USING THE FAMA-BLISS METHOD TO ESTIMATE THE TERM STRUCTURE OF INTEREST RATES

(A UTULIZAÇÃO DO MÉTODO DE FAMA-BLISS PARA ESTIMAR A ESTRUTURA TEMPORAL DAS TAXAS DE JURO)

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ABSTRACT

The objective of this paper is to provide a monthly estimation of term structure of spot interest rates and forward interest rates since the beginning of the European Monetary Union. In order to do this, we apply the Fama-Bliss method, the approximating functions of two of the methods most commonly applied by the central banks, the Nelson and Siegel method (1987) and the Svensson method (1994) and two objective functions. Then, we compare the four options to decide which the most satisfactory procedure is. Subsequently we provide the chosen term structures of spot and forward interest rates.

KEYWORDS

Term Structure of Interest Rates – Fama-Bliss Method – Bootstrapping Method – Estimation Methods.

RESUMO

O objectivo deste artigo consiste em proporcionar uma estimativa mensal, utilizando dados diários, da estrutura temporal das taxas de juro à ordem e a prazo desde o início da União Económica e Monetária. Para isso, aplicámos o método de Fama e Bliss, as funções de aproximação de dois dos métodos mais utilizados habitualmente pelos Bancos Centrais, o método de Nelson e Siegel (1987) e o método de Svensson (1994) e duas funções objectivas. Posteriormente, comparamos as quatro opções para decidir qual é o método mais satisfatório. Finalmente, apresentamos as estruturas temporais das taxas de juro a pronto e a prazo que foram seleccionadas.

PALAVRAS-CHAVE

Estrutura Temporal das Taxas de Juro – Método de Fama e Bliss – Método Bootstrapping – Métodos de Estimação

1. INTRODUCTION

The term structure of interest rates (IRTS from now on) is one of the main research topics regarding Financial Economy. In fact, it can be used for the prediction of macroeconomic magnitudes such as inflation, for the verification of several explanatory theories relating to IRTS, as well as for pricing fixed income securities, swaps and other derivatives.

The primary objective of this paper is to provide a monthly estimation of term structure of spot rates and forward interest rates since the establishment of the European Economic and Monetary Union (EMU) on January 1st 1999. In order to achieve this aim, we use daily euribor rates and euro vs euribor swaps interest rates. The data are collected from DataStream.

According to the nature of these data, we use the most frequent method, the *bootstrapping* method. Later, we will apply two of the most commonly approximating functions used by central banks to estimate IRTS, Nelson and Siegel's (1987) and Swenson's (1994), and two objective functions: minimization of the sum of squared errors of interest rates and minimization of the sum of squared errors of interest rates, but weighed by the inverse of the duration of each financial issue.

This paper is organised as follows: In Section 2 the background of research topic is introduced. Approximating functions are presented and interpreted in Section 3. The data and methodology are described in Section 4. In Section 5 four methods are compared and a statistical analysis is carried out to choose the most satisfactory procedure. In Section 6 the term structures of spot and forwar rates are estimated for the period January 1999-March 2006. Finally, conclusions and references are given.

2. BACKGROUND

In general, the complete spot interest rate curve is not directly observable, so it is necessary to estimate it. Several methods have been developed to obtain spot rates from par yields available in the market. One of the most simple methods for estimating an approximate function is the bootstrap method, based on a recurrent procedure which provides spot rates in successive dates. In order to apply this technique, it is necessary to adjust yields to a smooth curve, and linear or cubic splines are generally used.

Most of central banks apply Nelson and Siegel and Swenson methods to estimate IRTS. With this aim, information about government deb instruments is used because it refers to riskless issues. However, some countries, such as Italy, Switzerland and Norway, use data from the money market to estimate the short term curve, given the limited number of observations in this term.

As far as this aspect is concerned, Kloster (2000) estimates Germany´s IRTS and makes an analysis of differences between forward rates, estimated by applying information from diverse markets to them, combining data from the money market with data from the government debt market, using data from government debt issues for all maturities and using data from the money market for all maturities. However, it is also advisable to use data exclusively from the money market, as this also allows the comparison of term structures among different countries, since these data are more accessible and homogeneous. However, they lead to term structures which overestimate long term rates probably due to risk credit premium, which varies across the maturity spectrum and over time.

To estimate IRTS from the information given by financial markets, it is an usual practice to use the following tools: short term interbank deposit rates, forward rate agreements (FRAs) or interest rate futures contracts for the middle area of the curve (1-2 years), and swaps on interest rates for the long term, exactly as it is indicated in Ron (2000) and Alexander and Lvov (2003) for Canada and the United Kingdom, respectively.

However, since FRAs are not observable for most currencies or suffer from lack of liquidity, future contracts are usually used to extract the implied forward rates and it requires a convexity adjustment due to the difference in convexity characteristics of future contracts and forward rates. So as to avoid this restriction and use more homogeneous data, we also use swap rates for the period 1-2 years of the curve, a procedure also applied to the Italian market by Bank of Italy.

