

# Pollution Standards, Technology Investment and Fines for Non-Compliance\*

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## Abstract

Is it socially desirable that fines for exceeding pollution standards depend on the degree of non-compliance and also on the firm's level of investment in environmentally friendly technologies? We consider a representative firm that chooses the pollution level and the investment effort in response to a pollution standard, an inspection probability and a fine for non-compliance. The fine should not depend on the firm's investment effort if the optimal policy induces compliance. However, the fine should strictly decrease with investment effort under non-compliance. The optimal fine considers the relative importance of monitoring and sanctioning costs in the enforcement problem.

**JEL classification:** K32, K42, L51, Q28.

**Key words:** pollution standards, costly inspections, environmentally friendly technologies, non-compliance, optimal fines.

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# 1 Introduction

The structure of the fines for non-compliance with environmental rules has been extensively studied by economists in the enforcement literature.<sup>1</sup> A relevant research topic within this field is to question if regulators should impose maximum fines as a means of inducing compliance with the lowest possible monitoring efforts (Becker (1968)), or if fines should be kept low. In fact, an extensive number of papers suggest that optimum fines should not be maximum. Reasons that support this view include the possibility of targeting enforcement in dynamic settings (Harrington (1988), Raymond (1999) or Friesen (2003)), self-reporting of emission levels (Livernois and McKenna (1999)), penalty evasion (Kambhu (1989)), possible inverse relationships between fines and probabilities of conviction (Andreoni (1991)), hierarchical governments (Saha and Poole (2000), Decker (2007)), or others. Somehow, the final purpose of all these studies is to explain the stylized fact that in practise fines are relatively low and nevertheless compliance rates are still high, or at least, higher than predicted by the theory.<sup>2</sup>

An interesting feature of some nowadays environmental regulations is that they explicitly include the possibility that fines for non-compliance can be reduced if polluting agents have shown documented evidence of compliance-promoting activities. For example, in the Spanish legislation on hazardous waste (see Law 5/03 on Residuals and Law 10/93 on Liquid Industrial Waste of the Autonomous Community of Madrid), monetary sanctions depend on the degree of non-compliance with the required standards as well as on the efforts of firms to minimize the social pollution effects of their infractions. As a consequence, firms that invest in clean production processes associated with responsible water consumption are rarely inspected and, if inspected, they are rarely punished if found out of compliance. Another example can be found in the EPA's Audit Policy, where fines for non-compliance can be reduced up to 100% of the non-gravity-based part and up to 75% of the gravity-based component if firms promptly disclose

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<sup>1</sup>See the literature reviews by Polinsky and Shavell (2000) on public enforcement of the law, or by Heyes (2000) and Cohen (1999) within the environmental context.

<sup>2</sup>See Harrington (1988) and Livernois and McKenna (1999) for empirical evidence in the United States and Canada, respectively.

and correct any discovered violations or if they install enhanced emission control devices that simplify regulators' monitoring processes.

Penalty reductions in exchange for investment efforts by polluting firms are, therefore, possible in practise. However, an interesting (but not yet formulated) research question has to do with their convenience from an optimal perspective. The purpose of this paper is then to analyze under what conditions fines should be reduced in relation to the firms' investment effort in environmentally friendly technologies. This paper fits within the strand of the economics literature that defends the implementation of non-maximal fines. However, we are not aware of any paper that deals with the optimality of incorporating penalty discounts as a means of inducing investment efforts by the firms.<sup>3</sup>

We consider a simple game between a regulator and a polluting firm. We adopt a principal-agent setting where the regulator (principal) sets the environmental standard (or pollution limit), the inspection probability and the fine for non-compliance with the standard. Then, the firm (agent) reacts to this policy by selecting the pollution level and the investment effort in clean technology. When choosing the pollution level, the firm automatically selects the degree of non-compliance (if any) with the standard. The fine depends on the degree of violation and can also be contingent on the firm's investment effort. When designing the policy, the regulator considers the firm's compliance and investment costs, the pollution damages and the enforcement costs (that is, the monitoring and the sanctioning costs).

We find an intuitive necessary and sufficient condition that determines when it is optimal to set an environmental policy that induces compliance. This condition then characterizes the social preference for compliance and it balances sanctioning and monitoring costs. Obviously, the larger the sanctioning costs relative to monitoring costs, the more likely the optimal policy induces compliance. Under compliance, the optimal fine should not be reduced. The reason is

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<sup>3</sup>Arguedas and Hamoudi (2004) consider fines dependent on both the degree of violation and the environmental technology that the firm employs. However, in that setting fines are given, i.e., they are not a regulator's choice variable. Also, the possibility that fines depend on the firms' investment efforts is present in Arguedas (2005) but, in there, regulators and firms negotiate over the stringency of the fines and the possibility of clean technology investment, as opposed to the principal-agent framework adopted here.

that, in this case, there are no sanctioning costs. Therefore, the only way to save on monitoring costs is by setting fines as large as possible. However, under non-compliance, the optimal fine must reflect the balance between sanctioning and monitoring costs. Therefore, the larger the sanctioning costs relative to the monitoring costs, the smaller the fine.

