Optimal Redistribution with Heterogeneous Agents

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Abstract

In this paper we investigate what is the optimal timing of transfers when agents differ according to their productivity. We use a two period overlapping generations general equilibrium framework to assess the various ramifications of transferring resources from the rich to the poor. We consider whether the redistributive transfers should be in the form of income support to the young or pensions to the old. Our findings suggest that though redistribution imposes costs on the economy, mostly in terms of capital accumulation, intragenerational transfer is always welfare enhancing. Using intergenerational transfer can also enhance welfare albeit the distortionary costs to the economy are higher. The optimal timing and amount of transfer depends to a large extent on the initial endowment of the agents and the proportions of agents with low and high productivities respectively.

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1 Introduction

The role of governments in economies has been widely debated by the various schools of thought. However, there is some agreement that there are some basic functions that a government needs to undertake to ensure a smooth running of an economy. Whilst the need to address pervasive market failures in terms of externalities and inefficiencies are among the most important, redistributing income has also been at the fore of the policy debate. The need for redistribution can at least be traced back to Adam Smith (1776) who suggested, in The Wealth of Nations, "*No society can surely be flourishing and happy, of which the far greater part of the members are poor and miserable*".

There is a wide array of the literature that has focused, directly or indirectly, on inequality and the various channels through which it affects growth and welfare since Kuznets (1955). Since economies are seldom homogeneous, with agents differing across skills and asset endowments to highlight but two, there is a growing belief that redistribution can have major politico-economic ramifications and implications for policy. Redistribution is often considered as one of the routes through which social justice and efficiency can be promoted by reducing inequality and supporting those at the lower end of the economy. Besides the philanthropic arguments, there have been growing concerns that inequality can be harmful for growth and too skewed a distribution of assets and income can have damaging socio-economic ramifications. (Persson and Tabellini (1994); Alesina and Rodrik (1994); Alesina and Perrotti (1996)).

Whilst the most conventional way to redistribute from the rich to the poor is by taxing the rich and transferring it in the form of income support to the poor, in this paper we consider the additional possibility as to whether there are conditions under which it might be optimal to redistribute through a pay-as-you-go[1] (PAYG) pensions scheme. In so doing, we move from the intergenerational transfer to intragenerational transfer and consider whether there is an optimal level of transfer through the pensions system which yields a higher level of welfare than income support. Intragenerational transfer is the instrument *par excellence* to bridge the gap between the rich and the poor and there is thus a limited number of papers that deal explicitly with

[1]Although the term PAYG is used, in a somewhat general sense, the pensions system is in fact one with "targetting" where only the poor receive pensions.

how intergenerational transfers can be used to reduce inequality[2]. Haveman (1988) argues that intragenerational inequality reducing transfer works best since it increases the opportunities of the young poor. Our work could also be seen as a departure from the consumption smoothing role of pensions, as in Feldstein (1985), or cases where pensions is motivated by altruism (Hansson and Stuart, 1989) or that of risk-sharing as prescribed by Shiller (1999), Conesa and Krueger (1999) and Bohn(1999).

In our framework, pensions provides some form of social insurance and thus helps alleviate poverty and/or reduce inequality. We address this issue by developing an overlapping generations (OLG) general equilibrium model with endogenous capital formation to take account of some of the distortionary elements of redistribution. Our model also provides for a richer analytical framework in that we do not restrict our work to the existence and optimal level of transfer in a PAYG system but also consider the instances where a fully funded system yields a higher level of welfare. Crucial to our redistribution argument is an element of heterogeneity in the form of productivity of the poor relative to the rich. In this setting, the rich fund for their own pensions and they pay a proportion of their wages to the planner in the form of taxes. The planner is then faced with the problem of deciding how best to allocate a non-negative transfer to the poor. We assume this can take the form of either income support or alternatively a pay-as-you-go (PAYG) pensions scheme[3]. By definition, if the means of redistribution is taken to be income support, then the poor have to save for their own retirement and the system would be fully funded (FF). On the other hand, if the redistribution is done through pensions, then the system would be a hybrid with the pensions of the rich operating under a FF scheme whilst that of the poor would be under a PAYG scheme. Our paper thus bring together two approaches that have usually been considered on an individual basis.

We develop a two-period OLG model for a steady state economy to illustrate the various arguments. Though one of the main most referred

[2]Krueger and Kubler (2006) look at intergenerational transfers but more from the perspective where markets are missing.

