On welfare criteria and optimality in an endogenous growth model

Elena Del Rey* and Miguel-Angel Lopez-Garcia†

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Abstract

In this paper we explore the consequences for optimality of a social planner adopting two different welfare criteria. The framework of analysis is an OLG model with physical and human capital. We first show that, when the SWF is a discounted sum of individual utilities defined over consumption per unit of natural labour, the precise cardinalization of the individual utility function becomes crucial for the characterization of the social optimum. Also, decentralizing the social optimum requires an education subsidy. In contrast, when the SWF is a discounted sum of individual utilities defined over consumption per unit of efficient labour, the precise cardinalization of preferences becomes irrelevant. More strikingly, along the optimal growth path, education should be taxed.

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*Universidad de Girona, Spain

†Universidad Autonoma de Barcelona, Spain

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1 Introduction

In optimal growth theory, the choice of the social planner's objective function has not always been without controversy. Among the earliest contributions, Ramsey (1928), was primarily concerned with the implications of maximizing an infinite, undiscounted sum of present and future individual utility. For Ramsey, the discount of later enjoyments in comparison with earlier ones was an ethically indefensible practice. Instead, Cass (1965) was concerned with maximizing an infinite discounted sum of individual utilities. A different approach was adopted by Phelps (1961), who proposed that we should seek to maximize consumption per capita, rather than utilities.

Turning to an explicit OLG framework, in the late 50s, Samuelson (1958) advocated for the maximization of individual lifetime utility, while Lerner (1959) considered more appropriate the maximization of the current utility of individuals of different ages concurring at the same time period. This, of course, concerns the case where individuals are pure life-cyclers à la Diamond (1965). But if individuals are altruistic, as in Barro (1974), and behave as if they maximized dynastic utility, a new alternative appears between considering only the welfare level enjoyed by a representative child (Carmichael, 1982) or by all children (Burbidge, 1983). Clearly, each of these views of social welfare leads to a different optimal allocation.

All of the examples above refer to economies without productivity growth, in which a steady state is a situation where consumption levels per unit of (natural) labour are kept constant. In the presence of productivity growth that translates into consumption growth, however, these consumption levels will grow without any limit. Under these circumstances, if a social planner adopted a social welfare function whose arguments were utility functions defined over individual consumptions per unit of natural labour, it is clear that, for plausible specifications, the utility index would be growing without limit. Since utility will eventually be infinite along a balanced growth path, there would simply be no scope for utility maximization. A way to sidestep this is of course to assume that the planner maximizes a discounted sum of utilities. This is a standard procedure, and it is indeed the one adopted among others by Docquier, Paddison and Pestieau (2007) (henceforth DPP) to characterize the optimal balanced growth path in an endogenous growth setting. Focusing on optimal policies along the balanced growth path, DPP (2007) identify the subsidy that internalizes the externality associated with investing on education and the scheme of intergenerational transfers between old and middle-aged individuals. On the basis of a particular example, they claim that, on pure efficiency grounds, the case for public pensions is rather weak.

In this paper, we evaluate the consequences of the planner adopting a different welfare criterion. In particular, we will compare the results in DPP (2007) with those obtained when the planner maximizes a discounted sum of individual utilities defined over consumption levels per unit of *efficient* labour. As it will become clear, on the one hand, this new social welfare function depends on utility indices which, in turn, are obtained from a utility function that respects individual ordinal preferences for present and future consumption. On the other hand, like any SWF that embodies utility discounting, it does not treat individuals from different generations equally. More particularly, for a given discount factor, the more human capital a generation is endowed with, the lower its weight in this new social welfare function. This idea is not totally opposed to some notion of social justice.

We first show that, when, as in DPP (2007), the social planner maximizes a SWF whose arguments are utility levels derived from individual consumptions per unit of natural labour (which we will label "the standard approach"), the precise cardinalization of the individual utility function is crucial for both the characterization of the social optimum and the policies that support it. Decentralizing the social optimum requires an education subsidy that is definitely positive, but its size depends in a determinant way on the aforementioned cardinalization. In contrast, under "the alternative approach", when the planner maximizes a SWF whose arguments are individual utilities defined over individual consumptions per unit of efficient labour, the precise cardinalization of preferences becomes irrelevant. More strikingly, the optimal education subsidy is *negative*, i.e., the planner should tax rather than subsidize investments on human capital. The reason is that individuals choose their human capital investments accounting only for the effects on their earnings and loan repayment costs. Thus, in a laissez-faire economy, if individuals faced the optimal wage and interest rates, they would ignore the costs associated with maintaining these factor prices at their optimal balanced growth path level when human capital increases. Under these circumstances, they would over-invest in education. This is the reason why a tax is required to decentralize the optimum. With respect to the accompanying scheme of intergenerational transfers, we make patent that nothing can be said in general.

The rest of the paper is organized as follows. Section 2 presents the general framework and the decentralized solution in presence of the government. Section 3 analyzes the consequences of adopting the two alternative welfare criteria and Section 4 concludes.

2 The model and the decentralized solution

The basic framework of analysis is the overlapping generations model with both human and physical capital developed in Boldrin and Montes (2005) and DPP (2007). Individuals live for three periods. At period t, N_{t+1} individuals are born. They coexist with N_t middle-aged and N_{t-1} old-aged. A young individual at t is endowed with the current level of human capital (i.e., knowledge or labour efficiency), h_t, which, combined with the amount of output devoted to education, e_t , produces human capital at period $t+1$

according to the production function $h_{t+1} = \Phi(h_t, e_t)$. Assuming constant returns to scale, the production of human capital can be written in intensive terms as $h_{t+1}/h_t = \varphi(\bar{e}_t)$, where $\bar{e}_t = e_t/h_t$ and $\varphi(.)$ satisfies the Inada conditions. The middle-aged at period t, N_t, work and provide one unit of labour of efficiency h_t, and consume c_t. Finally, the N_{t-1} old individuals are retired and consume d_t. Population grows at the exogenous rate n so that $N_t = (1+n)N_{t-1}$ with $n > -1$.