This paper is similar in spirit to those of Stranlund (2007) or Arguedas (2008), in the sense of finding the optimal regulatory policy among the full rank of policies, that is, those that induce compliance and also those that induce non-compliance with the regulation. In these papers, there is also a key condition that determines the desirability of compliance versus non-compliance, although there the firm is not allowed to choose a level of investment effort.

The remainder of the paper is organized as follows. In the next section, we present the model. In Section 3, we study the optimal behavior of the firm. In Section 4, we find the optimal standard and inspection probability for given fines. In Section 5, we obtain the optimal fine. We conclude in Section 6. All the proofs are in the Appendix.

## 2 The Model

We consider a single firm that generates pollution as a by-product of its production activity. The firm can abate pollution at a cost, which depends on the pollution level  $e \in [0, e^0]$ , and on the firm's investment effort (or investment cost)  $z \geq 0$ . The abatement cost function is denoted as  $c(e, z)$ . We assume that abatement costs are fully convex in  $(e, z)$ , with the usual assumptions  $c_e(e, z) < 0$ ,  $c_z(e, z) < 0$  and  $c_{ez}(e, z) > 0$ , where subscripts denote partial derivatives.<sup>4</sup> For given  $(e, z)$ , the total costs of pollution abatement and technology investment are  $c(e, z) + z$ .

Pollution generates environmental damages measured by the function  $d(e)$ , such that  $d'(e) > 0$  and  $d''(e) \geq 0$ .

We assume that there exists a regulator concerned about environmental damages, who sets a pollution standard  $s \geq 0$ . This means that the firm is entitled to pollute  $e \leq s$ . Pollution is not observable without costs, and we assume that the regulator monitors with probability  $p \in [0, 1]$

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<sup>4</sup>For functions of one variable, we instead use the prime notation to denote full derivatives.

whether the firm actually complies with the standard. The cost per inspection is  $m > 0$ . If the firm is inspected and discovered to be exceeding the standard, then it is penalized with an amount  $f > 0$  per unit of pollution in excess of the standard.<sup>5</sup> The fine can be either a fixed quantity or it can negatively depend on the amount of the technology investment. Formally, we denote the fine as  $f(z)$ , such that  $f'(z) \leq 0$ . We also assume that sanctioning is socially costly, and  $t > 0$  represents the per-unit social cost of the sanction.<sup>6</sup>

Given the policy parameters  $(s, p, f)$ , the risk-neutral firm's expected costs of polluting the amount  $e \geq s$  and investing the quantity  $z$  are the following:

$$c(e, z) + z + pf(z)(e - s). \quad (1)$$

These costs include pollution abatement costs, technology investment effort and expected penalties for non-compliance. Obviously, the last term is 0 in case the firm chooses to comply, i.e., when  $e \leq s$ .

The regulator minimizes the firm's abatement and investment costs, environmental damages and also monitoring and sanctioning costs. Formally, the regulator's objective function is the following:

$$c(e, z) + z + d(e) + p[m + tf(z)(e - s)],$$

where  $(e, z)$  constitute the firm's optimal response to the regulatory policy.

Therefore, the timing we consider is the following. In the first step, the regulator simultaneously chooses the policy parameters (basically,  $s$  and  $p$  in the whole paper; also  $f$  in Section 5). In the second step, the firm simultaneously chooses  $e$  and  $z$  in response to the policy.<sup>7</sup> We

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<sup>5</sup>We consider fines linear in the degree of violation for two main reasons. First, because they are normally used in practise and also well understood by polluting agents. Second, because some recent theoretical works have shown their cost-effectiveness. The latter means that linear fines can achieve the same pollution target with lower social costs than strictly convex fines, see for example Stranlund (2007) and Arguedas (2008).

<sup>6</sup>We model sanctioning costs in the same way as in Stranlund (2007) or Arguedas (2008). Sanctioning costs may increase with the level of the fines since individuals can strongly resist to the imposition of larger fines (for example, by concealing assets), see for example Polinsky and Shavell (1992).

<sup>7</sup>In the terminology of Maxwell and Decker (2006), this type of regulation is *unresponsive*, in the sense that the regulator does not subsequently respond to the firm's investment decision. Here, the motivation for the firm to invest is given by the fact that penalties for non-compliance may depend on the investment effort.

solve the problem backwards to obtain the sub-game perfect equilibrium. In the next section, we study the optimal behavior of the firm.

### 3 Firm Behavior

Given the policy  $(s, p, f)$ , the firm solves the following problem:

$$\text{Min}_{e,z} c(e, z) + z + pf(z)(e - s). \quad (2)$$

The solution is presented next.