The most usual method to estimate IRTS by applying data from the money market is to use the so called bootstrapping method; in fact, several researchers have used and compared this method with others [Mansi and Phillips (2001), Jordan and Mansi (2003) and Bliss (1996)].

We carry out the estimation of term structure of spot and interest rates using the Smoothed model of Fama-Bliss (1987), which aims to smooth out the discount rates by fitting an approximating function through them. With this purpose, we employ the Nelson and Siegel (1987) and Svensson (1994) exponential functions because, according to the Bank for International Settlements (2005), most central banks have adopted these models to estimate the term structure of interest rates. The description of these models is given in the following section.

3. DESCRIPTION OF APPROXIMATING FUNCTIONS

Approximating functions which are used are the Nelson and Siegel (1987) and Svensson (1994) functional forms because they are two of the methods most used by central banks nowadays to estimate the term structure of interest rates. They are static models which aims to estimate the interest rate term structure at a specific time. They are also defined as parsimonious models because they lead to soft and flexible curves. With this aim, they consider that the instantaneous forward interest rate tends towards a constant value in the future, and assume that the instantaneous forward rates at any time t is given by the following functional form:

$$f(m) = \beta_0 + \beta_1 \cdot e^{-m/\tau_1} + \beta_2 \cdot \frac{m}{\tau_1} \cdot e^{-m/\tau_1} \quad (1)$$

where m is the time to maturity and the parameters to estimate have the following interpretation: β_0 is a positive constant which reflects the value towards which instantaneous forward interest rate tends; β_1 can be positive or negative, and indicates that the first exponential term is monotonically decreasing or increasing, respectively; β_2 can be positive or negative, and shows that the second exponential produces a hump or a trough, respectively and τ_1 are time constants associated with trend changes in the evolution of real interest rates. Small values of τ_1 correspond to rapid decay of the hump or trough towards the limiting value of β_0.

Spot interest rates can be represented by an average, obtained as an average of forward interest rates. When the time is continuous, they can be calculated as the integral of the instantaneous forward interest rates:

$$r(m) = \int_{t=0}^{t=m} f(t)\cdot dt = \beta_0 + \beta_1 \cdot \left[\frac{1-e^{-m/\tau_1}}{m/\tau_1}\right] + \beta_2 \cdot \left[\frac{1-e^{-m/\tau_1}}{m/\tau_1} - e^{-m/\tau_1}\right] \quad (2)$$

$$r(m) = \beta_0 + (\beta_1 + \beta_2)\cdot \left[\frac{1-e^{-m/\tau_1}}{m/\tau_1}\right] - \beta_2 \cdot e^{-m/\tau_1} \quad (3)$$

Nelson and Siegel (1987) demonstrate that this functional form allows to represent available shapes of the term structure, as it is shown in Table 1.The Svensson functional form (1994) is an extended version of the Nelson and Siegel functional form by an additional exponential term with two parameter β_3 and τ_2. It introduces further flexibility in possible curves in the form of a second possible hump or trough.

Table 1: Shapes of Nelson and Siegel curves

Curve shape	β_0	β_1	β_2	τ_1	Restriction
Increasing and concave	+	−	+	+	$\lvert\beta_1\rvert \geq \lvert\beta_2\rvert$
Increasing	+	−	−	+	$\lvert\beta_1\rvert \geq \lvert\beta_2\rvert$
Decreasing and convex	+	+	−	+	$\lvert\beta_1\rvert \geq \lvert\beta_2\rvert$
Decreasing	+	+	+	+	$\lvert\beta_1\rvert \geq \lvert\beta_2\rvert$
Hump, above β_0	+	+	+	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$
Hump, crosses β_0	+	−	+	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$
Trough, below β_0	+	−	−	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$
Trough, crosses β_0	+	+	−	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$

He assumes the following instantaneous forward interest rate and instantaneous spot interest rate:

$$f(m) = \beta_0 + \beta_1 \cdot e^{-m/\tau_1} + \beta_2 \cdot \frac{m}{\tau_1} \cdot e^{-m/\tau_1} + \beta_3 \cdot \frac{m}{\tau_2} \cdot e^{-m/\tau_2} \quad (4)$$

$$r(m) = \beta_0 + (\beta_1 + \beta_2)\cdot \left[\frac{1-e^{-m/\tau_1}}{m/\tau_1}\right] - \beta_2 \cdot e^{-m/\tau_1} + \beta_3 \cdot \left[\frac{1-e^{-m/\tau_2}}{m/\tau_2} - e^{-m/\tau_2}\right] \quad (5)$$

where β_3 and τ_2 explain the extended flexibility of the Svensson approach, but its interpretation is similar to that of parameters β_2 and τ_1.