A single good is produced by means of physical capital K_t and human capital H_t, using a neoclassical constant returns to scale technology, $F(K_t, H_t)$, where $H_t = h_t N_t$. Physical capital fully depreciates each period. If we define $k_t = K_t/N_t$ as the capital-labour ratio in natural units and $\bar{k}_t = K_t/H_t = k_t/h_t$ as the capital-labour ratio in efficiency units, this production function can be described as $h_t N_t f(\bar{k}_t)$, where $f(.)$ also satisfies the Inada conditions.

The lifetime welfare attained by an individual born at period $t-1$, U_t, can be written by means of the utility function

$$U_t = U(c_t, d_{t+1}) \tag{1}$$

As usual in consumer theory, (1) is assumed to be strictly quasi-concave. Furthermore, for the discussion of balanced growth paths to make sense, the utility function should also be homothetic. Boldrin and Montes (2005) do not explicitly refer to the shape of indifference curves, but use instead an equivalent condition (Assumption 2). The above refers to *consumer*'s behavior. In order to ensure that the *social planner*'s problem is well behaved, additional restrictions are needed. In particular, (1) is required to be homogeneous of degree $b < 1$, this guaranteeing both homotheticity and strict concavity. In section 3, this technicallity will be shown to fundamentally affect the social optimum (and thus the optimal policy) in DPP (2007)'s framework. However, it will also be argued therein that the degree of homogeneity of the utility function and the ensuing cardinalization of preferences is dispensable in an alternative framework.

Total output produced in period t, $F(K_t, H_t)$, can be devoted to consumption, $N_t c_t + N_{t-1} d_t$, investment on physical capital, K_{t+1}, and investment on human capital, $N_{t+1} e_t$. Thus, the aggregate feasibility constraint expressed in units of (natural) labour is

$$h_t f(k_t/h_t) = c_t + \frac{d_t}{1+n} + (1+n)e_t + (1+n)k_{t+1} \tag{2}$$

Alternatively, we can divide (2) by h_t and obtain the aggregate feasibility constraint in period t with all the variables expressed in terms of output per unit of efficient labour

$$f(\bar{k}_t) = \bar{c}_t + \frac{\bar{d}_t}{\varphi(\bar{e}_{t-1})(1+n)} + (1+n)\bar{e}_t + \varphi(\bar{e}_t)(1+n)\bar{k}_{t+1} \tag{3}$$

where $\bar{c}_t = c_t/h_t$ and $\bar{d}_t = d_t/h_{t-1}$.[1]

[1] Note that $c_t N_t$ and $d_t N_{t-1}$ are expressed in units of output. Since middle-aged individuals supply

We can now describe the behavior of the decentralized economy in the presence of government intervention. We focus on the three tax instruments considered in DPP (2007), namely, a subsidy on educational spending and two lump-sum taxes in the periods of work and retirement. For an individual born in $t-1$, let these instruments be σ_{t-1}, T_t^1 and T_{t+1}^2. Of course one can always recover the laissez faire equilibrium by forcing $\sigma_{t-1} = T_t^1 = T_{t+1}^2 = 0$.

In order to study the interaction among the individuals and the government, a careful description of their behavior is required. Under perfect competition, each factor is paid its marginal product, so that the rate of return on physical capital and the wage rate (per unit of efficient labour) are given by $1 + r_t = f'(\bar{k}_t)$ and $w_t = f(\bar{k}_t) - \bar{k}_t f'(\bar{k}_t)$. In their first period, individuals choose the amount of education that maximizes their lifetime resources. We assume that competitive credit markets exist in which young agents can borrow to buy education. They borrow $(1-\sigma_{t-1})\, e_{t-1}$ and are subsidised $\sigma_{t-1}e_{t-1}$. At time t, when they are middle-aged, they inelastically supply one unit of labour of efficiency h_t, for which they obtain the wage rate w_t. They also consume c_t, save s_t, repay the loan at the going interest rate r_t and pay the lump-sum tax T_t^1. In period $t+1$, when they are old, they retire and consume d_{t+1} out of their savings, after having paid T_{t+1}^2. The lifetime budget constraint of an individual born in period $t-1$ is therefore[2]

$$c_t + \frac{d_{t+1}}{1+r_{t+1}} = w_t h_t - (1+r_t)\,(1-\sigma_{t-1})\, e_{t-1} - T_t^1 - \frac{T_{t+1}^2}{1+r_{t+1}} \tag{4}$$

Individuals save for pure life-cycle reasons, i.e. to transfer purchasing power from the second to the third period. They maximize $U(c_t, d_{t+1})$ by the choice of e_{t-1}, c_t and d_{t+1} subject to (4). The first order conditions write:

$$w_t \varphi'(\bar{e}_{t-1}) = (1+r_t)\,(1-\sigma_{t-1}) \tag{5}$$

$$\frac{\partial U(c_t, d_{t+1})/\partial c_t}{\partial U(c_t, d_{t+1})/\partial d_{t+1}} = (1+r_{t+1}) \tag{6}$$

Since the government finances education subsidies by means of the lump-sum taxes T_t^1 and T_t^2, its budget constraint is, in period t,

$$T_t^1 + \frac{T_t^2}{1+n} = (1+n)\sigma_t e_t \tag{7}$$

Equilibrium in the market for physical capital is achieved when the physical capital stock available in $t+1$, K_{t+1}, equals savings made by the middle-aged in t, $s_t L_t$, minus the liabilities associated with the human capital investment by the young in t, $(1-\sigma_t)e_t L_{t+1}$:

$$(1+n)k_{t+1} = w_t h_t - (1+r_t)(1-\sigma_{t-1})e_{t-1} - T_t^1 - c_t - (1+n)\,(1-\sigma_t)\, e_t \tag{8}$$

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one unit of natural labour, c_t and d_t are expressed in units of output per unit of *natural* labour. The interpretation of $\bar{c}_t$ and $\bar{d}_t$ in terms of units of output per unit of *efficient* labour follows naturally.