**Lemma 1** *Given  $(s, p, f)$ , the firm's optimal pollution and investment levels  $(e^*, z^*)$  are given by the conditions:*

$$c_e(e^*, z^*) + pf(z^*) \geq 0; \quad e^* \geq s;$$

$$[c_e(e^*, z^*) + pf(z^*)][e^* - s] = 0;$$

$$c_z(e^*, z^*) + 1 + pf'(z^*)(e^* - s) = 0.$$

The firm simultaneously chooses the pollution level  $e$  and the investment cost  $z$ . For a given  $z$ , the firm complies with the standard ( $e = s$ ) as long as the additional expected penalty of infinitesimally exceeding the standard is larger than the corresponding additional abatement cost savings, that is, when  $pf(z) \geq -c_e(s, z)$ . The investment effort selected under compliance is such that marginal abatement cost savings and marginal costs are equal, i.e.,  $-c_z(s, z) = 1$ . Thus, if  $pf(z) \geq -c_e(s, z)$  at this particular investment effort, then the firm optimally decides to comply. Otherwise, the firm violates the standard ( $e^* > s$ ), and the optimal pollution level is such that marginal abatement costs and marginal expected penalty savings are equal, i.e.,  $c_e(e^*, z^*) + pf(z^*) = 0$ . Now, the optimal investment effort is such that  $c_z(e^*, z^*) + 1 + pf'(z^*)(e^* - s) = 0$ , where the last term  $pf'(z^*)(e^* - s)$  is the marginal expected penalty at the effort level  $z^*$ .

For a given standard  $s$ , we can define the minimum inspection probability which induces compliance. This threshold probability is the one that makes the firm indifferent between compliance and non-compliance. Technically, this probability is obtained when the first condition

in lemma 1 is satisfied with strict equality for  $e^* = s$ . In fact, as will become clear in the following section, the regulator has no reason to use more monitoring resources than those strictly required if the goal were compliance. Therefore, from now on, the set of policies  $(s, p)$  that will be considered for the analysis are such that:

$$c_e(e^*, z^*) + pf(z^*) = 0; \quad (3)$$

$$c_z(e^*, z^*) + 1 + pf'(z^*)(e^* - s) = 0. \quad (4)$$

The main novelty of our study is that fines depend not only on the degree of non-compliance, but on the investment effort  $z$  also.<sup>8</sup> This implies that the relationship between  $e$  and  $z$  given by the firm's optimal decision  $c_e(e, z) + pf(z) = 0$  could be either negative or positive, as can be shown by differentiating this expression with respect to  $e$  and  $z$ :

$$\frac{\partial e}{\partial z} = -\frac{c_{ez} + pf'}{c_{ee}}. \quad (5)$$

The sign of the term  $c_{ez} + pf'$  crucially determines whether the relationship between  $e$  and  $z$  is positive or negative. Obviously,  $\frac{\partial e}{\partial z} < 0$  as long as  $f' = 0$ . In this case, the larger the investment effort, the lower the pollution level. However, things may change if fines depend negatively on  $z$ . In particular,  $\frac{\partial e}{\partial z} > 0$  as long as  $pf' < 0$ .

To be more precise about these impacts on both marginal abatement costs and fines, we define  $\varepsilon_{c_e, z} = c_{ez} \frac{z}{c_e} < 0$  as the elasticity of marginal abatement costs with respect to the investment effort  $z$ . Similarly, we define  $\varepsilon_{f, z} = f' \frac{z}{f} \leq 0$  as the elasticity of the fine with respect to  $z$ . These elasticities reflect how sensible marginal abatement costs and fines are with respect to changes in the investment effort  $z$ . The values of these elasticities crucially affect the relationship between  $e$  and  $z$ , as shown in the following:

**Lemma 2**  $\frac{\partial e}{\partial z} = -\frac{c_{ez} + pf'}{c_{ee}} < 0$  if and only if  $\varepsilon_{c_e, z} \leq \varepsilon_{f, z}$ .

Therefore, there exists a negative relationship between the pollution level and the investment effort as long as the sensitivity of fines with respect to the investment effort is smaller (in

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<sup>8</sup>An exception can be found in Arguedas and Hamoudi (2004) or Arguedas (2005), but see footnote 3.

absolute terms) than that of marginal abatement costs. Otherwise, pollution increases with the investment effort. The reason is that an increase in the investment effort may substantially decrease fines as compared to marginal abatement costs, and therefore, may induce an increase in the pollution level.