4. DATA AND METHODOLOGY

To estimate the IRTS since the establishment of the European Economic and Monetary Union, data relative to the most liquid issues on every last Friday (day of close of business) from January 1999 through March 2006 were taken, making 87 monthly samples in all. Data are collected from DataStream and are the following:

- Eonia[1].
- Euribor for maturities of three, six and nine months and one year[2].
- Euro vs Euribor swap middle rates for maturities of two, ten, fifteen, twenty, twenty-four and thirty years[3].

Later we have proceeded to homogenize data:

- Eonia and euribor are quoted on an actual/360 day count convention. However, DataStream provides the 1 year-euribor on an actual/365 day count convention. All these interest rates are converted to the 30/360 day count convention of euro vs euribor swap interest rate, according to ISDA[4].

$$i_{1\,year(360)} = \frac{360}{365} \cdot i_{1\,year(365)} \quad (6)$$

where $i_{1\,year\,(360)}$ is the euribor-1 year on an actual/360 day count convention and $i_{1\,year\,(365)}$ is the euribor-1 year on an actual/365 day count convention.

Eonia and euribor represent the simple interest rate on a short term loan without coupon, i.e. spot interest rates. Then we calculate the annually-compounded interest rate and the continuously-compounded interest rate with the following expressions:

$$i_c = \left(1 + i_s \cdot \frac{t}{360}\right)^{360/t} - 1 \quad (7)$$

$$r(t) = \ln(1+i) \quad (8)$$

where i_c is the annually-compounded interest rate, i_s is the simple interest rate, t is the time to maturity (in days) and $r(t)$ is the continuously-compounded interest rate or instantaneous interest rate.

- The most liquid interest rates swaps are available only for two, five, ten fifteen, twenty, twenty-five and thirty years. Thus, we utilize an exponential interpolation to achieve an estimate of the missing swap rates for the maturities of three, four, six, seven, eight, nine, eleven, twelve, thirteen, fourteen, twenty-one, twenty-two, twenty-three, twenty-four, twenty-six, twenty-seven, twenty-eight and twenty-nine years. With these data the spot interest rates are calculated using the bootstrap method. With this aim, we apply the following expression:

$$i_t = \frac{C_t}{100 - \sum_{j=1}^{n-1} \frac{C_j}{(1+i_j)}} - 1 \quad (9)$$

where i_t is the spot interest rate at time t and C_t is the coupon at time t.

- Finally, we apply the Nelson and Siegel and Svensson approximating functions to estimate parameters by minimizing two objective functions:

1. The sum of squared deviations between estimated and observed spot interest rates:

$$Min \sum_{j=1}^{35} \left[\left(r_j - \hat{r}_j(\beta_0, \beta_1, \beta_2, \tau_1)\right)\right]^2 \quad \text{(Nelson and Siegel).} \quad (10)$$

$$Min \sum_{j=1}^{35} \left[\left(r_j - \hat{r}_j(\beta_0, \beta_1, \beta_2, \tau_1, \beta_3, \tau_2)\right)\right]^2 \quad \text{(Svensson).} \quad (11)$$

2. The sum of squared deviations between estimated and observed spot interest rates weighted by the inverse of Macaulay duration:

$$Min \sum_{j=1}^{35} \left[\left(r_j - \hat{r}_j(\beta_0, \beta_1, \beta_2, \tau_1)\right)\right]^2 \cdot 1/\Phi_j \quad \text{(Nelson and Siegel).} \quad (12)$$

$$Min \sum_{j=1}^{35} \left[\left(r_j - \hat{r}_j(\beta_0, \beta_1, \beta_2, \tau_1, \beta_3, \tau_2)\right)\right]^2 \cdot 1/\Phi_j \quad \text{(Svensson).} \quad (13)$$

where r_j are the observed instantaneous spot interest rates, $\hat{r}_j$ are the estimated instantaneous spot interest rates and Φ_j is the inverse of Macaulay duration for the maturity j.

These models coincide with Smoothed Fama-Bliss (1987) method, which applies Nelson and Siegel and Svensson functions to estimate spot and forward interest rates using government bond data. However, we utilize money market interest rates. Parameters are estimated by an iterative extraction method and not directly from bond prices, as it happens in Nelson and Siegel and Svensson models. This is due to the fact that these models cannot be applied to swap interest rates because of the nature and the features of data.

In order to obtain estimated spot interest rates we have used Visual Basic and a non-linear least squares optimization method, the algorithm quasi-Newton of Excel Solver Program. Moreover, it is necessary to start from an initial parameter vector in these two optimization criteria. We have assigned them the following values: β_0 is 30 year-swap interest rate, β_1 is the difference between the 30 year-swap interest rate and the overnight rate, β_2 is established according to table 1 values, τ_1 is the time when a trend change is observed in the spot observed interest rate and τ_2 is the time when a second trend change is observed in the observed spot interest rate.