[2] The periodical budget constraints in periods t and $t+1$ are $c_t = w_t h_t - (1+r_t)\,(1-\sigma_{t-1})\, e_{t-1} - T_t^1 - s_t$ and $d_{t+1} = (1+r_{t+1})s_t - T_{t+1}^2$ respectively. From them, (4) can be directly found.

Notice that (7), (8) and the individual budget constraints in middle age and old age, allow to recover the aggregate feasibility constraint (2). Equilibrium condition (8) can be written, in units of output per efficient labour,

$$(1+n)\varphi(\bar{e}_t)\bar{k}_{t+1} = w_t - \frac{(1+r_t)(1-\sigma_{t-1})\bar{e}_{t-1}}{\varphi(\bar{e}_{t-1})} - \bar{T}_t^1 - \bar{c}_t - (1+n)\,(1-\sigma_t)\,\bar{e}_t \qquad (9)$$

The homogeneity of preferences implies that c_t and $\bar{c}_t$ are, respectively, a fraction depending on r_{t+1} of the right hand side of (4) and of the right hand side of (4) divided by h_t. On the other hand, (5) characterizes $\bar{e}_{t-1}$ for given values of $\bar{k}_t$ and σ_{t-1}, i.e., $\bar{e}_{t-1} = \phi(\bar{k}_t, \sigma_{t-1})$. Allowing for the government budget constraint (7) to be expressed in terms of output per unit of efficient labour, it then follows that (9) implicitly provides $\bar{k}_{t+1}$ as a function $\Psi(\bar{k}_t; \bar{T}_t^1, \bar{T}_{t+1}^1, \sigma_t, \sigma_{t+1})$. Along a balanced growth path, all variables expressed in terms of output per unit of natural labour will be growing at a constant rate. Thus all variables expressed in terms of output per unit of efficient labour, including factor payments, will remain constant over time. We can then delete the time subscripts and write $\bar{k} = \Psi(\bar{k}; \bar{T}^1, \sigma)$. An equilibrium ratio of physical capital to labour in efficiency units, along a balanced growth path and in the presence of government intervention, will then be a fixed point of the Ψ function. Assuming that such a balanced growth path is unique and locally stable, we can write $\bar{k} = \bar{k}(\bar{T}^1, \sigma)$.

We can now turn to the determination of $\bar{e}$ or, since $1+g = \varphi(\bar{e})$, the characterization of the growth rate g of any variable expressed in terms of output per unit of natural labour. The amount of output devoted to education per unit of inherited human capital along a balanced growth path will be governed by (5), so that $\bar{e} = \phi\left(\bar{k}(\bar{T}^1, \sigma), \sigma\right)$ or, $\bar{e} = \bar{e}(\bar{T}^1, \sigma)$. It then follows that $1+g = \varphi(\bar{e}(\bar{T}^1, \sigma))$. Finally, the growth rate of all variables expressed in absolute terms (physical capital, human capital and output) is $(1+g)(1+n)$.

3 The planner's problem

As we have already mentioned, along a balanced growth path, all variables expressed in units of natural labour will be growing at a constant rate, g. Under these circumstances, if a social planner adopted a social welfare function whose arguments were utility functions defined over consumption per unit of natural labour, it is clear that, for plausible specifications, the utility index U_t in (1) would be growing without limit. Since utility will eventually be infinite along a balanced growth path, there is simply no scope for utility maximization. A way to sidestep this is to assume that the planner maximizes a discounted sum of utilities. This is a standard procedure, and it is indeed the one adopted by DPP (2007) to characterize the optimal balanced growth path. But even if we accept that future utilities should be discounted at an arbitrary rate (i.e., if discounting is not deemed immoral, in Ramsey's words), one can ask whether there are some other uncomfortable consequences of this way to tackle the issue. One of the purposes of this section is precisely to show that, in this case, the specific cardinalization of the individual utility function (i.e., the degree of homogeneity of (1)) fundamentally affects the optimal solution to the planner's problem. As a result, the optimal policy *varies* when we use different utility specifications representing the *same* individual ordinal preferences. One could reasonably claim that the crucial dependence of the results of an entirely arbitrary cardinalization of the utility function is an unpleasant feature of this approach.

The second purpose of this section is to present an alternative approach. We start by noticing that, with homogeneous (of any degree) utilities, the *same* individual ordinal preferences can be represented using as variables consumptions per unit of natural labour or consumptions per unit of efficient labour. Further, along a balanced growth path, consumptions per unit of efficient labour, $\bar{c}_t$ and $\bar{d}_t$, will be kept constant. It is then clear that if the social planner adopted a social welfare function whose arguments were utility functions defined over consumptions per unit of efficient labour, the utility index, $U(\bar{c}_t, \bar{d}_t)$, would be finite. More importantly, as we show below, the optimal balanced growth path is *independent* of the precise cardinalization of the utility function. Hence, the optimal policy is invariant to transformations of the utility function that preserve individual ordinal preferences. As a consequence, positing a social welfare function whose arguments are consumptions per unit of labour efficiency emerges as a reasonable alternative or at least one that deserves some attention. We explore the consequences of doing so, focusing on the different resource allocations associated with the social planner adopting the two welfare criteria outlined above and, particularly, the value of the optimal education subsidy in each case.