Conditions (3) and (4) when  $e^* = s$  define the set of policies  $(s, p)$  which induce the firm to comply. Using the chain rule, the implicit function that links the two variables satisfies the following condition:

$$\left. \frac{dp}{ds} \right|_c = - \frac{c_{ee} - (c_{ez} + pf') \frac{c_{ez}}{c_{zz}}}{f} < 0. \quad (6)$$

This relationship is strictly negative, since the abatement cost function is strictly convex (i.e.,  $c_{ee}c_{zz} - c_{ez}^2 > 0$ ) and  $f' \leq 0$ . Therefore, the laxer the standard, the smaller the minimum required inspection probability to induce compliance. Compared to the situation where the firm cannot choose the investment effort (and consequently, fines cannot vary accordingly), the relationship defined in (6) can be more or less negative depending on the sign of the term  $c_{ez} + pf'$ . When  $c_{ez} + pf' > 0$  (or, equivalently,  $\varepsilon_{ce,z} < \varepsilon_{f,z}$ , see lemma 2), the possibility that the firm chooses  $z$  makes  $\left. \frac{dp}{ds} \right|_c$  less negative than when  $z$  is fixed. In other words, for a one unit increase in the standard, the regulator can reduce the inspection probability less than what she could if the investment effort were not a choice variable. The reason is that a one unit increase in the standard reduces the investment effort<sup>9</sup>, and this further reduces the marginal abatement costs ( $c_{ez} > 0$ ) and increases the fine level ( $f' \leq 0$ ), such that the absolute change in the former is larger. This then causes a larger increase in the inspection probability to maintain compliance than that where the investment effort is fixed. However, the opposite reasoning applies when  $c_{ez} + pf' < 0$  (or, equivalently,  $\varepsilon_{ce,z} > \varepsilon_{f,z}$ ).

From expressions (3) and (4), we can now deduct how the firm's optimal choices  $(e^*, z^*)$  depend on the policy parameters  $(s, p)$ .

**Lemma 3** *The firm's optimal choices  $(e^*, z^*)$  are related to the policy parameters  $(s, p)$  as*

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<sup>9</sup>Simply apply the implicit function theorem in expression (4) when  $e^* = s$  to obtain  $\frac{dz}{ds} = - \frac{c_{ez}}{c_{zz}} < 0$ .



follows:

$$\begin{aligned}
\frac{\partial e^*}{\partial p} &= -\frac{f(c_{zz} + pf''(e-s))c_{zz} - f'(e-s)(c_{ez} + pf')}{|A|} \geq 0; \\
\frac{\partial z^*}{\partial p} &= -\frac{c_{ee}f'(e-s) - f(c_{ez} + pf')}{|A|} \geq 0; \\
\frac{\partial e^*}{\partial s} &= -\frac{pf'(c_{ez} + pf')}{|A|} \geq 0; \\
\frac{\partial z^*}{\partial s} &= \frac{c_{ee}pf'}{|A|} \leq 0;
\end{aligned}$$

$$\text{where } |A| = \begin{vmatrix} c_{ee} & c_{ez} + pf' \\ c_{ez} + pf' & c_{zz} + pf''(e-s) \end{vmatrix} > 0.$$

Again, the results are crucially affected by the dependence of the fine on the technology investment. In fact, if the fine does not depend on  $z$  ( $f' = 0$ ), neither the pollution level nor the technology investment are affected by the standard, since the fine is linear in the degree of non-compliance (and, therefore, the marginal fine is constant). Also, in this case, the pollution level is negatively affected by the inspection probability, while the investment effort is positively affected.

Things change when  $f' < 0$ . Again, the sensitivities of the fine and the marginal abatement costs with respect to the investment effort  $z$  are key in determining the sign of the relationships between the firm's choices and the policy variables. If the fine is less sensible (in absolute terms) to the investment effort than marginal abatement costs (or  $c_{ez} + pf' > 0$ , see lemma 2), the results obtained are the intuitive ones: a tighter policy (an increase in the inspection probability and/or a decrease in the standard) decreases the pollution level and increases the investment effort. If, to the contrary, the fine is more sensible to the investment effort than marginal abatement costs (i.e.,  $c_{ez} + pf' < 0$ ), a smaller standard increases both the investment effort and the pollution level (the latter because an increase in the investment effort causes a large decrease in the marginal fine, and therefore reduces deterrence significantly), while a larger inspection probability could affect the pollution level and the investment effort in either way.

For later purposes, it is very useful to see how the policy variables  $(s, p)$  can be combined, such that the pollution level is held constant. Somehow, this helps in finding the trade-off

between the policy variables for a given pollution objective. We take expressions  $\frac{\partial e}{\partial p}$  and  $\frac{\partial e}{\partial s}$  from lemma 3 above to obtain:

$$\left. \frac{dp}{ds} \right|_e = - \frac{\frac{\partial e}{\partial s}}{\frac{\partial e}{\partial p}} = - \frac{pf'(c_{ez} + pf')}{f(c_{zz} + pf''(e - s)) - f'(e - s)(c_{ez} + pf')} \geq 0. \quad (7)$$

This expression informs about how the inspection probability should be changed for a one unit change in the pollution standard, to keep the induced pollution level constant. If  $f' < 0$ ,  $\left. \frac{dp}{ds} \right|_e > 0$  as long as  $c_{ez} + pf' > 0$  and  $e = s$ . The policy parameters in this case are substitutes: pollution can be kept constant by decreasing both the standard and the inspection probability (i.e., by tightening the former instrument and relaxing the latter). The reason is that a decrease in the standard increases the optimal investment effort (see footnote 9) and this decreases the pollution level, by (5). Thus, the inspection probability needs to be decreased to compensate for this effect. However,  $\left. \frac{dp}{ds} \right|_e < 0$  when  $c_{ez} + pf' < 0$  and  $e = s$ . In this case, the policy variables are complements: since a decrease in  $s$  causes an increase in  $z$  that results in an increase in  $e$ , by (5),  $p$  needs to be increased to keep pollution constant. Finally, note that  $\left. \frac{dp}{ds} \right|_e = 0$  as long as  $f' = 0$  or  $c_{ez} + pf' = 0$ .

