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NOTAS CRÍTICAS

Visualisation and Logic

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Euler-Diagramme: Zur Morphologie einer Repräsentationsform in der Logik, PETER BERNHARD, MENTIS, PADERBORN, 2001, pp. 162, 42.00 €.

The book being reviewed here combines two subjects which, for related reasons, are no longer viewed by the majority of logicians as being central to the logical enterprise: syllogisms and visualisation. Before commenting in detail on the contents of the book in any detail, it is appropriate to indicate why these subjects have lost their places in the logical and scientific canon.

Syllogistic logic concerns the validity of inferences from two premisses (called the major and minor premiss) to a conclusion. Both premisses and conclusions are of the forms "all As are B", "some As are B", "no As are B" and "some A is not B". Traditionally these forms are known by the abbreviations A, I, E, O respectively. To use examples from the book "all apple trees are fruit trees", "no fruit trees are conifers" therefore "no conifers are apple trees" is a valid syllogism, while "some broad leafed trees are not apple trees", "all apple trees are fruit trees", therefore "some fruit trees are not broad leafed trees" is invalid. In a syllogistic inference, a common term in both premisses is eliminated, e.g. "fruit tree" in the valid syllogism above. There are four ways in which this can happen depending on whether the eliminated term, known as the middle term, is subject or predicate in the premisses. These four combinations give rise to the four figures of the syllogism. For each figure some of the forms will be valid. For example in the fourth figure, where the middle term is a predicate of the major premiss and a subject of the minor premiss, AEE, EIO, IAI, AAI, AEO and EAO are traditionally taken to be generally valid syllogistic forms or *moods*. The syllogism "all apple trees are fruit trees", "no fruit trees are conifers" therefore "no conifers are apple trees" is a valid AEE syllogism of the fourth figure.

The origin of syllogistic logic was due to Aristotle in the *Prior Analytics*. The subject was developed throughout the middle ages and was commonly taught in universities until the early 20th century. Bernhard notes that the research programme into syllogistic logic continues to the present day, citing the *New Syllogistic* of Fred Sommers and George Englebretsen (p. 14) [New York, Peter Lang, 1987].

The reason why syllogistic logic is generally no longer taught at universities is that the symbolic logic developed by Frege, Hilbert, Peano and Russell amongst others has been seen to have a far broader scope than syllogistic logic (dealing with complex relations such as "for all B and C there is an A such that A is between B and C" as well as simple relations between A and B) and to be applicable to mathematics and science as well as arguments in general discourse. In fact, syllogisms can be represented in monadic first-order predicate logic, i.e. in predicate logic with only one place predicates. For example, "no fruit trees are conifers" can be represented symbolically by $(\neg\exists x)(Fx \,\&\, Cx)$, where Fx represents "x is a fruit tree" and Cx represents "x is a conifer": in words "there is no x such that x is a fruit tree and x is a conifer".

The power of modern logic has led to syllogistic logic being replaced by symbolic logic in most logic courses, although those courses that are historically aware may introduce the concepts of the syllogistic logic by way of background. Since its beginnings symbolic logic has become an extremely well researched area with a large number of well known results. The true sentences of first order predicate logic, for example, can be completely described by a finite set of axiom schemas and a finite set of deductive rules, while there is no recursive function that provides a generally applicable decision process for the truth of sentences of first order predicate logic [see, e.g., David Hilbert & Willhelm Ackermann *Principles of Mathematics*, New York, Chelsea, 1950; first published as *Grundzüge der theoretischen Logik*, Berlin, Springer, 1938]. (Coincidentally, Hilbert and Ackermann's book also contains a proof of the validity of 15 of the traditional 19 distinct moods of syllogism.)