3.1 The standard approach

Let $\gamma < 1$ be the social planner's discount factor. Since the planner now maximizes a discounted sum of utilities of consumption per unit of natural labour, the social welfare function W can be written as

$$W = \sum_{t=0}^{\infty} \gamma^t U(c_t, d_{t+1}) \tag{10}$$

which is maximized subject to the sequence of aggregate feasibility constraints (2) and the sequence of human capital production functions $h_{t+1} = h_t \varphi(\bar{e}_t)$, for given initial values of k_0, h_0 and d_0.

The optimum of this planner's problem can be characterized by the first-order conditions with respect to c_t, d_t, k_{t+1}, h_{t+1} and e_t and the transversality conditions. Let the subscript $*$ stand for optimal values and the superscript N refer to the case where the planner cares for utility defined over consumption per unit of natural labour. Following

DPP (2007), the optimum balanced growth path can be characterized by the following conditions

$$\frac{\partial U(\bar{c}_*^N, \bar{d}_*^N)/\partial \bar{c}}{\partial U(\bar{c}_*^N, \bar{d}_*^N)/\partial \bar{d}} = f'(\bar{k}_*^N) \tag{11}$$

$$\frac{\gamma f'(\bar{k}_*^N)}{(1+n)} = [\varphi(\bar{e}_*^N)]^{1-b} \tag{12}$$

$$w(\bar{k}_*^N)\varphi'(\bar{e}_*^N) = f'(\bar{k}_*^N)\left(1 - \frac{1+n}{f'(\bar{k}_*^N)}\left[\varphi(\bar{e}_*^N) - \bar{e}_*^N\varphi'(\bar{e}_*^N)\right]\right) \tag{13}$$

and the balanced growth path version of (3). These conditions provide four equations that, together, determine the optimal levels of $\bar{c}$, $\bar{d}$, $\bar{k}$, $\bar{e}$.

Equation (11) is the equality of the marginal rate of substitution between second and third period consumptions and the marginal product of physical capital. Observe that it is expressed in terms of the marginal utilities of consumptions per unit of efficient labour. The reason why this can be done is a mere consequence of the assumption of homothetic preferences, which implies that the slope and curvature of the indifference curves in (c_t, d_{t+1}) space are the same as the those of the indifference curves in $\left(\bar{c}_t, \bar{d}_{t+1}\right)$ space.[3] In turn, (12) is the Modified Golden Rule derived in this environment. As stressed by DPP (2007), the infinite sum in (10) will converge provided that $\gamma[\varphi(\bar{e}_*^N)]^b < 1$. This implies that $f'(\bar{k}_*^N) > (1+n)\varphi(\bar{e}_*^N)$, i.e., that, along the optimum balanced growth path, the marginal product of physical capital exceeds the economy's growth rate. Finally, (13) provides the optimal accumulation of human capital. This equation reflects the fact that, when a marginal unit of output is invested on education, not only next period human capital but also future generation's labour efficiency increases. In other words, there is a positive intergenerational externality from investing on education. From this point of view, when one compares (13) and (5), it becomes apparent that the optimal education *subsidy* that internalizes this externality is given by

$$\sigma_*^N = \frac{1+n}{f'(\bar{k}_*^N)}\left[\varphi(\bar{e}_*^N) - \bar{e}_*^N\varphi'(\bar{e}_*^N)\right] \tag{14}$$

and is *positive* by virtue of the concavity of the $\varphi(.)$ function.

It is also clear from expression (12) that the degree of homogeneity of the individual utility function b emerges as a vital element in the determination of $\bar{k}_*^N$ and $\bar{e}_*^N$. This is tantamount to saying that different *cardinalizations* of the preferences underlying the

[3] Formally, since $U(c_t, d_{t+1})$ is homogeneous of degree b its first partial derivatives are homogeneous of degree $b-1$. Then

$$\frac{\partial U(c_t/h_t, d_{t+1}/h_t)}{\partial(c_t/h_t)} = \left(\frac{1}{h_t}\right)^{b-1}\frac{\partial U(c_t, d_{t+1})}{\partial c_t}$$

With a similar result applying to $\partial U()/\partial d_{t+1}$, it follows that the marginal rates of substitution are the same when expressed in terms of c_t, d_{t+1} or $\bar{c}_t, \bar{d}_{t+1}$.

individual utility function (1) will yield different values of $\bar{k}_*^N$ and $\bar{e}_*^N$ and, as a result, different tax parameters supporting them.

In particular, the optimal subsidy σ_*^N will depend on the precise cardinalization. This point can be illustrated with the "triple Cobb-Douglas case" used in Appendix C of DPP (2007). They write the individual utility function as $U(c_t, d_{t+1}) = ln\ c_t + \beta\ \ ln\ d_{t+1}$, with $\beta > 0$. Clearly, this function is not homogeneous, although it can be interpreted to represent the case where $b = 0$. To emphasize that this specific cardinalization hides the relevance of the parameter b, consider the utility function $U(c_t, d_{t+1}) = c_t^{b\delta} d_{t+1}^{b(1-\delta)}$, with $0 < \delta < 1$, which is strictly concave and homogeneous of degree $b < 1$. Notice that the ordinal representation of these preferences is the same as the logarithmic one when we take $\beta = (1-\delta)/\delta$. With the production function of human capital $\varphi(\bar{e}) = B\bar{e}^\lambda$, where $B > 0$ and $0 < \lambda < 1$, one has $\varphi'(\bar{e}) = \lambda B \bar{e}^{\lambda-1} = \lambda(1+g)/\bar{e}$. Then, using (12), (13) becomes

$$w(\bar{k}_N^*)\varphi'(\bar{e}_N^*) = f'(\bar{k}_N^*)\left(1 - \gamma(1-\lambda)\varphi(\bar{e}_N^*)^b\right) \tag{15}$$