5. COMPARISON OF MODELS

Once the term structure of spot and forward interest rates is obtained by applying the Smoothed Fama-Bliss (1987) method with Nelson and Siegel (1987) and Svensson (1994) approximating functions and two objective functions, the following statistical measures are analysed to determine the goodness of fit:

1) Coefficient of determination which measures the proportion of the total variation in a dependent variable that can be explained by the model. It takes values in the range 0 to 1. The critical point accepted in the econometric literature is 0,7 or 70 per cent.

$$R^2 = 1 - \frac{S_e^2}{S_y^2} \qquad 0 \le R^2 \le 1 \quad (14)$$

2) Root mean square yield error which can be interpreted as the standard deviation of yield errors.

$$RMSE = \sqrt{\frac{\sum_{j=1}^{35}(r_j - \hat{r}_j)^2}{35}} \quad (15)$$

3) The average absolute yield error which is less sensitive than the previous measure to outliers.

$$AABSE = \frac{\sum_{j=1}^{35} \left| r_j - \hat{r}_j \right|^2}{35} \qquad (16)$$

4) The best value of objective function which also measures the robustness of the solution.

$$SES = \sum_{j=1}^{35} \left(r_j - \hat{r}_j \right)^2 \qquad (17)$$

Previous statistical measures are calculated to choose the most satisfactory estimation method. Results are included in Table 2.

Table 2: Statistical results

		FBNS	FBNSP	FBSV	FBSVP
R2	Average	99,4605	97,9499	99,6547	99,1955
	Standard deviation	0,6680	4,5841	0,2979	1,2750
	Maximum	99,9375	99,9251	99,9424	99,9106
	Minimum	96,0622	69,6952	98,3368	90,5116
SES	Average	0,1079	0,0022	0,0555	**0,0003**
	Standard deviation	0,2717	0,0143	0,0383	**0,0013**
	Maximum	2,5544	0,1306	0,1864	0,0120
	Minimum	0,0161	0,0000	0,0001	0,0000
RMSE	Average	0,0475	0,0037	0,0375	**0,0023**
	Standard deviation	0,0289	0,0071	0,0133	**0,0021**
	Maximum	0,2702	0,0611	0,0730	0,0185
	Minimum	0,0215	0,0008	0,0013	0,0007
AABSE	Average	3,9290	6,3333	3,1581	4,4699
	Standard deviation	2,5828	5,2686	1,0543	1,8199
	Maximum	24,9912	29,5183	6,1262	11,7827
	Minimum	1,8652	1,8839	1,3587	1,6740

Source: Personal research based on DataStream data

where:

FBNS: Smoothed Fama-Bliss method which applies Nelson and Siegel approximating function with minimization of the sum of yield square errors.

FBNSP: Smoothed Fama-Bliss method which applies Nelson and Siegel approximating function with minimization of the sum of weighted yield square errors.

FBSV: Smoothed Fama-Bliss method which applies Svensson approximating function with minimization of the sum of yield square errors.

FBSVP: Smoothed Fama-Bliss method which applies Svensson approximating function with minimization of the sum of weighted yield square errors.

As regards the four versions of the Smoothed Fama-Bliss method (1987), coefficient of determination exceeds the critical point of 0,7 widely and, therefore, their values are excellent. The terminal value of objective function is introduced by the FMSVP method, i.e., difference between estimated and observed values was minimised. As for root mean square yield error, the minimum value is also introduced by the FMSVP method. Therefore, since the aim is to estimate interest rate term structure, and as statistical tests take very similar values, in principle, any of these methods is valid to estimate interest rate term structure; however, the best results are appreciated in the FMSVP method.

Table 3 shows a summary of estimated parameters. It is necessary to highlight that coherence with pri-

Table 3: Estimated parameters

		FBNS	FBNSP	FBSV	FBSVP
β_0	Average	5,5943	5,5231	5,6028	5,5906
	Standard deviation	0,6817	0,6910	0,6667	0,6656
	Maximum	6,5173	6,5435	6,5468	6,5795
	Minimum	3,9779	3,6523	4,0430	3,9629
β_1	Average	-2,5801	-2,5323	-2,5874	-2,6037
	Standard deviation	0,7626	0,8048	0,7270	0,7495
	Maximum	-1,2521	-0,9602	-1,2960	-1,0716
	Minimum	-3,9125	-3,8691	-3,8318	-3,9106
β_2	Average	-2,1486	-2,2373	-1,5170	-1,2121
	Standard deviation	1,4930	1,5700	1,1826	1,0537
	Maximum	0,0000	0,2261	1,7940	1,2441
	Minimum	-5,0148	-5,8777	-4,5872	-3,4028
τ_1	Average	2,1816	1,8102	1,9964	2,0150
	Standard deviation	0,6858	0,7001	1,1489	1,0703
	Maximum	4,1892	3,0011	7,6809	6,7622
	Minimum	0,8650	0,2256	0,2204	0,0624
$\beta_0 + \beta_1$	Average	3,0142	2,9907	3,0154	2,9869
	Standard deviation	0,9459	0,9428	0,9466	0,9454
	Maximum	4,9613	5,0305	4,9549	5,0306
	Minimum	1,9966	2,0075	2,0190	1,9428
β_3	Average			-1,1449	-1,2527
	Standard deviation			1,1390	0,9762
	Maximum			3,5329	0,9194
	Minimum			-5,4222	-7,0304
τ_2	Average			1,7097	1,9982
	Standard deviation			1,1415	1,0365
	Maximum			7,0620	6,3739
	Minimum			0,0706	0,0567

Source: Personal research based on DataStream data

ori hypothesis on coefficient signs is observed in the above mentioned Table 3. Since parameters standard deviations are considerably lower in the FMSVP than in other methods, it seems to be logical to choose this method.