Whilst the power and scope of symbolic logic is hard to deny, one might think that symbolic logic is too powerful for deciding the validity of arguments in general discourse. The appropriateness of applying symbolic reasoning to argument in natural languages is also open to question given the focus of the development of symbolic logic was initially on mathematics and later on science [see Bernhard p. 14 and for the use of symbolic logic as a foundation of science see, e.g., Rudolf Carnap *The Logical Structure of the World*, Chicago, Open Court; originally published as *Der Logische Aufbau der Welt*, Berlin, 1928]. A related concern is that the rules of logic are not rich enough to express the complex inferential patterns in natural language. Usage of natural language often has inferential rules for handling uncertainty and expectation, necessity and contingency, beliefs and desires, modes of address (such as imperatives) and counterfactual conditionals that are not obviously expressible in symbolic terms. It has been a major research programme in analytic philosophy and artificial intelligence to try to capture the usage of natural language in symbolic terms. This research programme has generated a great deal of important results, but a serious charge against it is that in some cases (e. g. modal logic) the models of the formal systems used to provide semantical frameworks for those systems have been more philosophically questionable than the inferential rules that the systems are formalising [see, e. g., David Lewis's views on modal realism in *On the Plurality of Worlds*, Oxford, Blackwell, 1986]. With these concerns in mind, there may be room for a study of syllogistic logic. How far this can be achieved will be discussed later in this review.

Visualisation had for a long time also been neglected as a technique in symbolic logic and consequently overlooked as a subject for debate in the philosophy of logic. Until recently the main techniques in logic were algebraic and set-theoretical: the nearest that modern logic came to using visual techniques was in the treatment of proofs as structures (graphs/tableaux of) syntactic objects constructed according to rules of inference and subject to transformations while preserving the validity of the conclusion. The most well known examples relate to the work of Gentzen in developing sequent and natural deduction formulations of logic systems [see "Investigations into logical deduction" in *The Collected Papers of Gerhard Gentzen* ed. M. E. Szabo, Amsterdam, North-Holland, 1969, pp. 68-131] and the work of Beth in developing the tableaux formulation [see, e. g., the development of first-order predicate logic in John Bell and Moshé Machover *A Course in Mathematical Logic*, Amsterdam, North-Holland, 1977]. However, an original aim of proof theory as a subject was to make use of intuitive, visualisable methods in studying formal axiomatic systems. Both of Gentzen's proofs of the consistency of first order arithmetic [translated by M.E. Szabo as "The consistency of elementary number theory" and "New version of the consistency proof for elementary number theory" in *The Collected Papers of Gerhard Gentzen*] are examples of this, as they can be visualised by means of explicit operations on tree structures (although the visualisability of mathematical induction to the first fixed-point ordinal, ε_0, used to establish the consistency of number theory has been the subject of much debate ever since).

As Bernhard notes [p. 12], the recent work of Jon Barwise and John Etchmendy has helped to stimulate interest in the value of visual reasoning [see, eg, *Logical Reasoning with Diagrams*, New York, Oxford University Press, 1996]. Based on experience of producing computer packages to help students learn logic, their view was that visual reasoning using diagrams and graphical representations can provide a richer and more natural way to reason in certain situations because the amount of information carried by a diagram may be greater than the amount carried by a sentence or a collection of sentences. As they say in ["Computers, Visualization and the Nature of Reasoning" in *The Digital Phoenix: How Computers are Changing Philosophy*, T. W. Bynum and James H. Moor (eds.), London, Blackwell, 1998, pp. 93-116]:

> Diagrams, like sentences, carry information: they carve up the same space of possibilities, though perhaps in very different ways. A good diagram, for example, may represent information in a form that is particularly appropriate for the subject matter at hand, one that allows you to visualize and manipulate the information more readily than would a collection of sentences or even a different sort of diagram.

The antipathy to the use of visualisation derived in part from the trend that originated in nineteenth century mathematical thought to remove appeals to "intuition" from mathematical proof and thence to rigorise mathematics, that subsequently became a norm in the teaching of mathematics generally and mathematical logic in particular. For more details of this trend see, for example, the chapter on geometry in Friedman's [*Kant and the Exact Sciences*, Cambridge Massachusetts, Harvard University Press, 1992]. Another, related reason is that historically images were used in mathematics predominantly in Euclidean geometry: the discovery of other geometries and broader classes of spatial forms (such as projectivities and topologies) led to a distrust of spatial reasoning in general. In mathematics it is not too much of an exaggeration to say that formalisation took the place of visualisation from the early 20th Century onwards.