Therefore, the optimal subsidy (14) is now given by $\sigma_N^* = \gamma(1-\lambda)\varphi(\bar{e}_N^*)^b$, and clearly depends on the degree of homogeneity of the utility function. Of course, when $b = 0$, which as stated above can be taken to be equivalent to assuming logarithmic preferences, we are back to expressions (C.5) and (C.6) in DPP (2007): $w(\bar{k}_N^*)\varphi'(\bar{e}_N^*) = f'(\bar{k}_N^*)\,(1 - \sigma_N^*)$ and $\sigma_N^* = \gamma(1-\lambda)$, which is independent of b. As a matter of fact, this is the only case where the parameter b becomes irrelevant when the planner's social welfare function depends on consumptions per unit of natural labour.

To stress this point again, the education subsidy is definitely positive, but its size depends on the precise cardinalization of preferences (i.e., the parameter b). From a mathematical point of view, this fact is not odd, since different objective functions will entail different solutions and thus different tax parameters that will decentralize them. However, this is a rather uncomfortable feature because different mathematical specifications representing the *same* economic problem translate into *different* optimal choices. Indeed, while the behavior of the individual is the same for all b, the social optimum and the optimal policy are not. Admittedly, it is unclear how the planner is going to posit one specific value of this parameter. Thus, the optimal policy becomes somewhat arbitrary. In contrast, as we show in the next subsection, this inconvenience does not emerge when the planner's objective function changes slightly.[4]

Before turning to the next subsection, however, it is worth stressing that the dependence of the optimal policy on the degree of homogeneity of the utility function is not inherent to the endogenous growth nature of the model. To see this, consider a model

[4]With respect to the optimal values of the lump sum-taxes T^1 and T^2 along the balanced growth path, although DPP (2007) argue that, in the triple Cobb-Douglas case, T^2 can have any sign, they also claim that, for reasonable weights given to future generations, the case for public pensions is weak (p. 373).

à la Diamond (1965) where, in addition to population growth at rate n, there is growth of labour efficiency, h_t, at a constant rate g, as a consequence of labour-augmenting or Harrod-neutral technical progress: $h_t = (1+g)h_{t-1}$. Since g is now exogenously given, and without any loss of generality, we can force $e_t = 0$ in (2) so that the planner chooses c_t, d_t, k_{t+1} that maximize (10) subject to the sequence of aggregate feasibility constraints (2) dismissing the amounts e_t. From the first order conditions, we obtain the Modified Golden Rule, i.e., $\gamma f'(.) = (1+g)^{1-b}(1+n)$ which amounts to (12) in the current context. Once again, the role of the parameter b is determinant for the characterization of the optimal allocation. This is in contrast with what happens in Diamond (1965)'s framework when technical progress is assumed away: the Modified Golden Rule then writes $\gamma f'(.) = 1+n$, this being independent of the precise cardinalization of the individual utility function.

3.2 An alternative approach

The preceding discussion invites to consider some alternative welfare criterion. In particular, it seems legitimate to explore the consequences of the social planner adopting a social welfare function that is respectful with individual preferences and gives rise to a social optimum which is independent of the specific cardinalization of such preferences. In this section we show that this can be done by postulating a social welfare function that is based on utilities of consumptions per unit of efficient labour. Observe that this was actually the procedure followed when writing expression (11) above: marginal utilities from consumption therein (and thus the marginal rate of substitution between them) were written in terms of consumptions per unit of efficient labour, i.e., the only variables that it makes sense to consider along a balanced growth path.

Since the utility function is homogeneous of degree b, we can take the monotonic transformation of U_t resulting from dividing by h_t in (1), and obtain a new utility index while ensuring that ordinal preferences are respected:

$$\bar{U}_t = U_t/h_t^b = (1/h_t^b)U(c_t, d_{t+1}) = U\left(c_t/h_t, d_{t+1}/h_t\right) = U(\bar{c}_t, \bar{d}_{t+1}) \tag{16}$$

It is worth emphasizing, first, that h_t is given at the beginning of period t, so that this procedure is nothing else but a mere change of variable. And, second, that the utility function in (16) has exactly the same functional form as (1), and thus continues to be homogenous of degree b. Otherwise, we can continue to posit that the social planner's objective is to maximize a discounted (with a discount factor $\gamma < 1$) sum of individual utilities, but now derived from consumptions per unit of efficient labour:

$$\bar{W} = \sum_{t=0}^{\infty} \gamma^t U(\bar{c}_t, \bar{d}_{t+1}) \tag{17}$$

A natural question is, of course, what is the relationship between W and $\bar{W}$. Using (16) and allowing for the fact that $h_t = \prod_{i=1}^{t}(1+g_i)h_0$, where the productivity growth rate g_i verifies $g_i = (h_i - h_{i-1})/h_{i-1}$, one has:

$$\bar{W} = \sum_{t=0}^{\infty} \frac{\gamma^t}{h_t^b} U(c_t, d_{t+1}) = \frac{1}{h_0^b} U(c_0, d_1) + \frac{1}{h_0^b} \sum_{t=1}^{\infty} \frac{\gamma^t}{\prod_{i=1}^{t}(1+g_i)^b} U(c_t, d_{t+1}) \tag{18}$$

Clearly, in addition to the conventional constant discount factor γ, the social welfare function (17), or, equivalently, (18) discounts the utility of future generation's consumptions at a variable rate that explicitly accounts for the level of human capital they are endowed with. In other words, the weight of successive generations in the new social welfare function is inversely related to their inherited knowledge.