6. RESULTS OF ESTIMATION

The method FBNS provides us with the following parameters: β_0 indicates that the instantaneous forward interest rate tends to 5,59%; β_1 is negative (-2,58), it indicates that the first exponential term is monotonically increasing and allows us to determine the short term segment of the curve; β_2 is negative (-2.14), which shows that the curve contains a through; τ_1 is a temporary constant which indicates a trend change, in this case in about two years (2,18) and it must be positive to guarantee a long term convergence with β_0. From these estimates does $\beta_0 + \beta_1$ derive, which can be interpreted as the shortest-term interest rate (3,01%), in this case it would be the overnight rate. As far as validation statistics are concerned, the coefficient of determination reaches a value superior to the minimum one required by the literature (99,46), and shows a good adjustment to the estimated function. With regards to the best value of the objective function (0,1079) and to RMSE (0,0475), it reaches minimal values; but, if compared with the other three methods, we can see that is values are higher, and, therefore, the estimation of parameters resulting form the assessment with this method is less reliable. Finally, the less representative indicator, the AABSE (3,92) shows that the estimation methods of Nelson and Siegel and of Svensson without weighting produce better results.

The parameters estimated by the method FBNSP are the following ones: β_0 indicates that the instantaneous forward interest rate tends to 5,52%; β_1 is negative (-2,53) and shows us that there is a increasing short term segment; β_2 is negative too (-2.23) which shows that the curve contains a through; τ_1 is a temporary constant which shows a change in trend, in this case at the end of the second year (1,81), and it must be positive to guarantee the long term convergence with β_0. From these estimates does $\beta_0+\beta_1$ derive, which can be interpreted as the shortest-term interest rate (2,99%). As far as validation statistics are concerned, the coefficient of determination reaches a value superior to the minimum one required by the literature (97,94), and reaches a good adjustment to the estimated function. With regards to the best value of the objective function (0,0022) and to RMSE (0,0037), it reaches better values than with methods without weighting/minimization, but anyway superior to that of FBSVP. Finally, the less representative indicator, the AABSE (6,33) allows us to deduce that its result is the worst one of the four methods.

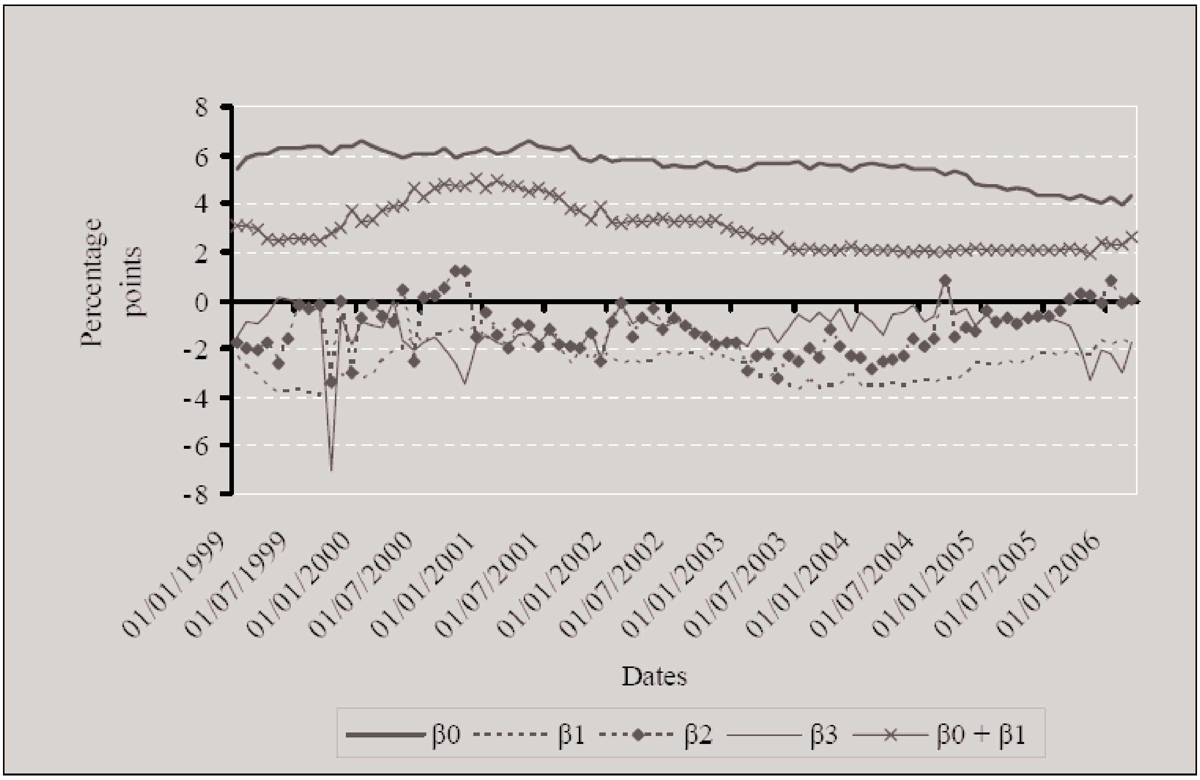
The parameters estimated by the method FBSV are the following ones: β_0 indicates that the instantaneous forward interest rate tends to 5,61%; the negative value of β_1 (-2,58) shows us a increasing short term segment; β_2 is negative too (-1.51), which shows a through in the curve; the change in trend in about two years (1,99) and the convergence with the long term segment provides us with the parameter τ_1; $\beta_0+\beta_1$ generates the shortest-term interest rate (3,01%). Svensson's methods introduce two new parameters: β_3 (-1,14) and τ_2 (1,71), which allow a second change in the estimated curve trend. As far as validation statistics are concerned, the coefficient of determination reaches a value superior to the minimum one required by the literature (99,65), and reaches a good adjustment to the estimated function. With regards to the best value of the objective function (0,0055) and to RMSE (0,0375), this method provides us with better results than Nelson and Siegel's ones. Finally, the less representative indicator, the AABSE (3,15) is better than in Nelson and Siegel's methods.