All this analysis has important implications in deriving the optimal policy, as we show next.

## 4 The Optimal Policy with Given Fines

The objective of this section is to analyze the characteristics of the optimal policy when the fine  $f(z)$  is given. Since the regulator cannot affect the amount of the fine, the problem she faces is the following:

$$\begin{aligned} \min_{p,s} \quad & c(e, z) + z + d(e) + p[m + tf(z)(e - s)], \\ \text{s.t.} \quad & c_e(e, z) + pf(z) \geq 0, \\ & c_z(e, z) + 1 + pf'(z)(e - s) = 0, \\ & e \geq s, \end{aligned} \quad (8)$$

where the constraints in this problem characterize the firm's optimal decision in response to the policy  $(s, p)$ , analyzed in the previous section.

The following result presents the intuitive condition which guarantees that a policy which induces compliance is socially desirable.

**Proposition 1** *Let  $(s^*, p^*)$  be the solution of problem (8). Then,  $(s^*, p^*)$  induces compliance if and only if:*

$$p^* t f(z^*) \geq m \left. \frac{dp}{ds} \right|_e, \quad (9)$$

where  $c_e(s^*, z^*) + p^* f(z^*) = 0$ ,  $c_z(s^*, z^*) + 1 = 0$  and  $\left. \frac{dp}{ds} \right|_e = -\frac{p^* f'(c_{ez} + p^* f')}{f c_{zz}}$ .

This result has a nice interpretation. Assume that the policy  $(s^*, p^*)$  induces compliance, that is,  $e^* = s^*$ . Now, consider a one unit decrease in the standard and adapt the inspection probability such that the pollution level is kept constant (in the way expressed in (7)). Note that this alternative policy induces non-compliance by exactly one unit. Therefore, the firm will be penalized on the amount  $f$  with probability  $p$ . The issue is to see whether this alternative policy which induces non-compliance decreases social costs, as compared to the policy which induces compliance,  $(s^*, p^*)$ . Thus, we have to balance the additional sanctioning and monitoring cost savings of this alternative policy. The additional sanctioning costs are given by the left hand side of (9), since the policy which induces compliance has zero sanctioning costs. The additional monitoring cost savings are given by the right hand side of (9). Clearly, if the former is larger than the latter, the alternative policy which induces non-compliance is more expensive (in social terms) than the policy which induces compliance. Otherwise, the policy which induces non-compliance is socially preferred.

Note that, in the event that  $f' = 0$ , we have  $\left. \frac{dp}{ds} \right|_e = 0$  and, consequently, expression (9) is always met. This means that the solution to problem (8) is always a policy which induces compliance if fines do not depend on the investment effort. This result is consistent with previous works by Stranlund (2007) and Arguedas (2008): if the fine is linear in the degree of non-compliance, the pollution level is not affected by the standard. Therefore, a decrease in the standard (keeping the pollution level constant) only results in additional sanctioning costs.

Another situation where the optimal policy induces compliance is when  $\left. \frac{dp}{ds} \right|_e < 0$ . As analyzed in the previous section, this corresponds to the case where  $c_{ez} + pf' < 0$  and  $e = s$ , see (7), that is, when the standard and the inspection probability act as complements in keeping the pollution level constant. In this case, a one unit reduction in the standard not only causes sanctioning costs, but also additional monitoring costs, because the inspection probability has to be increased to keep the pollution level constant.

The situation where  $c_{ez} + pf' > 0$  and  $e = s$  (i.e., when  $\left. \frac{dp}{ds} \right|_e > 0$ ) is the one where we may have non-compliance as the best possible alternative. Here, the standard and the inspection probability are substitutes in keeping pollution constant. Therefore, the necessary and sufficient requirement for the optimal policy to induce non-compliance is that the additional monitoring cost savings associated with reducing the inspection probability off-set the additional sanctioning costs of reducing the standard.

The characterization of the optimal policy with given fines is presented next.