The planner's problem is to maximize (17) subject to the sequence of aggregate feasibility constraints (3), for given values of $\bar{k}_0$, $\bar{d}_0, \bar{e}_{-1}$. As before, the optimum can be characterized by the first order conditions with respect to $\bar{c}_t$, $\bar{d}_t$, $\bar{k}_{t+1}$ and $\bar{e}_t$, and the transversality conditions. The derivation of the first order conditions follows the lines of DPP (2007). Let us write the Lagrangian:

$$L = \sum_{t=0}^{\infty} \gamma^t \left[U(\bar{c}_t, \bar{d}_{t+1}) - \frac{\lambda_t}{\gamma^t} \left(\bar{c}_t + \frac{\bar{d}_t}{\varphi(\bar{e}_{t-1})(1+n)} - f(\bar{k}_t) + (1+n)\varphi(\bar{e}_t)\bar{k}_{t+1} + (1+n)\bar{e}_t \right) \right] \tag{19}$$

where λ_t is the Lagrange multiplier associated with the resource constraint at time t. From the first order conditions corresponding to $\bar{c}_t$, $\bar{d}_t$, $\bar{k}_{t+1}$, $\bar{e}_t$ and λ_t we obtain

$$\frac{\partial U(\bar{c}_t, \bar{d}_{t+1})/\partial \bar{c}_t}{\partial U(\bar{c}_t, \bar{d}_{t+1})/\partial \bar{d}_{t+1}} = f'(\bar{k}_{t+1}) \tag{20}$$

$$\frac{\partial U(\bar{c}_t, \bar{d}_{t+1})/\partial \bar{c}_t}{\partial U(\bar{c}_{t-1}, \bar{d}_t)/\partial \bar{d}_t} = \frac{(1+n)\varphi(\bar{e}_{t-1})}{\gamma} \tag{21}$$

$$\varphi'(\bar{e}_t) \left(\frac{\bar{d}_{t+1}}{f'(\bar{k}_{t+1})\varphi(\bar{e}_t)(1+n)} - \bar{k}_{t+1} \right) = 1 \tag{22}$$

as well as (3). The interpretation of (20) and (21) is straightforward. The first one reflects the equality of the intertemporal rates of substitution in consumption and of transformation in production between periods t and $t+1$. The second one captures the static conditions of optimal distribution of consumption available in period t between middle-aged and old-aged individuals. Expression (22) can be interpreted as an arbitrage condition between the returns from investing on physical capital and on education. The intuition can be grasped making use of the fact that λ_t (resp. λ_{t+1}) is the shadow value (in terms of social welfare $\bar{W}$) of a unit of output per efficient labour in period t (resp. $t+1$). Suppose that in period t the social planner slightly increases $\bar{k}_{t+1}$. It is clear from (3) that this will affect the feasibility constraint at periods t and $t+1$: a higher $\bar{k}_{t+1}$ implies a reduction in the resources left for consumption in period t, given by $(1+n)\varphi(\bar{e}_t)$, and an increase of the resources available for consumption in period $t+1$, captured by $f'(\bar{k}_{t+1})$. Thus, the marginal cost of investing in physical capital is $\lambda_t(1+n)\varphi(\bar{e}_t)$ and the marginal benefit is $\lambda_{t+1}f'(\bar{k}_{t+1})$. The first order condition for $\bar{k}_{t+1}$ imposes that these marginal cost and benefit should be equal at the optimum. If, instead, the planner increases $\bar{e}_t$, the feasibility constraints at periods t and $t+1$ will also be modified. The cost, incurred in period t, has now two components. On the one hand, there is a direct cost, $(1+n)$, that reduces consumption possibilities. But there is also an indirect cost, given by $\varphi'(\bar{e}_t)(1+n)\bar{k}_{t+1}$: as a consequence of the effect of $\bar{e}_t$ on the growth rate, the amount of output devoted to investment in physical capital must be increased if we are to achieve the optimal value of $\bar{k}_{t+1}$. Using the shadow value λ_t, the marginal cost of an increased investment on education is thus $\lambda_t[(1+n)+\varphi'(\bar{e}_t)(1+n)\bar{k}_{t+1}]$. The benefits, however, do not take place until period $t+1$. Indeed, evaluating (3) at $t+1$, the increased growth rate lowers the marginal rate of transformation between third and second period consumption in the RHS. This amounts to an expansion of consumption possibilities, so that the marginal benefit is $\lambda_{t+1}\varphi'(\bar{e}_t)(1+n)\bar{d}_{t+1}/[\varphi(\bar{e}_t)(1+n)]^2$. As before, the first order condition for $\bar{e}_t$ imposes that these marginal costs and benefits should be equal at the optimum. Both first order conditions involve the same ratio of shadow values, λ_t/λ_{t+1}, so that an arbitrage condition between the returns from investing on $\bar{k}_{t+1}$ and $\bar{e}_t$, measured in units of resources at $t+1$ per unit of resources at t, can be derived. This is precisely the way expression (22) is obtained.

In order to identify the nature of the education externality it proves practical to use the feasibility constraint (3) evaluated at $t+1$ to write the arbitrage condition (22) in a way that resembles (5):

$$w(\bar{k}_{t+1})\varphi'(\bar{e}_t) = f'(\bar{k}_{t+1})\left(1+\frac{\varphi'(\bar{e}_t)}{f'(\bar{k}_{t+1})}\left[(1+n)\varphi(\bar{e}_{t+1})\bar{k}_{t+2}+(1+n)\bar{e}_{t+1}+\bar{c}_{t+1}\right]\right) \quad (23)$$

Since the individual always behaves according to the utility function (1) regardless of the social welfare function adopted by the planner, one has to compare (23) with (5). Then, it becomes clear that we are facing a situation that we can characterize as a *negative* externality. Needless to say, this is in sharp contrast with DPP (2007).