The formalisation of Euclidean geometry as an axiomatic formal system was accomplished by David Hilbert in 1899 [see *Foundations of Geometry,* Chicago, Open Court, 1971; originally published as *Grundlagen der Geometrie*, Leipzig, Teubner, 1899], which may be supposed to have removed the need for intuitions in Euclidean geometry. However, much later in his 1932 preface to *Geometry and the Imagination* [trans. P. Nemenyi, New York, Chelsea, p. iii; originally published as *Anschauliche Geometrie*, Berlin, Springer, 1932] Hilbert says:

> As to geometry, in particular, the abstract tendency has here led to the magnificent systematic theories of Algebraic Geometry, of Riemannian Geometry, and of Topology; these theories make extensive use of abstract reasoning and symbolic calculation in the sense of algebra. Notwithstanding this, it is still as true today as it ever was that *intuitive* understanding plays a major role in geometry. […] With the aid of visual imagination we can illuminate the manifold facts and problems of geometry, and beyond this, it is possible in many cases to depict the geometric outline of the methods of investigation and proof without necessarily entering into the details connected with the strict definitions of concepts and with the actual calculations.

In this quotation Hilbert captures one essential advantage of visual reasoning that also seems to underlie Barwise and Etchmendy's view: whether it is in geometry or in logic, visual reasoning can provide the realisation of a truth in a speedy and concise way, reducing the amount of symbolic manipulation. But visualisation is not only a way to shorten the lengths of proofs; it may lead to knowledge of new truths that may not be captured by a formal system if the subject matter has not been completely formalised or if the truth were infeasible to deduce from the axioms.

Of course the value of visual reasoning depends on whether we can trust visualisation to convey information faithfully. After all it is arguable that human representation of objects is in terms of conceptual frameworks (such as geometries and scientific theories) that enable us to communicate sensibly about the things we see. A foundationalist response to this problem is to give a general characterisation of the concept of space and time in general [see Kant's metaphysical expositions of space and time in Section I of the "Transcendental Aesthetic" of the *Critique of Pure Reason*, A23/B37] and show that a particular conceptual framework is compatible with that characterisation. Paul Lorenzen has done this for Euclidean geometry by using the logical concept of homogeneity (i. e. object X is homogeneous with respect to object Y if every part of X has the same properties with respect to Y) [see "The Foundation Problem of Geometry" in *Constructive Philosophy*, Amherst, The University of Massachusetts Press, 1987, pp. 257-273]. The compatibility of other geometries can be shown by giving relative consistency proofs or by arguing the case for compatibility directly. Those who see no value in foundations can ignore this line of argument, but it does lend plausibility to the inherent value of visual information.

Based on the foregoing, it seems reasonable to agree with Bernhard that visualisation ought not to be a poor relative of other methods in logic, but constitutes a powerful tool for effective understanding complex concepts, methods and proofs.

A case in point is the subject of the book reviewed, viz Euler diagrams. Euler diagrams exploit a close analogy between predication, set membership (an object satifisfying a predicate being a member of the extension of that predicate) and of one space form being part of another.

The idea behind Euler diagrams is to represent propositions in subject predicate form as relationship between the sets of things that satisfy the subject and the set of things that satisfy the predicate. Thus "all apples trees are fruit trees" could be represented by a disc standing for the set of all fruit trees with a smaller disc wholly inside it standing for the set of all apple trees. Similarly, "some apple trees are fruit trees" and "some apple trees are not fruit trees" can be represented by two partially intersecting discs, "no apple trees are fruit trees" by two non-intersecting discs. These diagrams may be used to verify syllogisms and to suggest counterexamples. For example, "all apple trees are fruit trees", "no fruit trees are conifers" may be represented by three discs, one for fruit trees containing another for apple trees and an entirely separate one for conifers. It can be seen that "no apple trees are conifers". As Bernhard amply illustrates [s2.2, pp. 22-38], Euler's method is much simpler than the various medieval rules and dicta used to indicate the truth of a syllogism.