**Proposition 2** *With given fines, the (interior) optimal policy  $(s^*, p^*)$  is characterized by the following conditions:*

$$\begin{aligned} c_e(e^*, z^*) + d'(e^*) - c_{ee} \frac{m}{f} - c_{ez} \frac{tf}{f'} &= 0, \\ c_z(e^*, z^*) + 1 + p^* tf'(z^*)(e^* - s^*) - \frac{m}{f}(c_{ez} + pf') - \frac{tf}{f'}(c_{zz} + pf''(e - s)) &= 0, \\ c_e(e^*, z^*) + p^* f(z^*) &= 0, \\ c_z(e^*, z^*) + 1 + p^* f'(z^*)(e^* - s^*) &= 0. \end{aligned}$$

These four equations contain the two possibilities for the optimal policy (compliance or non-compliance), depending on whether expression (9) is met or not. The first equation balances additional abatement cost savings and external damages of a one unit increase in pollution against additional monitoring and sanctioning costs. The second equation balances additional abatement and investment costs of a one unit increase in the investment level against additional monitoring and sanctioning costs. The remaining two equations are the firm's optimal responses to the policy, analyzed in the previous section.

In the particular case of compliance,  $e^* = s^*$ , the optimality conditions easily reduce to

$$\begin{aligned} c_e(s^*, z^*) + d'(s^*) + m \left. \frac{dp}{ds} \right|_c &= 0, \\ c_e(s^*, z^*) + p^* f(z^*) &= 0, \\ c_z(s^*, z^*) + 1 &= 0, \end{aligned}$$

where  $\left. \frac{dp}{ds} \right|_c$  is given by (6). Remember that the optimal policy is characterized in this way if and only condition (9) is met. As expected, the optimal policy under compliance balances abatement costs, investment costs and external damages (i.e., efficiency) versus monitoring costs. As a result, the optimal standard level is always above the one that minimizes the sum of abatement costs and external damages, since  $\left. \frac{dp}{ds} \right|_c < 0$ , see (6). Again, expression  $c_{ez} + pf'$  is key in determining how negative  $\left. \frac{dp}{ds} \right|_c$  is and, therefore, how far the optimal standard is from the efficient pollution level (i.e., the one that minimizes the sum of abatement costs, investment costs and external damages). As explained in the previous section,  $\left. \frac{dp}{ds} \right|_c$  is less negative when  $c_{ez} + pf' > 0$  than when  $c_{ez} + pf' < 0$ . Therefore, if  $c_{ez} + pf' > 0$  holds, the resulting optimal standard level is closer to the efficient level than if  $c_{ez} + pf' < 0$  holds.

## 5 Should Fines Depend on the Investment Effort?

In this section, we analyze whether it is socially convenient that fines depend on the investment effort level. For that purpose, we assume that there exists a maximum of the fine given by law associated to an investment level  $z = 0$ ,  $f(0)$ , and then the regulator is allowed to decrease the fine contingent on the selection of a particular investment level, that is,  $f(z)$ , where  $f'(z) < 0$ .

In the following proposition, we provide the expression for the optimal fine.

**Proposition 3** *The optimal fine is given by the expression:*

$$f(z^*) = f(0) \frac{m}{m + tf'(0)(e^* - s^*)}. \quad (10)$$

Interestingly, the optimal fine considers the relative importance of monitoring costs with respect to total enforcement costs, that is, monitoring plus sanctioning costs. As a result, the

optimal fine is a proportion of the maximum fine,  $f(0)$ . Intuitively, the larger the monitoring costs, the larger the resulting fine (since larger monitoring costs result in a smaller inspection probability). Conversely, the larger the sanctioning costs, the smaller the resulting fine.

Note that, if the optimal policy induces compliance ( $e^* = s^*$ ), then the fine per unit of the violation should not decrease with the investment effort  $z$ , that is,  $f(z) = f(0)$ . Under compliance, only monitoring costs matter. Thus, the larger the fine, the smaller the resulting optimal inspection probability and the smaller the social costs.

Therefore, non-compliance is a necessary requirement to have fines decreasing in the investment effort level. However, even in the case where the optimal policy induces non-compliance, the optimal decrease in the fine is zero as long as there are no sanctioning costs, i.e., when  $t = 0$ . Thus, the larger the sanctioning costs, the larger the fine for non-compliance.

## 6 Conclusions

In this paper, we have analyzed the conditions under which fines should decrease with the investment effort and, therefore, should not be set at the maximum level. These conditions are that the optimal policy induces non-compliance and that there exists sanctioning costs associated with fines collection. Only under these conditions the firm can be induced to make a larger investment effort through a smaller marginal penalty, which then induces a social cost savings.

Since the purpose of this paper is to link the dependence of the fines on the investment effort and analyze the desirability of imposing non-maximal fines in this context, we have deliberately simplified the paper in many other aspects of the enforcement problem. For example, the timing we consider here is one of *unresponsive* regulation, that is, one in which the regulator first sets the terms of the policy and then the firm chooses the investment effort and the pollution level. We do not allow the regulator to react to the firm's choices afterwards, that is, we assume that the regulator commits to the specific policy levels she announced in the first place, applying the specific penalty discounts previously announced (if any). There are

alternative possibilities for modeling the relationship between the regulator and the firm. For example, Arguedas and Hamoudi (2004), Maxwell and Decker (2006) or Decker (2007), among others, analyze the case of *responsive* regulation, where the regulator selects the enforcement policy reacting to the investment effort chosen by the firms. A common conclusion in all these studies is the over-investment effect generated by the firm knowing that it can affect the enforcement policy, in comparison with the case of unresponsive regulation. Also, we consider the simplifying assumption of a non-hierarchical government. This means that the regulator that is responsible for setting the environmental policy is also responsible for enforcing the law (that is, responsible for setting inspection probabilities and fines). An alternative assumption would allow different regulators to be responsible of these two different goals, such as in Saha and Poole (2000) or Decker (2007), among others. These and other more relaxed alternative assumptions, such as allowing for dynamic enforcement (Harrington (1988)) or for self-reporting (Livernois and McKenna (1999)), only reinforce the result that fines for non-compliance should not be maximum.