[3]Though we only consider the transfer in either one form or another, it is also possible for the planner to provide the income to the poor during the two periods of their lives. A poor agent would thus receive income support when young and pensions when old. However, though this may add to the analytical part of the paper, it is not entirely pertinent to the question we are trying to answer: whether there is a role for pensions in this economy.

to characteristics of OLG models is the double infinity of agents and time (Shell, 1971) whilst the planning horizon of each agent is finite (Galor and Polemarchakis, 1985), we nonetheless focus on the steady state, rather than the whole economy, since the steady state can be considered as the representative framework within which infinitely many generations evolve (Galor, 1992). The main advantage of this simple, yet insightful, set up is that with some simplifying assumptions, it allows for a greater degree of analytical tractability that yield clear and intuitive policy insights. One of the main differences between income support and PAYG is the generational implications. Whilst income support consists of a transfer from young to young, that is an intragenerational transfer, a PAYG scheme is a transfer from young to old, that is an intergenerational transfer. These obviously have implications for the various economic aggregates we consider in this paper, the most important of which is capital formation. Intragenerational redistribution can adversely affect capital formation when the poor discount the future at a higher level than the rich. Intergenerational redistribution unambiguously leads to a lower level of savings for the poor and reduces the capital stock. Given the dynamics generated by the choice of redistribution from the rich to the poor, we aim to find out which of the two schemes yields the highest welfare.

Our results confirm earlier findings that the existence of a PAYG system leads to a lower level of savings and a crowding out of capital (Feldstein, 1974). Whilst this might be the best policy for an economy characterised by dynamic inefficiency, that is one where there has been an overaccumulation of capital and the prevailing interest rate is less than the population growth rate, and welfare can be increased by reducing capital (Samuelson, 1975), in a dynamically efficient economy, a PAYG social security scheme leads to a lower capital stock and a higher level of interest rate and a lower level of output and wages. With redistribution, we find that the capital stock is unchanged when there is intragenerational redistribution(as opposed to intergenerational transfers) with the agents having the same discount rate. Under a given set of parameters, there is a potential for overaccumulation and dynamic inefficiency to creep in. Although redistribution introduces some elements of distortion in the economy, we find that intragenerational transfers always lead to an increase in welfare compared to the competitive equilibrium. This can be rationalised on account of the fact that the welfare gains of the poor as a result of the transfers at least exceeds the loss of the rich as a result of the taxes imposed to fund the redistribution and the other distortions this induces. Though transfers through pensions are also welfare enhancing, there can be a set of (extreme) parameters for which the competitive equilibrium yields a higher welfare. Our main policy findings suggest that it is optimal to transfer resources from the young (rich) to the young (poor) when an economy is characterised by a small proportion of rich and the poor have a low productivity. Otherwise, in an economy where there is no wide differences in the wealth of the rich and the productivity of the poor, a PAYG might deliver a higher level of aggregate welfare.

The rest of this paper is as follows: in Section 2 we describe the set up of the economy. Section 3 derives the competitive equilibrium whilst Section 4 considers the planner's problem in terms of finding the optimal level and timing of transfer. Section 5 concludes.

2 The Economy

Economic activity takes place over infinite discrete time $t\{0, 1..., \infty\}$ without uncertainty. The economy consists of two types of agents $h(r, p)$, firms and a central planner. At each time t, two factors, an amount of capital, k, and labour, l, are available as inputs to production and a homogeneous good is produced.

2.1 Social Planner

The social planner is assumed to be a benevolent welfare maximiser. Its role is limited to that of raising taxes from the currently active rich and transferring it to the poor, young or old, depending on which mode yields the highest welfare level. The total taxes (T) raised can be represented as a proportion of wages as follows: $T_t = \pi L_t \tau w_t^r$. If this is redistributed to the young (poor), the total benefits (B) they receive is: $B_t = (1-\pi) L_t \tau w_t^r$. If the transfer is done as a PAYG pensions scheme, the poor thus receive: $B_t = (1-\pi) L_{t-1} \tau w_t^r$. With a stationary population and commitment device, a poor agent is thus faced with the following benefit levels:

$$\text{FF:} \qquad b_1 = \frac{\pi \tau_1 w_1^r}{(1-\pi)} \tag{1}$$

$$\text{PAYG:} \qquad b_2 = \frac{\pi (1+n) \tau_2 w_2^r}{(1-\pi)} \tag{2}$$

Since the redistribution affects the capital accumulation process, we can safely assume that the wages will differ in both settings and the optimal level of tax rate will also be different. Hence we can also infer that the level of benefits, b, would be different under both settings. Some basic comparative statics reveal that under both settings, $\partial b/\partial \pi = 1/(1-\pi)^2 > 0$ suggesting that as π, the proportion of rich increases, the level of benefits b increase as well. For the PAYG system, $\partial b/\partial n = \pi\tau w^R/(1-\pi) > 0$, implying that as the population growth rate n increases, the generosity of the pensions scheme increases as well.

2.2 Firms

A large number of identical firms produce a homogeneous good using an identical economy-wide Cobb Douglas production function of the form $y = k^\alpha$, where α is the share of capital in production. Firms maximise profit by taking factor prices, which are paid their marginal products in a competitive setting, as given. It is assumed that the labour market clears such that labour demand equals labour supply and the wages received by a worker depends on his level of productivity. The economy is endowed with an initial capital stock $K_0 > 0$ and capital depreciates fully from one period to the next. Without loss of generality, we assume no technological change. The production function satisfies the usual conditions such that $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$ and the Inada conditions: $\lim\limits_{k\to 0} f'(k) = \infty$ and $\lim\limits_{k\to\infty} f'(k) = 0$.

2.3 Agents

At t, two generations live simultaneously - one generation is young and the other is old. Population (P) grows at a constant rate n and therefore, at any time t, there are $(1+n)$ more (young) workers than (old) retirees. The population at t can thus be expressed as: $P_t = L_t + L_{t-1} = (2+n)L_t$. L_t refers to the agents born at t. Following Samuelson (1958), the population is considered to be stationary. Therefore, the proportion and type of individuals remains the same across generations.

Each young agent is endowed with one unit of labour which he provides inelastically when young. The labour endowment in the second period of life is zero. Agents differ according to their productivity $\psi \in (0, 1]$, which in turn determines the wage they receive. To simplify the argument, the productivity of the rich (r) is normalised to 1 and hence any agent with $\psi < 1$

is considered as poor (p). We assume that the rich make up a proportion π of the economy and hence by definition, the poor make up the remaining $(1-\pi)$.

Agents derive utility solely out of consumption and they are non-altruistic. They are thus born without assets and do not leave bequests. At time t the young agent chooses his level of consumption and savings to maximise utility whilst the old agent lives off his savings (and any transfer).

The inter-temporal optimisation problem can be expressed as maximising $u^h(c_t^h, c_{t+1}^h)$ subject to the budget constraints which vary according to the individual's type and mode of transfer in operation. The intertemporally additive lifetime utility function is taken to be log-linear. The utility function is strictly concave, since more consumption is preferred to less, and twice differentiable: $u'(c) > 0$ and $u''(c) < 0$. The function also satisfies $\lim_{c\to 0} u'(c) = \infty$ such that subject to its disposable income, the household will always choose a positive level of consumption when maximising life-cycle utility. The rich agent's problem can be expressed as:

$$\underset{\{c_t^y, c_{t+1}^o, s_t\}}{Max} : U^r = \ln c_t^{y,r} + \beta \ln c_{t+1}^{o,r} \tag{3}$$

subject to:

$$\begin{aligned} c_t^{y,r} &= w_t^{y,r}(1-\tau) - s_t^{y,r} && (4) \\ c_{t+1}^{o,r} &= R_{t+1} s_t^{y,r} && (5) \end{aligned}$$

Let us consider in the first case, the transfer from the young rich to the young poor. The poor agent's problem will thus be of the following form:

$$\underset{\{c_t^y, c_{t+1}^o, s_t\}}{Max} : U^p = \ln c_t^{y,p} + \theta \ln c_{t+1}^{o,p} \tag{6}$$