The parameters estimated by the method FBSVP are the following ones: β_0 indicates that the instantaneous forward interest rate tends to 5,59%; the negative value of β_1 (-2,58) reveals a increasing short term segment; β_2 is negative too (-1.51), which shows a through in the medium-term segment; the change in trend in two years the convergence with the long term segment provides us with the parameter τ_1. $\beta_0+\beta_1$ generates the shortest-term interest rate (2,98%);the parameters B_3 (-1,25) and τ_2 (1,99), allows us to introduce a greater flexibility in the estimated curve, creating thus a second change in trend. As far as validation statistics are concerned, the coefficient of determination reaches a value superior to the minimum one required by the literature (99,19) and reaches a good adjustment to the estimated function. With regards to the best value of the objective function (0,0003) and to RMSE (0,0023), they obtain better results than with the other methods. Finally, the less representative indicator, the AABSE is 4,46. We can state that Svensson's methods provides us with better results than Nelson's ones; nevertheless, it has the disadvantage of the calculation and interpretation of the two additional parameters.

All the estimated functions show similar curve trends since the signs estimated in the four methods are maintained. Nevertheless, the best reliability indicators are shown by the method FBSVP.

Fig. 1 and 2 show the parameters estimated with the FMSVP method. The advantage of this method is mainly the fact that, once the parameters have been estimated, it is possible to analyse the evolution of the interest rate for any maturity. In general, fit in long term is rather good, the function is asintotic and changes in curve shape and slope are basically caused in short and medium term. The parameter β_0, which provides the long term interest rate, is very stable. The variability logically appears in the short term ($\beta_0+\beta_1$); in addition, the spot and forward curves converge to parameter β_0. Estimated short term interest rate varies along the analised period between 5,41% and 4,35%.