## 7 Appendix

**Proof of Lemma 1.** First note that  $e < s$  is not an optimal decision, since the firm can save on abatement costs by infinitesimally increasing pollution without being penalized. This reduces the set of possible firm's pollution choices to  $e \geq s$ . Then, the problem to be solved is:

$$\begin{aligned} \text{Min}_{e,z} \quad & c(e, z) + z + pf(z)(e - s), \\ \text{s.t.} \quad & e \geq s. \end{aligned}$$

The optimality conditions of this problem are:

$$c_e(e, z) + pf(z) - \lambda = 0 \tag{11}$$

$$c_z(e, z) + 1 + pf'(z)(e - s) = 0 \tag{12}$$

$$\lambda(e - s) = 0; \lambda \geq 0; e \geq s, \tag{13}$$

where  $\lambda \geq 0$  is the Kuhn-Tucker multiplier of the inequality restriction  $e \geq s$ . From (13),  $\lambda \geq 0$  implies  $e = s$ . From (11) and (12), this implies  $c_e(s, z) + pf(z) \geq 0$ , where  $z$  is such that  $c_z(s, z) + 1 = 0$ . If conversely,  $\lambda = 0$ , we then have  $e > s$ . Now, conditions (11) and (12) reduce to  $c_e(e, z) + pf(z) = 0$  and  $c_z(e, z) + 1 + pf'(z)(e - s) = 0$ . Combining both possibilities, we obtain the desired result.

**Proof of Lemma 2.** Since at the optimum  $c_e + pf = 0$ , we have  $c_{ez} + pf' = c_{ez} - \frac{c_e}{f} f'$ . Since  $c_e < 0$ , we have  $c_{ez} - \frac{c_e}{f} f' \geq 0$  if and only if  $\frac{c_{ez}}{c_e} - \frac{f'}{f} \leq 0$ . Multiplying this expression by  $z$ , we have  $\frac{c_{ez}}{c_e} z - \frac{f'}{f} z \leq 0$ , and applying the definitions  $\varepsilon_{c_e, z} = c_{ez} \frac{z}{c_e}$  and  $\varepsilon_{f, z} = f' \frac{z}{f}$ , we obtain the desired result.

**Proof of Lemma 3.** Fully differentiating expressions (11) and (12) with respect to  $(e, z, s, p)$ , we obtain:

$$\begin{aligned} c_{ee}de + (c_{ez} + pf')dz + fdp &= 0; \\ (c_{ez} + pf')de + (c_{zz} + pf''(e - s))dz + f'(e - s)dp - pf'ds &= 0; \end{aligned}$$

or put differently,

$$\begin{pmatrix} c_{ee} & c_{ez} + pf' \\ c_{ez} + pf' & c_{zz} + pf''(e - s) \end{pmatrix} \begin{pmatrix} de \\ dz \end{pmatrix} = - \begin{pmatrix} f & 0 \\ f'(e - s) & -pf' \end{pmatrix} \begin{pmatrix} dp \\ ds \end{pmatrix}.$$

Let  $A = \begin{pmatrix} c_{ee} & c_{ez} + pf' \\ c_{ez} + pf' & c_{zz} + pf''(e - s) \end{pmatrix}$ . Note that  $|A| > 0$  (this ensures that sufficient conditions for the firm's optimal choices hold). Applying Crammer's rule, we then have:

$$\begin{aligned} \frac{\partial e}{\partial p} &= - \frac{\begin{vmatrix} f & c_{ez} + pf' \\ f'(e - s) & c_{zz} \end{vmatrix}}{|A|} = - \frac{f(c_{zz} + pf''(e - s)) - f'(e - s)(c_{ez} + pf')}{|A|}; \\ \frac{\partial z}{\partial p} &= - \frac{\begin{vmatrix} c_{ee} & f \\ c_{ez} + pf' & f'(e - s) \end{vmatrix}}{|A|} = - \frac{c_{ee}f'(e - s) - f(c_{ez} + pf')}{|A|}; \\ \frac{\partial e}{\partial s} &= - \frac{\begin{vmatrix} 0 & c_{ez} + pf' \\ -pf' & c_{zz} + pf''(e - s) \end{vmatrix}}{|A|} = - \frac{pf'(c_{ez} + pf')}{|A|}; \\ \frac{\partial z}{\partial s} &= - \frac{\begin{vmatrix} c_{ee} & 0 \\ c_{ez} + pf' & -pf' \end{vmatrix}}{|A|} = \frac{c_{ee}pf'}{|A|}; \end{aligned}$$

as desired.