A derivation of a syllogism in the first-order predicate calculus (taken here in linear form) can also be complicated by comparison to Euler's method. Take the derivations of the AEE syllogism of the fourth figure. This has premisses $(\forall x)(Px \rightarrow Mx)$ and $(\forall x)(Qx \rightarrow \neg Mx)$. By universal instantiation, $Px \rightarrow Mx$ and $Qx \rightarrow \neg Mx$. By contraposition and double negation elimination, $Mx \rightarrow \neg Qx$. Consider the general schema, $(B \rightarrow C) \rightarrow (A \rightarrow (B \rightarrow C))$, which is a logical axiom. Then if $A \rightarrow B$ and $B \rightarrow C$ are axioms, $A \rightarrow (B \rightarrow C)$ by modus ponens, $(A \rightarrow B) \rightarrow (A \rightarrow C)$ by virtue of another standard logical axiom, and hence $A \rightarrow C$ by modus ponens. Substituting Px for A, Mx for B and $\neg Qx$ for C, we see that $Px \rightarrow \neg Qx$ follows from $Px \rightarrow Mx$ and $Mx \rightarrow \neg Qx$. But by contraposition and double negation elimination, $Qx \rightarrow \neg Px$ follows. By universal generalisation, $(\forall x)(Qx \rightarrow \neg Px)$, which establishes the validity of AEE in the fourth figure. However if double negation elimination is not taken as an inference rule (and it is usually not in linear formulations of first-order logic) but $(\neg B \rightarrow \neg C) \rightarrow (C \rightarrow B)$ is, then take $\neg\neg A \rightarrow (\neg\neg\neg\neg A \rightarrow \neg\neg A)$ as an axiom. $(\neg\neg\neg\neg A \rightarrow \neg\neg A) \rightarrow (\neg\neg A \rightarrow A)$ follows by two applications of $(\neg B \rightarrow \neg C) \rightarrow (C \rightarrow B)$ with $B=\neg\neg\neg A$ and $C=\neg A$ and then $B=A$ and $C=\neg\neg A$. Hence $\neg\neg A \rightarrow (\neg\neg A \rightarrow A)$ by modus ponens, and $(\neg\neg A \rightarrow \neg\neg A) \rightarrow (\neg\neg A \rightarrow A)$ by distribution. To establish $\neg\neg A \rightarrow \neg\neg A$ we can use $(B \rightarrow (C \rightarrow B)) \rightarrow ((B \rightarrow C) \rightarrow (B \rightarrow B))$, and deduce $(B \rightarrow C) \rightarrow (B \rightarrow B)$ as $B \rightarrow (C \rightarrow B)$ is an axiom and then substitute $C \rightarrow B$ for C, to obtain $B \rightarrow B$ by modus ponens, and finally substitute $\neg\neg A$ for B. The double elimination inference, $\neg\neg A \rightarrow A$, then follows by modus ponens.

Although one may argue that the general inference rules used in the above proof need only be derived once, derivations in first-order predicate logic can be complicated and are not especially intuitive (generally being much easier to work out if you start at the conclusion and work backwards), even if they do possess a kind of beauty. Some of this awkwardness is removed in natural deduction and tableau proofs, but there is still a sense in which the axiomatic system does not help the theorem prover. A standard proof by Euler diagrams by contrast requires the consideration of various configurations of the diagrams (to exhaust the logical possibilities), but it is extremely intuitive.

Bernhard identifies three stages a proof of the truth of a syllogism by Euler diagrams [p. 46]: construction of a diagram representing the first premiss, integration of discs that represent both premisses, and inspection of the diagram to derive the conclusion. One theme of the book is the diversity of methods and views on determining the validity of syllogisms and which syllogistic forms are valid. Chapter 4 of the book [pp. 55-82] is devoted to different visualisations of Euler diagrams in particular comparing the intensional and extensional interpretations of syllogism, the extensional view being the one developed in the book. Chapter 5 [pp. 83-95] is devoted to different views on the number and nature of valid syllogisms. The controversy surrounds the extent to which the existence of subjects and predicates is assumed. The traditional view is that the domains of subjects and predicates are not empty. This gives rise to 19 distinct moods. The modern view is that only the subject domain needs to be non-empty and then only when existential claims are made (i. e. in the I and O forms). On this basis there are 15 valid moods. Consider AAI in the fourth figure, i. e. "all Ps are M", "all Ms are S", therefore "some Ss are P". In modern logic, both premisses can be true if there are no Ps and no Ms and there are Ss. But then no Ps are Ss, invalidating the syllogism. However if there are Ps and Ms by assumption, the truth of the syllogism follows. Bernhard provides a nice illustration of the intermediate views of William of Ockham and John Stuart Mill, who dropped the assumption that the predicate domain is empty in the general case, although they had different views of the E and O cases, Ockham believing that negative judgements do not require the existence of the subject.