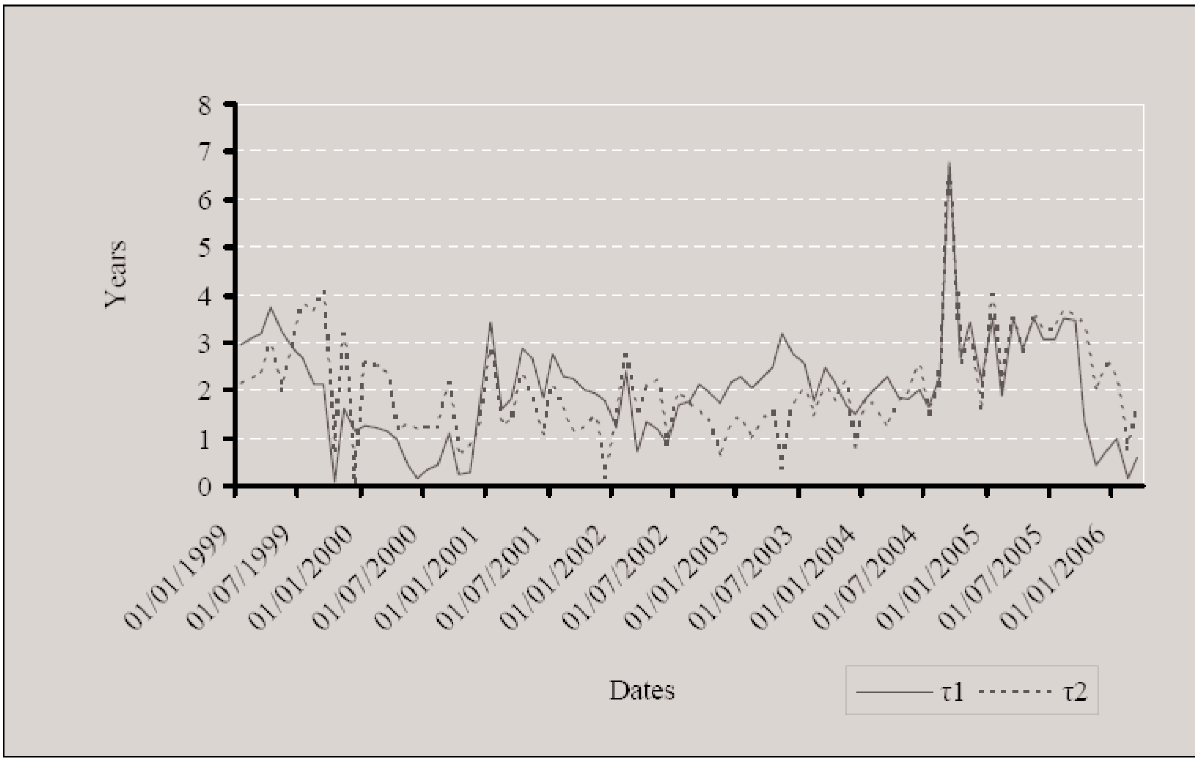
Figure 1. Estimated parameters (Betas)



Source: Personal research based on DataStream data

The parameter τ_1 can only take positive values in order to guarantee convergence to parameter β_0.

Figure 2. Estimated parameters (Taus)

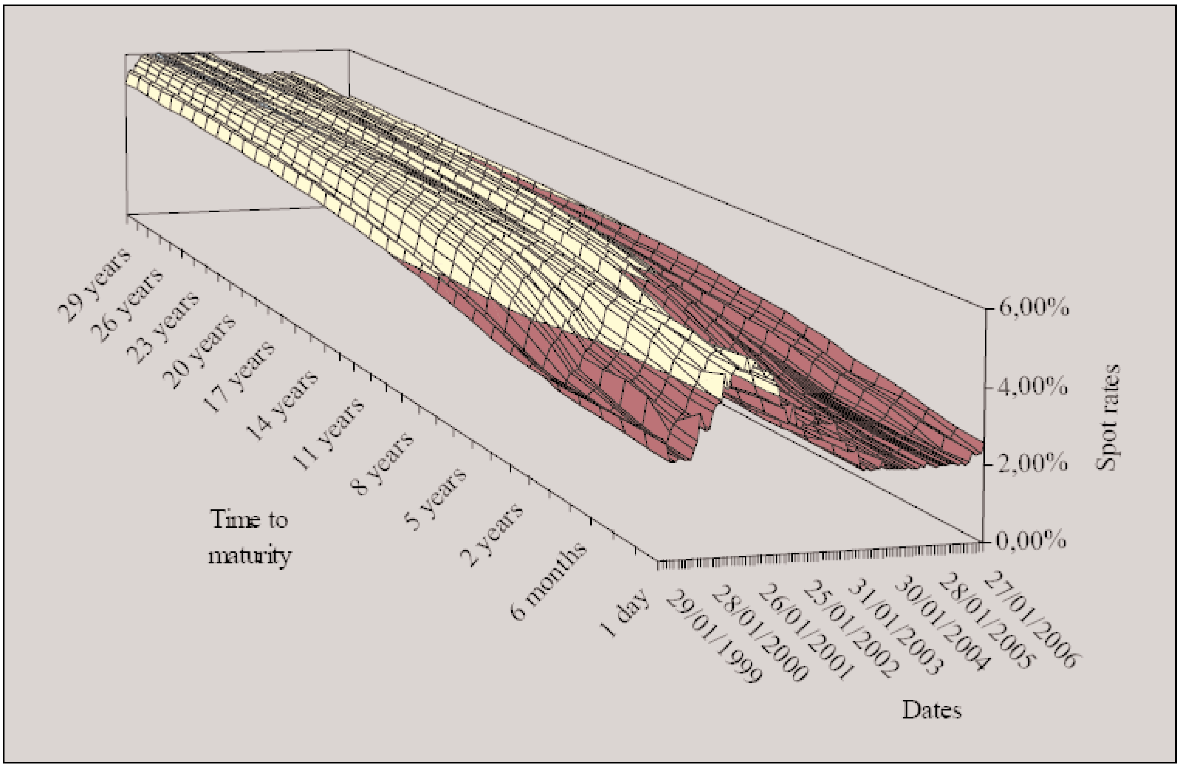


Source: Personal research based on DataStream data

The method chosen to estimate the curve of interest rates is FMSVP. It provides a parsimonious function, sufficiently flexible and which allows the representation of possible curve shapes. The representation of the whole curves of interest rates allows the establishment of a monthly series composed of 87 samples of the period being studied.