**Proof of Proposition 1.** The first order conditions of this problem are the following:

$$\begin{aligned}
(w.r.e) \quad & c_e + d' + ptf + \lambda c_{ee} + \gamma (c_{ez} + pf') + \mu = 0; \\
(w.r.z) \quad & c_z + 1 + ptf' (e - s) + \lambda (c_{ez} + pf') + \gamma (c_{zz} + pf'' (e - s)) = 0; \\
(w.r.p) \quad & m + tf (e - s) + \lambda f + \gamma f' (e - s) = 0; \\
(w.r.s) \quad & -ptf - \gamma pf' - \mu = 0;
\end{aligned}$$

where  $\lambda \leq 0, \gamma \geq 0, \mu \leq 0$  are the respective Kuhn-Tucker multipliers.

Under compliance, we have  $e = s, \mu \leq 0$  and  $c_z + 1 = 0$ , by (12). The optimality conditions reduce to:

$$\begin{aligned}
(w.r.e) \quad & c_e + d_e + ptf + \lambda c_{ee} + \gamma (c_{ez} + pf') + \mu = 0; \\
(w.r.z) \quad & \lambda (c_{ez} + pf') + \gamma c_{zz} = 0; \\
(w.r.p) \quad & m + \lambda f = 0; \\
(w.r.s) \quad & \mu = -ptf - \gamma pf';
\end{aligned}$$

from which we obtain  $\lambda = -\frac{m}{f} < 0$  and  $\gamma = \frac{m}{fc_{zz}} (c_{ez} + pf')$ . Thus,  $\mu \leq 0$  if and only if

$$tf + \frac{m}{fc_{zz}} (c_{ez} + pf') f' \geq 0. \quad (14)$$

From expression (7), we have  $\frac{dp}{ds}\big|_e = -\frac{pf'(c_{ez}+pf')}{fc_{zz}}$  when  $e = s$ . Therefore, (14) can be rewritten as:

$$ptf \geq m \frac{dp}{ds}\bigg|_e,$$

as desired.

**Proof of Proposition 3.** Consider the solution of problem (8),  $(s^*, p^*)$ . Substituting this policy in the Lagrangian of problem (8), we have:

$$\begin{aligned}
L(s^*, p^*) = & c(e^*, z^*) + z^* + d(e^*) + p^* [m + tf(z^*)(e^* - s^*)] \\
& + \lambda^* [c_e(e^*, z^*) + p^* f] + \gamma^* [c_z(e^*, z^*) + 1 + p^* f'(z^*)(e^* - s^*) + \mu^* (e^* - s^*)],
\end{aligned}$$

where  $(\lambda^*, \gamma^*, \mu^*)$  are the respective Kuhn-Tucker multipliers.

Now, consider the second order degree polynomial approximation of the fine as follows:

$$f(z) = f(0) + f'(0)z + f''(0)\frac{z^2}{2}.$$

Applying the envelope theorem in the above Lagrangian expression, we have:

$$\frac{\partial L(s^*, p^*)}{\partial f'(0)} = p^* t z^* (e^* - s^*) + \lambda^* p^* z^* + \gamma^* p^* (e^* - s^*), \quad (15)$$

$$\frac{\partial L(s^*, p^*)}{\partial f''(0)} = p^* t \frac{(z^*)^2}{2} (e^* - s^*) + \lambda^* p^* \frac{(z^*)^2}{2} + \gamma^* p^* z^* (e^* - s^*). \quad (16)$$

Assuming  $e^* \geq s^*$ , we have  $\mu^* = 0$  and  $\gamma^* = -\frac{tf}{f'} > 0$ . Also,  $\lambda^* = -\frac{m}{f} < 0$ . Substituting these terms in the above expression, the condition for an optimal interior level of  $f'(0)$  is such that condition (15) is met with strict equality:

$$t z^* (e^* - s^*) - \frac{m}{f} z^* - \frac{t f}{f'} (e^* - s^*) = 0.$$

But if this condition holds, condition (16) is strictly positive, which leads to  $f''(0) = 0$ .

Then,  $f(z) = f(0) + f'(0)z$  and  $f'(z) = f'(0)$ , which results in:

$$f'(0) = -\frac{f(0)}{z} \frac{t f(0) (e^* - s^*)}{m + t f(0) (e^* - s^*)}.$$

Therefore,

$$f(z) = f(0) - f(0) \frac{t f(0) (e^* - s^*)}{m + t f(0) (e^* - s^*)} = f(0) \frac{m}{m + t f(0) (e^* - s^*)},$$

as desired.<sup>10</sup>

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<sup>10</sup>The reader can easily check that the same conclusion is achieved if we instead consider an interior solution for  $f''(0)$ , where condition (16) is satisfied with equality and condition (15) is strictly negative.

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