The maximisation problem of the two agents are broadly similar except for the subjective discount rate, proportionally related to patience: $\theta \leq \beta \in (0,1)$. Whilst both agents provide one unit of labour inelastically, the budget constraints differ in the way the rich are taxed a proportion $\tau \in (0,1)$ out of their income and this is redistributed to the poor. Disposable income is then allocated *à la* Diamond (1965) between present consumption and savings. $R \equiv (1+r)$ is the gross rate of return on savings. Irrespective of the transfer mode, the rich consume only their savings in retirement. In turn, the budget constraints of the poor will depend on the mode of transfer in operation. With redistribution in the first period, their consumption is as follows:

$$
\begin{aligned}
c_t^{y,P} &= w_t^{y,p} + b_t - s_t^{y,p} && (7)\\
c_{t+1}^{o,P} &= R_{t+1} s_t^{y,P} && (8)
\end{aligned}
$$

where b_t represents the redistribution from the rich to the poor. We find that under this scheme, the poor also have only their savings to rely on when old. The pensions system in this set up thus approximates to a fully-funded scheme where everyone is responsible for the provision of their own consumption in retirement through theis savings. On the other hand, if the redistribution takes place in the second period of the lifetime, the consumption of the poor will be:

$$
\begin{aligned}
c_t^{y,P} &= w_t^{y,P} - s_t^{y,P} && (9)\\
c_{t+1}^{o,P} &= R_{t+1} s_t^{y,P} + b_{t+1} && (10)
\end{aligned}
$$

where b_{t+1} represents the redistribution from the young rich to the old poor.

3 Competitive Equilibrium

Given the households' and the firms' objectives, a competitive equilibrium for the economy can be defined as a sequence of consumption $\{c_t^y, c_t^o\}_{t=0}^{\infty}$ such that:

1. Given a sequence of taxes and transfers, $\{\tau w_t, b_t\}_{t=0}^{\infty}$, and the prevailing competitive wages, w_t, and interest rate, r_t, solves the individual's optimisation problem subject to satisfying the Euler equation;
2. Factors of production are paid their marginal products ($w_t = (1-\alpha)k_t^{\alpha}; R_t = \alpha k_t^{\alpha-1}$) and labour and capital markets clear such that $L_t^D = L_t$ and $S_t = K_{t+1}$;
3. Irrespective of the mode of transfer, the planner's budget is always under balance hence taxes raised is redistributed as benefits in the same period $T_t = B_t$;

4. The economy's resource constraint is always satisfied. In intensive form, the constraint which is defined as the allocation of current output, y_t,

$$y_t = c_t^y + \frac{c_t^o}{(1+n)} + (1+n)k_{t+1} \tag{11}$$

The budget constraint suggests that output at any time is divided between consumption and capital formation. Consumption consists of that of the young and the old.

Given the above definition of competitive equilibrium, the agent has to choose his level of savings subject to the budget constraint to maximise utility. The intertemporal budget constraint (IBC) suggests the present value of lifetime consumption equals lifetime income. The IBC of the rich can be expressed as:

$$c_t^{y,r} + \frac{c_{t+1}^{o,r}}{R_{t+1}} = w_t^r(1-\tau) \tag{12}$$

The IBC of the poor will vary with the mode of transfer - with income support, there is no discounting, whilst any pensions received as an elderly will be discounted. The IBCs under the two settings can thus be written as:

$$\text{FF} \quad : \quad c_t^{y,p} + \frac{c_{t+1}^{o,p}}{R_{t+1}} = w_t^{y,p} + b_t \tag{13}$$

$$\text{PAYG} \quad : \quad c_t^y + \frac{c_{t+1}^o}{R_{t+1}} = w_t^h + \frac{b_{t+1}}{R_{t+1}} \tag{14}$$

Given the intertemporal budget constraint, we can solve for the competitive equilibrium for the poor when the transfer takes place in the first period. The Lagrangian and the first order conditions are:

$$\underset{\{c_t^y, c_{t+1}^o\}}{Max} : \ell = \ln c_t^{y,p} + \theta \ln c_{t+1}^{o,p} - \lambda[c_{t+1}^{o,p} - R_{t+1}\{w_t^{y,p} + b_t - c_t^{y,p}\}] \tag{15}$$

$$\frac{\partial \ell}{\partial c_t^y} \quad : \quad \lambda R_{t+1} c_t^y = 1 \tag{16}$$

$$\frac{\partial \ell}{\partial c_{t+1}^o} \quad : \quad \theta c_{t+1}^o = \lambda \tag{17}$$

where λ represents the Lagrangian multiplier. Combining the two first order conditions results in the Euler equation, which is the optimal allocation of consumption during the two life periods of the agents:

$$c_{t+1}^{o} = \theta R_{t+1} c_t^y \tag{18}$$

Using the Euler equation, the optimal consumption and savings of a poor utility maximising agent who receives a transfer in the first period of his life can be described as:

$$\begin{aligned} c_t^{y,p} &= \left(\frac{1}{1+\theta} \left[w_t^p + b_t \right] \right) && (19) \\ c_{t+1}^{o,p} &= \left(\frac{\theta R_{t+1}}{1+\theta} \left[w_t^p + b_t \right] \right) && (20) \\ s_t^{y,p} &= \left(\frac{\theta}{1+\theta} \left[w_t^p + b_t \right] \right) && (21) \end{aligned}$$

