretative framework for grasping the context of the problem, which is necessarily associated with a change in underlying assumptions and beliefs.

In comparison to Type I, Type II paradoxes provoke contradictory propositions and contradictory interpretive frameworks for the problem. In order not to complicate the matter, I will not differentiate between the Type II paradox and the antinomy. The latter, an example of which is the Liar paradox, creates a Batesonian "double bind." Another example would be Cantor's paradox, which can be offered in the following version: Let's consider the following infinite sets of numbers: [1, 2, 3, 4, 5, 6, . . .] and [2, 4, 6, . . .]. Do they have equal or a different number of elements? If you think the latter is the case, which set has more elements? In fact these were two paradoxes triggered a crisis in the foundations of mathematics, and caused a series of modifications and innovations in set theory and classical logic. So historically, they did create untenable situations of "double bind" variety, which brought about major structural changes in the system. Mathematical paradoxes, or what we termed Type II paradoxes, are perfect demonstrations of "borderline cases" that can't be resolved within the formal system, and thus call implicitly for reorganization of the whole system of mathematical knowledge.

It should be remembered that the Type II paradox is a necessary but not a sufficient condition for Learning III, since it requires cognitive maturity and knowledge; but it at least creates conditions for meta-analysis, and for deliberation about

restructuring the set of interpretive frames under consideration. In fact my own research so far (Kennedy, 2005) suggests that communal discussions in which paradoxes are deconstructed in an attempt to restore the consistency broken by the appearance of a logical contradiction echo the historical struggle of mathematicians in their efforts to salvage the foundations of mathematics after they had been seriously shaken by various paradoxes. And such discussions also implicitly challenge the old belief in "certainty," and the idea that a math statement has a "stamp of incontestability" (Wittgenstein, 1969, par. 655). As a pedagogical device, paradox is useful, not as a means of devaluing standard logic and logical reasoning, but to demonstrate the limits of standard logic, and to open students to alternatives. The benefits of perspectival flexibility are clear enough—if one need not conform to one particular method or mode of reasoning because it is the accepted standard, he or she is in a position to make a better-informed decision after critical examination of the alternatives. The possibility of cultivating such flexibility means, of course, the transformation of our understanding of teaching from a matter of delivering facts and/or "the true method"—whatever that happens to be—to understanding it as the business of conducting critical inquiry within a spectrum of methods and dimensions of reasoning.

Cognitive scientists Francisco Varela, Evan Thompson & Eleanor Rosch (1999) integrate Buddhist phenomenology and Western cognitive science in their development of the idea of "embodied mind." A central aspect of their description of this form of cognition is *mindful, open-ended reflection*, one which is not just *on* experience, but *is* a form of experience itself—and such A reflective form of experience precludes habitual thought patterns and responses based on preconception. The goals of being mindful of one's own mental processes expressed in the notion of "embodied mind" and of "enlightenment" in Zen Buddhism are progressively reached, according to their proponents, through observation and consequent unexpected discovery—or to put it in more secular terms, through systematic doubt and inquiry. Contrary to Köestler (1969), who sees the phenomenon of "bisociation" as unpredictable and rare, a phenomenon so

elusive that to "manage" it is inconceivable, Zen doctrine claims that the cultivation of paradox, combined with meditation, can not only confound the habit of discursive thought and shock the mind into awareness, but also alter the habitual mind-body relation. The mystery of the koan appears as a precondition--much like the untenability of the personal paradox in paradoxical psychotherapy, or the radical puzzlement encountered in classroom inquiry into paradoxes—which compels students to reduce (but not necessarily to eliminate) uncertainties, a phenomenon which, on these accounts, is associated with awakening and insight, and with the consideration of possibilities which may not otherwise have been conceived of.

Conclusion

This paper explores the effects of paradox and the mediation of cognitive conflict it creates in a variety of *topoi,* including double-bind theory in psychotherapy and learning—in particular Bateson's three-tiered theory of learning—and the role of the koan in Zen Buddhist practice. It argues that paradox—which plays a major role in all of these forms of theory and practice, promises as well to support and stimulate the conditions for higher order learning in the mathematics classroom. The study has concerned to justify the use of paradoxes quite specifically in the pedagogical context of a community of mathematical inquiry—a discursive location in which participants are invited to justify their positions and to examine their claims in the light of the claims and positions of others, and to balance both justification and negotiation processes in a sort of "third way."

The experience of being exposed to radical contradiction can lead to significant restructuring of participants' understanding through the alteration of reasoning frameworks, and result in learning which is of a higher order than rote or instrumental. Understood as the unavoidable self-presentation of opposing ideas, of cognitive confusion, and in its ultimate form as a contradictory statement, paradox offers an ideal

didactic context for open-ended group discussion, for the intensive practice of reasoning, acquiring dispositions critical for mathematical thinking, and learning associated with cognitive change.

Community of inquiry pedagogy suggests the benefits of a form of practice in mathematics education significantly different from the traditional teaching and learning paradigm. It takes the notion of distributed learning and thinking with the utmost seriousness, which amounts to the epistemological claim that knowledge constructed in an inquiring system—a group whose chosen activity is carried out through collaborative, dialogical deliberation—has qualitative differences from knowledge attained individually, or even as a result of a dyadic interaction. CMI and its forms of learning demand a form of pedagogy informed by positive humanistic belief that unites theory and practice, philosophy and application, argumentation and calculation, conflict and its mediation in the concrete, problem-based context of the classroom. The application of this learning and teaching model poses a profound challenge to mathematics education, given both the nature of the discipline and the pedagogical traditions which still dominate it, but it is capable of developing into a form of classroom practice which has the potential of transforming the field.

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Recebido em: 14/09/2006
Aprovado em: 21/10/2006