Before turning to explain the intuition underlying this result, we can characterize the optimal balanced growth path, where all variables expressed in terms of output per unit of efficient labour, $\bar{c}$, $\bar{d}$, $\bar{k}$ and $\bar{e}$, remain constant. Letting the superscript E refer to the case where the planner cares for utility defined over consumption per unit of efficient labour, the optimal balanced growth path can be characterized by the following equations:

$$\frac{\partial U(\bar{c}_*^E,\bar{d}_*^E)/\partial\bar{c}}{\partial U(\bar{c}_*^E,\bar{d}_*^E)/\partial\bar{d}} = f'(\bar{k}_*^E) \quad (24)$$

$$\frac{\gamma f'(\bar{k}_*^E)}{1+n} = \varphi(\bar{e}_*^E) \tag{25}$$

$$w(\bar{k}_*^E)\varphi'(\bar{e}_*^E) = f'(\bar{k}_*^E)\left(1 + \frac{\varphi'(\bar{e}_*^E)}{f'(\bar{k}_*^E)}\left[(1+n)\varphi(\bar{e}_*^E)\bar{k}_*^E + (1+n)\bar{e}_*^E + \bar{c}\right]\right) \tag{26}$$

in addition to the balanced growth path version of (3). Together, these four equations provide the optimal levels of $\bar{c}$, $\bar{d}$, $\bar{k}$ and $\bar{e}$.

Conditions (24) and (25) have the exact same interpretation as their counterparts (11) and (12). The second one, in particular, states that, at the Modified Golden Rule, the marginal product of physical capital will be greater than the economy's growth rate. Yet, the Modified Golden Rule (25) is independent of the specific cardinalization of the individual preferences. In other words, the degree of homogeneity b is now irrelevant. The reason why different Modified Golden Rules result under the two approaches is simply that the social planner is applying different discount factors to individual utility levels defined over consumption per unit of natural labour. To see this, assume for a moment that the rate of productivity growth is exogenously given at g (although, of course, a similar reasoning applies when g is endogenous). Observe that, when the social welfare function (17) is rewritten as (18), i.e., in terms of consumptions per unit of natural labour, the discount factor is $\gamma' = \gamma/(1+g)^b$. Thus, for a given rate of productivity growth, plugging γ' into (12) one obtains $[\gamma/(1+g)^b]f'(.)/(1+n) = (1+g)^{1-b}$, which is nothing else but $\gamma f'(.)/(1+n) = (1+g)$, that is, condition (25). This is the counterpart, with productivity growth, of other well known instances in optimal growth theory without productivity growth, where using different discount factors yields different optimum allocations.[5]

Coming back to our endogenous growth framework, and as we have already pointed out, condition (26) is nothing else but the arbitrage condition (22) written in a way that allows to compare it with the individual decision. It is then clear that, if $\sigma = 0$ and the individual were confronted with the (optimal) wage and interest rates, $w(\bar{k}_*^E)$ and $f'(\bar{k}_*^E)$, she would fail to take into account the following costs associated with an increase in the investment on education: (i) the investment in physical capital (per unit of efficient labour) required to keep the optimal $\bar{k}$, $\bar{k}_*^E$, constant along the balanced growth path, $(\varphi(\bar{e}_*^E)(1+n)\bar{k}_*^E)$, (ii) its counterpart, referred to $\bar{e}_*^E$, i.e., the investment in human capital required to keep $\bar{e}$ constant at its optimal balanced growth path level $\bar{e}_*^E$ $((1+n)\bar{e}_*^E)$, and (iii) the total consumption of the middle aged (per unit of efficient labour) necessary to keep $\bar{c}$ constant. In these circumstances, as the individual does not account for these

[5] In a model à la Diamond (1965) without productivity growth, consider the alternative social criteria of maximizing a discounted sum of the utility indices attained by a *representative* individual in every generation, $\sum_{t=0}^{\infty}\gamma^t U_t$, or maximizing a discounted sum of the utility levels enjoyed by *all the members* of every generation, $\sum_{t=0}^{\infty}\gamma^t N_0(1+n)^t U_t$. One can easily show that, in the former case, the Modified Golden Rule entails $\gamma f'(.) = (1+n)$. With respect to the latter, it can be analyzed in terms of the former by rewriting the discount factor as $\gamma' = \gamma(1+n)$. Thus, under "the more, the merrier" approach, the modified Golden Rule becomes $\gamma f'(.) = 1$. This parallels the reasoning given in the main text.

costs, she over-invests in education and a tax is required.

From the above discussion we can identify the value of the tax parameter addressed to education decisions along the optimal balanced growth path. Observe that by virtue of the homotheticity of preferences, (24) allows to express the ratio $\bar{c}^E/\bar{d}^E$ as a function of $f'(\bar{k}_*^E)$ and, using (25), of the growth rate of the economy $\varphi(\bar{e}_*^E)(1+n)$. Using the balanced growth path version of (3), one readily obtains $\bar{c}_*^E$ as a function of $\bar{k}_*^E$ and $\bar{e}_*^E$, i.e., $\bar{c}(\bar{k}_*^E, \bar{e}_*^E)$. Then, comparing (26) with (5), it becomes apparent that the optimal subsidy σ_*^E is *negative*:

$$\sigma_*^E = -\frac{\varphi'(\bar{e}_*^E)}{f'(\bar{k}_*^E)}\left[(1+n)\varphi(\bar{e}_*^E)\bar{k}_*^E + (1+n)\bar{e}_*^E + \bar{c}(\bar{k}_*^E, \bar{e}_*^E)\right] < 0 \tag{27}$$

It is also important to note that education expenditures should not only be taxed along the optimal *balanced* growth path but also along the *entire* optimal growth path. This follows in an obvious way from the second term in parentheses in the RHS of (23).