Using the same approach, the optimal level of consumption and savings of the poor who receive a transfer in the second period of their life in the form of a PAYG pensions scheme can be expressed as:

$$\begin{aligned} c_t^{y,p} &= \left(\frac{1}{1+\theta} \left[w_t^p + \frac{b_{t+1}}{R_{t+1}} \right] \right) && (22) \\ c_{t+1}^{o,p} &= \left(\frac{\theta R_{t+1}}{1+\theta} \left[w_t^p + \frac{b_{t+1}}{R_{t+1}} \right] \right) && (23) \\ s_t^{y,p} &= \left(\frac{1}{1+\theta} \left[\theta w_t^p - \frac{b_{t+1}}{R_{t+1}} \right] \right) && (24) \end{aligned}$$

The main difference between the two modes of transfer for the poor is the fact that pensions causes savings to be lower - potentially crowding out capital. We first consider the optimal level of consumption and savings of the rich - which can be expressed in a similar manner under both settings since the rich only pay out taxes and receive no benefits. However, the fundamental difference will arise when the tax rates τ_1 and τ_2 are different under the two settings.

$$c_t^{y,r} = \left(\frac{1}{1+\beta}\left[w^h(1-\tau)\right]\right) \tag{25}$$

$$c_{t+1}^{o,r} = \left(\frac{\beta R_{t+1}}{1+\beta}\left[w^h(1-\tau)\right]\right) \tag{26}$$

$$s_t^{y,r} = \left(\frac{\beta}{1+\beta}\left[w^h(1-\tau)\right]\right) \tag{27}$$

It can be seen that if the agents do not discount the future, that is, $\beta = \theta = 1$, then the agents consume exactly half of their income and save the remaining half. Conversely, if $\beta = \theta \approx 0$, then most of the consumption takes place during the first period of lifetime.

Having derived the optimal levels of consumption and savings, it is now possible to consider the capital stock formation. For comparative purposes, the capital stocks under three settings are derived: without redistribution (k_0); with redistribution in first period (k_1) and second period (k_2) respectively. In all three cases, with complete depreciation of capital from one period to the next, the capital formation process follows:

$$(1+n)k_{t+1} = \pi s_t^{y,r} + (1-\pi)s_t^{y,p} \tag{28}$$

implying that the capital available per worker in the current period is the savings of the rich and the poor carried over from the previous period. We thus find that the proportions of rich and poor play an important role in capital formation.

The steady state capital stock in the three cases are:

$$k_0 = \left(\frac{(1-\alpha)}{(1+n)}\left[\left(\frac{\beta\pi}{1+\beta}\right) + \left(\frac{\theta(1-\pi)\psi}{1+\theta}\right)\right]\right)^{\frac{1}{(1-\alpha)}} \tag{29}$$

$$k_1 = \left(\frac{(1-\alpha)}{(1+n)}\left[\left(\frac{\beta\pi(1-\tau_1)}{1+\beta}\right) + \left(\frac{\theta\left[(1-\pi)\,\psi + \pi\tau_1\right]}{1+\theta}\right)\right]\right)^{\frac{1}{(1-\alpha)}} \tag{30}$$

$$k_2 = \left(\frac{(1-\alpha)}{(1+n)}\left[\frac{(1+\theta)\,\alpha}{(1+\theta)\,\alpha + \pi\tau_2(1-\alpha)}\right]\left[\left(\frac{\beta\pi(1-\tau_2)}{1+\beta}\right) + \left(\frac{\theta(1-\pi)\psi}{1+\theta}\right)\right]\right)^{\frac{1}{(1-\alpha)}} \tag{31}$$