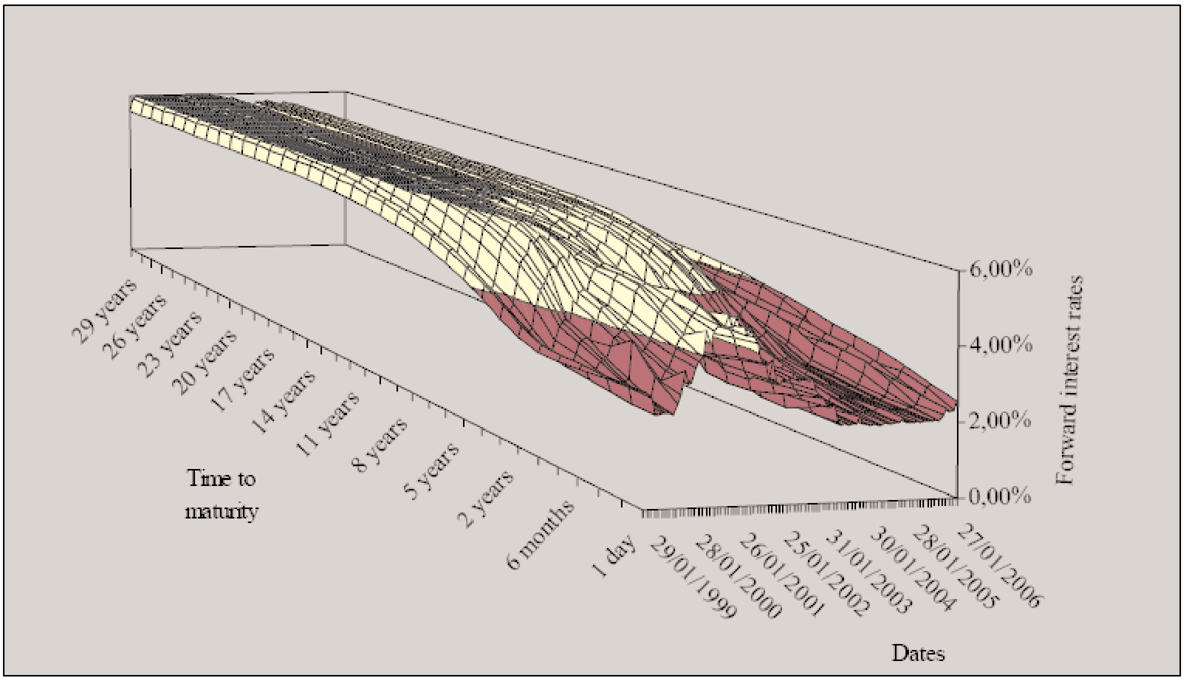
Finally, Fig. 3 and 4 show selected term structures of interest rates.

Figure 3: Estimated spot rates



Source: Personal research based on DataStream data

Figure 4: Estimated forward rates



Source: Personal research based on DataStream data

7. CONCLUSIONS

In this paper it is introduced an estimation of IRTS since the establishment of the European Economic and Monetary Union. With this purpose, it is applied the Smoothed Fama-Bliss method (1987), which aims to smooth out the spot rates obtained by applying bootstrapping method. To reach this aim, we apply approximating functions of Nelson and Siegel (1987) and Svensson and two objective functions, we compare results obtained with the four versions by using four statistical measures, and we conclude that all provide good fits. But the best version is the one which applies Svensson approximating function and uses the minimization of the sum of weighted yield square errors as an objective function the minimization of the sum of weighted yield square errors. We can state that Svensson's methods provides us with better results than Nelson's ones; nevertheless, it has the disadvantage of the calculation and interpretation of the two additional parameters.

Notes:

[1] Eonia (Euro Overnight Index Average) is the effective overnight reference rate for the euro. It is computed as a weighted average of all overnight unsecured lending transactions undertaken in the interbank market, initiated within the euro area by the contributing banks. It is reported on an actual/360 day count convention.

[2] Euribor (European Interbank Offered Rate) is the rate at which euro interbank term deposits are being offered within the EMU zone by one prime bank to another at 11.00 a.m. Brussels time (the best price between the best banks). It is quoted for spot value T+2 (two Target days) and on actual/360 day count convention.

[3] As the underlying variable rate instruments is by definition traded at par when the swap is entered into, the same must also apply to the underlying fixed-rate instrument. This implies that rates observed in the market are par yields, i.e. interest rate swaps are par instruments with zero net present value. Used data are middle rate, i. e., the arithmetical mean of the bid price and the offered price.

[4] International Swaps and Derivatives Association, Inc.

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