Bernhard then considers two different ways of determining the truth of syllogisms, which he calls the segment semantics and the surface semantics. A segment semantics attempts to represent a single syllogistic form, A, I, E or O, by a single Euler diagram, while the surface semantics uses multiple Euler diagrams that represent distinct logical possibilities. Bernhard draws out a number of historical attempts to produce complete segment semantics, from Euler through William Thompson and Charles Peirce to John Neville Keynes. Bernhard notes that the approach of Richard Purtill [see *The Cambridge Dictionary of Philosophy*, Cambridge, Cambridge University Press, 1995, pp. 251-252] is the best currently available that does not use additional symbols to indicate existence assumptions. However even this approach is not completely satisfactory as it requires an additional rule to be added (p. 102). If additional symbols are allowed, Bernhard notes that the approach taken by Keith Stenning provides a complete semantics if no existence assumptions are made for the predicate domain [p. 103]. Surface semantics is less problematic because all logical possibilities are considered separately, but this can be time consuming because of the number of combinations involved. Bernhard suggests a falsification method, where the integration of the premisses with the contradictory of the conclusion is attempted [pp. 113-114]. If integration is possible the syllogism is not logically valid; otherwise the syllogism is valid. The falsification method can be more efficient than verifying all the combinations and can be used as a general method of determining the truth of syllogisms, although the reviewer found the flow diagrams that Bernhard used to express the falsification process [pp. 118-121] difficult to follow.

Although it is an efficient diagrammatic method, the falsification method is based on a prior logical principle, namely *reductio ad absurdum*, since if the proposition $A\wedge B\wedge\neg C$ is not possible, i. e. is logically false, then $\neg A\vee\neg B\vee C$ is a logical truth, i. e. $(A\wedge B)\rightarrow C$ is true (with the material reading of $P\rightarrow Q$ as $\neg P\vee Q$). For the falsification method to be a truly diagrammatic method, we would need a diagrammatic derivation of the *reductio ad absurdum* rule. We can use Venn diagrams as a model, interpreting $<\vee,\wedge,\rightarrow,\neg>$ by the set–theoretic operations $<\cup,\cap,\subseteq,->$ where set complement is written as a minus sign and, as you would expect, $x\subseteq y$ if and only if $-x\cup y$ covers the plane. For each proposition P, taking P to be P'c for constant c, $(\exists x)(P'x)$ or $(\forall x)(P'x)$, P can be represented by the set of objects for which P is true, written $\{x\}(P'x)$, and usually called the *extension* of P. Then we can verify the axioms of propositional logic using the interpretation that a logical truth corresponds to coverage of the plane for an arbitrary non-empty set of objects. For example, $(\neg A\rightarrow\neg B)\rightarrow(B\rightarrow A)$ becomes $(-\{x\}A'(x)\cap\{x\}B'(x))\cup(-\{x\}B'(x)\cup\{x\}A'(x))$, which covers the whole plane. Modus ponens can be written $A\rightarrow((A\rightarrow B)\rightarrow B)$ and shown to be true as a Venn diagram. Given that propositional logic is complete (deduces all propositional truths), Venn diagrams can represent all propositional truths. Moreover all Venn diagrams can be represented as propositional formulae remembering that the empty set is equivalent to the false (proposition), e. g. $x\cap y=\emptyset$ maps to $\neg(X\wedge Y)$, $x\cap y\neq\emptyset$ to $X\wedge Y$. It follows that the set of propositional truths deducible by Venn diagrams and propositional logic is the same. Then, to return to *reductio ad absurdum*, if $\neg(A\wedge B\wedge\neg C)$ is logically true, $-(\{x\}(A'x)\cap\{x\}(B'x)\cap-\{x\}(C'x))$ covers the plane, hence $-\{x\}(A'x)\cup-\{x\}(B'x)\cup\{x\}C'(x)$ covers the plane, which is true if $\{x\}(C'x)$ covers the intersection of $\{x\}(A'x)$ and $\{x\}(B'x)$, ie $\{x\}(A'x)\cap\{x\}(B'x)\subseteq\{x\}(C'x)$, which also deals with the case where $\{x\}(A'x)$ does not intersect $\{x\}(B'x)$. Hence $(A\wedge B)\rightarrow C$ is true.