Some of the basic comparative statics can be summarised as: $\frac{\partial k_0}{\partial n} < 0$ obviously implying that as the population growth rate increases, capital available per work falls; $\frac{\partial k_0}{\partial \beta} > 0$; $\frac{\partial k_0}{\partial \theta} > 0$ increases in the discount rates will lead to increased savings and therefore increased capital formation ; $\frac{\partial k_0}{\partial \pi} > 0$ implying that as the proportion of rich increases, capital formation is higher; $\frac{\partial k_0}{\partial \psi} > 0$ suggests that an increase in the productivity of the poor leads to higher capital formation; $\frac{\partial k_1}{\partial \tau_1} < 0$ for $\beta \neq \theta$; $\frac{\partial k2}{\partial \tau_2} < 0$ reflect the distortionary costs of taxation.

To simplify the comparisons between the different levels of capital stock, consider the special case where all the agents have the same discount rate such that $\beta = \theta = \delta$. Then, the steady capital stocks can be simplified to:

$$k_0 = \left(\frac{\delta(1-\alpha)\left[\pi + (1-\pi)\psi\right]}{(1+\delta)(1+n)}\right)^{\frac{1}{(1-\alpha)}} \tag{32}$$

$$k_1 = \left(\frac{\delta(1-\alpha)\left[\pi + (1-\pi)\psi\right]}{(1+n)(1+\delta)}\right)^{\frac{1}{(1-\alpha)}} \tag{33}$$

$$k_2 = \left(\frac{\delta(1-\alpha)\alpha\left[\pi(1-\tau_2) + (1-\pi)\psi\right]}{(1+n)\left[(1+\delta)\alpha + \pi\tau_2(1-\alpha)\right]}\right)^{\frac{1}{(1-\alpha)}} \tag{34}$$

In this special case, it can be seen that the steady state capital stock k_0 and k_1 are the same. Hence it can be inferred that intragenerational redistribution does not affect the capital stock when the discount rates of the two types of agents are the same. For $\beta = \theta = \delta$; $k_0 = k_1 = k_2$ iff $\tau_2 = 0$.

It can also be inferred that, if $\beta \neq \theta$ such that $\beta > \theta$, then for any $\tau_1 > 0$ transfer in the first period results in $k_0 > k_1$. This can be shown by another extreme case such that by setting $\beta \approx 1$ and $\theta \approx 0$, then it can be seen that any $\tau > 0$ reduces k_1 whilst k_0 remains unaffected.

One of the basic tenets of critiques of a PAYG system is the fact that it reduces the capital stock (Feldstein, 1974). One of the channels through which this takes place is reduced savings. If this is the case, then $k_1 - k_2 > 0$. For the general case, this results in:

$$\underline{\{[\beta(1+\theta)\pi(1-\tau_1)] + [\theta(1+\beta)[(1-\pi)\psi + \pi\tau_1]]\}} > \frac{\{(1+\theta)\alpha[\beta(1+\theta)\pi(1-\tau_2) + \theta(1+\beta)(1-\pi)\psi]\}}{\{[(1+\theta)\alpha + \pi\tau_2(1-\alpha)]\}}$$

The special case $\beta = \theta = \delta$ can be reconsidered and it simplifies to the following condition:

$$\delta\pi\tau_2\left[\pi(1-\alpha)+(1-\pi)(1-\alpha)\psi+\alpha\left(1+\delta\right)\right]>0 \tag{35}$$

It follows that for all $\tau_2 > 0$, all the above conditions are always satisfied and hence $k_1 > k_2$.

Since, $k_1 > k_2$, and $w = (1-\alpha)k^{\alpha}$ and $R = \alpha k^{\alpha-1}$, it follows that $w'(k) > 0$ and $R'(k) < 0$.

4 The Planner's Problem

Given the competitive equilibrium, the planner's problem is to choose the tax rate τ so as to maximise $\{c_t^y, c_t^o, k_{t+1}\}_{t=0}^{\infty}$ subject to the allocation $\{y_t\}_{t=0}^{\infty}$. Abstracting from Ramsey's criticism of planners discounting of the future, the planner weighs the utility of all agents living in the economy and each generation, born or unborn, is given a weight, $\rho \in (0,1)$, which is decreasing in time. In the Benthamite tradition, we assume that the planner attaches the same weight to all agents living at a given point in time. Assuming the utility function remains the same across generations, the planner's social welfare function is thus:

$$W=\sum_{t=0}^{\infty}\rho^{t}\left(U^{h}\left(c^{h}\right)\right) \tag{36}$$

To enable a clearer comparison between the two modes of transfers, the economy will be considered to be in steady state. This assumption enables us to avoid the issue of having to include the first generation of retirees (beneficiaries) and workers (taxpayers) when the PAYG pensions scheme is set up at $t = 0$. Following Feldstein (1985), unless the future is discounted at a very high rate, the effect of the initial period over the long term will be relatively unimportant. Moreover, the steady state can reasonably be considered as the framework within which most of the agents will operate in an infinitely lived economy (Galor, 1992).