To conclude, one can ask if there is a systematic underlying relationship between $\bar{k}_*^N$ and $\bar{k}_*^E$, on the one hand, and $\bar{e}_*^N$ and $\bar{e}_*^E$, on the other. Such a systematic relationship does, however, not exist. Comparing these values is not an easy task, because $\bar{k}_*^N$ (resp. $\bar{k}_*^E$) and $\bar{e}_*^N$ ($\bar{e}_*^E$) are simultaneously determined by different systems of equations. We have used the "triple Cobb-Douglas case" to undertake these comparisons. The simulation results suggest that any combination of relative orderings is possible. This is not surprising for two reasons. First, the degree of homogeneity b is present in (12) but not in (25). Second, the rate of time preference δ, used in the Cobb-Douglas example as reported in subsection 3.1, which plays no role whatsoever in (11)-(13), is however present in (24)-(26) through the $\bar{c}(\bar{k}_*^E, \bar{e}_*^E)$ function.

Two further comments seem in order. The first one concerns the parallel of DPP (2007)'s claim that in their framework the case for public pensions (i.e., $\bar{T}_*^{2N} < 0$) is weak. As shown in the Appendix, under the alternative approach discussed in this subsection, nothing can be said with generality about the sign of $\bar{T}_*^{2E}$. The second comment refers to the consequences of the social planner adopting the social welfare function (17) in a model with exogenous productivity growth (i.e., an exogenously given value of g and $e_t = 0$ for all t). It can easily be verified that the Modified Golden Rule is then given by $\gamma f'(.) = (1+g)(1+n)$. As in (25), the degree of homogeneity b of the individual utility function is irrelevant for the characterization of the optimal balanced growth path. The fact that the Modified Golden Rule does not depend on the parameter b is clearly not a characteristic of the endogenous growth nature of the model above.

4 Concluding remarks

In this paper, we have used an OLG model with physical and human capital to ascertain the consequences for optimality of a social planner adopting two different welfare criteria that are respectful of individual preferences. In both cases, the SWF is a discounted sum of individual utilities, the difference being whether consumption is considered in terms of output per unit of natural or efficient labour. The results suggest that, in a sense, we are forced to choose between two alternative scenarios. On the one hand, if the planner maximizes a SWF with individual utility functions defined over consumption per unit of natural labour, we require a precise cardinalization of preferences, this cardinalization playing a crucial role in the resulting size of the education subsidy. On the other hand, if the planner maximizes a SWF with individual utility functions defined over consumption in terms of units of efficient labour, we do not require any cardinalization but, now, along the balanced growth path, the government should impose a tax (instead of a subsidy) on education.

To conclude, it should be stressed that although this paper has focused on the implications of the social planner adopting two different welfare criteria, one should not forget that some of the assumptions underlying the model are quite unrealistic. In particular this can be said of the assumption that individuals have access to perfect credit markets. Indeed, the insights emerging from the analysis may be different depending on whether or not individuals face constraints when trying to borrow to finance their education investments. We leave these issues for further research.

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Appendix: The sign of $\bar{T}^2$

Along a balanced growth path, (4) can be writen using (7)

$$\frac{\bar{d}}{f'(\bar{k})} = w - \frac{f'(\bar{k})\bar{e}(1-\sigma)}{\varphi(\bar{e})} - \bar{T}^1 - \frac{\varphi(\bar{e})(1+n)(1+n)\sigma\bar{e}}{f'(\bar{k})} + \frac{\varphi(\bar{e})(1+n)\bar{T}^1}{f'(\bar{k})} - \bar{c} \tag{28}$$

From (9), also along a balanced growth path,

$$\varphi(\bar{e})(1+n)\bar{k} + (1+n)(1-\sigma)\,\bar{e} = w - \frac{f'(\bar{k})\bar{e}(1-\sigma)}{\varphi(\bar{e})} - \bar{T}^1 - \bar{c} \tag{29}$$

Evaluating (28) and (29) along the optimal balanced growth path, and using (25):

$$\frac{\varphi'(\bar{e}_*^E)}{\varphi(\bar{e}_*^E)/\bar{e}_*^E} - 1 - \varphi'(\bar{e}_*^E)\left(\frac{\sigma_*^E\bar{e}_*^E}{\varphi(\bar{e}_*^E)} + \frac{(1+n)\sigma_*^E\bar{e}_*^E}{f'(\bar{k}_*^E)} - \frac{\bar{T}_*^{1E}}{f'(\bar{k}_*^E)}\right) = 0 \tag{30}$$

On the one hand, the concavity of $\varphi(\bar{e})$ implies that $\frac{\varphi'(\bar{e}_*^E)}{\varphi(\bar{e}_*^E)/\bar{e}_*^E} - 1 < 0$. Then, it has to be the case that

$$\frac{\sigma_*^E\bar{e}_*^E}{\varphi(\bar{e}_*^E)} + \frac{(1+n)\sigma_*^E\bar{e}_*^E}{f'(\bar{k}_*^E)} - \frac{\bar{T}_*^{1E}}{f'(\bar{k}_*^E)} < 0$$

From the government budget constraint (7) expressed in efficiency units and along the optimal balanced growth path, this is equivalent to

$$\frac{\sigma_*^E\bar{e}_*^E}{\varphi(\bar{e}_*^E)} + \frac{\bar{T}_*^{2E}}{f'(\bar{k}_*^E)\varphi(\bar{e}_*^E)(1+n)} < 0$$

which, since $\sigma_*^E < 0$, is compatible with any sign of $\bar{T}_*^{2E}$.