Bernhard then moves on to consider how syllogistic logic can cope with non-categorical syllogisms. He considers singular syllogisms such as "Voltaire is a philosopher; Voltaire is a poet; therefore there is a poet who is also a philosopher", the so-called indirect moods, where the order of the subject and predicate in the conclusion is reversed, and contraposition inferences where subject and predicate are replaced with their negatives and swapped. Singular propositions are best treated as a type of universal propositions, ie "Voltaire is a poet" would be treated as "for all x, if x is Voltaire, then x is a poet", although this is not ideal as a claim about Voltaire does not seem like a universal claim. Indirect moods can also be accommodated by the standard verification method or by the falsification method for Euler diagrams although general symmetry rules for E and I can be used with direct moods. Contraposition inferences require the complement of a subject or predicate's extension to be carefully distinguished. For this he uses boxes that fit the entire plane to indicate that subject or predicate extensions are exactly complementary or overlap. Bernhard then considers the a metric for information content of syllogisms suggested by Rudolph Carnap, viz. the ratio of false cases over total number of cases [pp. 137-139]. Before his final summary, Bernhard considers extensions to Euler diagrams that have more expressive power than standard Euler diagrams, such as being able to express "each S is M or P" in a single diagram, which can be handled by using boxes rather than discs.

The content of this book forms a very interesting survey, Bernhard having demonstrated a good understanding of the history of syllogistic logic. There are however three limitations of the work that the author should consider in future revisions to this work. The first is a detail, viz that various types of non-categorical syllogism are not mentioned in the work. Standard works on syllogisms distinguish hypothetical and disjunctive syllogisms such as modus ponens, modus tollens and argument by dilemma. Although these forms have long been superseded by propositional logic, there existence should perhaps have been acknowledged. The second limitation is fairly minor again, but it does point to a problem with diagrammatic methods. Diagrammatic methods find it difficult to handle empty sets (see for example Bernhard p. 131). Diagrams have to resort to symbolic representations of emptiness because diagrams can only represent something rather than nothing. This failure shows that there are elements of human reasoning which cannot be adequately expressed through diagrams alone.

The third limitation is more fundamental. The book is a case study of visual methods applied to syllogistic logic. But there did not seem to be a strong sense of why visual methods are legitimate and how far the work done on syllogistic logic relates to modern logic. Ideally there should be more discussion in the book on the role of formal analogy in logic and an indication of what can be carried over from monadic predicate logic to modern logic. Although analogies in the sense of structural embeddings or equivalences have placed a significant part in modern logic and mathematics, and there are many results on classifications of common structures such as groups and topological manifolds, the book does not aim to explain the relationship of the diagrammatic theory of parts through the possible mediation of naïve set theory to the theory of syllogisms. In a sense it would suffice to give a simple structure preserving mapping (e. g. $<\vee,\wedge,\rightarrow,\neg>$ to $<\cup,\cap,\subseteq,->$ as provided above in the case of propositional logic), but a treatment of what the structural equivalence amounts to in the sense of satisfying the axioms of a Boolean algebra would also be useful. As far as what could be carried over to modern logic, a discussion of visualisation techniques in more general Boolean algebra would be interesting as would a survey of visual techniques in modern predicate logic (some views on which have been sketched in the course of this review) with emphasis on those that can be carried across from syllogistic logic. In conclusion, Bernhard's book does provide a good deal of scholarship on Euler diagrams and motivation for further research into visual methods, but does not tackle some of the broader issues alluded to above.

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