Normalising the first period's population to 1, the aim of the planner at t can be considered as choosing a sequence of $\{\tau, b\}_{t=0}^{\infty}$ so as to maximise welfare for all $t > 0$. The welfare function at t can thus be expressed as that of maximising the welfare of all living generations, young and old, rich and poor, simultaneously. Hence, $V_t = C = C^{y,h} + C^{o,h}$. This can be summarised as:

$$V = (1+n)\left[\pi \ln(c^{y,r}) + (1-\pi)\ln(c^{y,p})\right] + \left[\pi \ln(c^{o,r}) + (1-\pi)\ln(c^{o,p})\right] \tag{37}$$

The first part of the welfare function refers to the young generation, of which there are $(1+n)$ more than the elderly and proportions π and $(1-\pi)$ of rich and poor, respectively. The second part of the function refers to the elderly living in retirement. Based on the optimal levels of consumption from the competitive equilibrium and the equilibrium conditions for the transfers from the planner, the welfare functions for the two modes of transfers can now be elaborated.

Let V_1 be the welfare functions if the transfer is from young to young and V_2 represent welfare from young to old. In their simplified forms, the welfare functions V_1 and V_2 can be expressed as:

$$V1 = \left[\begin{array}{c} (2+n)\left[\pi \ln(1-\tau_1) + (1-\pi)\ln\left([\psi(1-\pi) + \pi\tau_1]\right)\right] \\ +\left[\alpha(3+n) - 1\right]\ln(k_1) + \mathbf{z_1} \end{array}\right] \tag{38}$$

$$V2 = \left[\begin{array}{c} (2+n)\left[\pi \ln(1-\tau_2) + (1-\pi)\ln\left(\psi(1-\pi)R_2 + \pi(1+n)\tau_2\right)\right] \\ +\left[(2+n)\left[(1+\pi)\alpha - \pi\right] + (1-\alpha)(1+n)\right]\ln(k_2) + z_2 \end{array}\right] \tag{39}$$

where $\mathbf{z_1} = {z_0}^{4} + \ln(\alpha)$ and $\mathbf{z_2} = z_0 + \left[(2+n)\pi - (1+n)\right]\ln(\alpha)$.

If redistribution increases welfare such that the $V1$ and $V2$ are at least equal to welfare under competitive equilibrium, the decision criteria for the planner can be summarised as follows:

(i) $V_1 - V_2 > 0$

If $V_1 - V_2 > 0$, it implies that the transfer in the first period yields the highest welfare for the economy and τ_1 is optimal. The system then approximates to a FF system, where each individual is responsible for providing for his own income in retirement.

[4]

$$z_0 = \pi\left[\ln(\beta) - \ln(1+\beta)\right] + (1-\pi)\left[\ln(\theta) - \ln(1+\theta) - \ln(1-\pi)\right]$$
$$-(1+n)\left[\pi \ln(1+\beta) + (1-\pi)\left\{\ln(1+\theta) + \ln(1-\pi)\right\}\right] + (2+n)\ln(1-\alpha)$$

(ii) $V_1 - V_2 = 0$

If $V_1 - V_2$, then both modes of transfer generate the same welfare and whether the redistribution takes place through τ_1 or τ_2 does not matter.

(iii) $V_2 - V_1 > 0$

Alternatively, if $V_2 - V_1 > 0$ then τ_2 is the optimal instrument to redistribute with. The transfer promotes a kind of a hybrid system where the rich fund their pensions under the FF scheme whilst the redistribution provides for the pensions of the poor under a PAYG scheme. Before analysing the decision rules, the optimal level of transfer is considered.

4.1 The Optimal Level of Redistribution

The optimal level of redistribution or transfer τ from the rich to the poor can be found by setting $\frac{dV}{d\tau} = 0$. Two cases are considered with a variation in assumptions about the discount rates. These simplifying assumptions, although at times considering extreme cases, enable to derive analytical solutions. The general cases are then solved via simulations.

4.1.1 The Golden Rule: Intragenerational Transfer

The Golden Rule, where the returns to capital, $R = (1+n)$, Samuelson's biological rate of interest, is considered. The capital stock when the transfer is from young to young is considered first and the steady state capital stock is defined as previously by Eqn. (30) and the welfare function $V1$ now summarises to:

$$V1 = \left[\begin{array}{c} (2+n)\left[\pi \ln(1-\tau_1) + (1-\pi)\ln\left([(1-\pi)\psi + \pi\tau_1]\right)\right] \\ +\frac{\alpha(2+n)}{(1-\alpha)} \ln\left[\beta\left(1+\theta\right)\pi - \theta\left(1+\beta\right)(1-\pi)\psi + (\theta-\beta)\,\pi\tau_1\right] + z_1 \end{array} \right] \tag{40}$$

where z_1[5] is a set of parameters independent of the policy term. The optimal tax can now be characterised as follows:

[5] $\mathbf{z}_1 = \mathbf{z}_0 + (2+n)\ln(1-\alpha) + \frac{\alpha(2+n)}{(1-\alpha)}\left[\ln(1-\alpha) - \ln(1+n) - \ln(1+\beta) - \ln(1+\theta)\right] + \ln(1+n)$

$$\frac{\partial V1}{\partial \tau} = (2+n)\pi \left[\begin{array}{c} \left(\frac{-1}{(1-\tau_1)} + \frac{(1-\pi)}{([(1-\pi)\psi+\pi\tau_1])} \right) + \\ \frac{\alpha}{(1-\alpha)} \left(\frac{(\theta-\beta)}{([\beta(1+\theta)\pi-\theta(1+\beta)(1-\pi)\psi]+(\theta-\beta)\pi\tau_1)} \right) \end{array} \right] \quad (41)$$

Case 1 $\beta = \theta = \delta$

A special case where $V1 = 0$ for $\beta = \theta = \delta$, that is, when all agents have the same discount rate, can now be considered and this yields $\frac{-1}{(1-\tau_1)} + \frac{(1-\pi)}{([(1-\pi)\psi+\pi\tau_1])} = 0$ and hence the optimal tax is:

$$\tau_1^* = (1-\pi)(1-\psi) \quad (42)$$

For $\beta = \theta$, the optimal level of transfer is simply a function of the level of "productivity deficiency" of the poor and the proportion of poor in the economy. Eqn. (42) suggests that $\frac{\partial \tau_1^*}{\partial \pi} < 0$ and $\frac{\partial \tau_1^*}{\partial \psi} < 0$. This implies that as the proportion of rich in the economy increases, the optimal level of transfer from the rich to the poor falls - and this can be rationalised to the extent that if there are fewer poor people in the economy, the contribution to be made by the rich for redistribution towards the poor falls relatively. Similarly, if the productivity gap between the rich and the poor falls, the optimal level of contribution by the rich declines. This can again be explained by the fact that as the wedge between the rich and the poor declines, the amount of funds needed to bridge the gap between the rich and the poor narrows.

$\tau_1^* \in (0,1)$ is always satisfied[6]. One of the clear implications of the optimal tax above is that the minimum rate of tax is achieved under conditions of near homogeneity. If all the agents are classed as rich ($\pi = \psi = 1$), then the optimal level of redistribution from young to rich is zero. Obviously, if all agents have the same level of earnings and the same discount rate, then there is no need to transfer resources from one to another. Conversely, the lower the level of π and ψ, the higher the tax rate. This suggests that in a highly unequal society with a small proportion of rich and where the poor have a lower productivity, the rich will have to bear a high taxation burden.

[6]For $\tau_1 > 0, \pi < 1$ and $\psi < 1$.For $\tau_1 < 1, \pi > 0$ and $\psi > 0$. Since the above conditions on π and ψ are always satisfied, it follows that $\tau_1^* \in (0,1)$.

4.1.2 The Golden Rule: Intergenerational Transfer

The case where the transfer is from young to old is now considered under the golden rule. The steady state capital stocks and simplified welfare functions are:

$$k_2 = \left(\frac{(1-\alpha)}{1+n} \left[\frac{\beta\pi(1+\theta)(1-\tau_2) + (1+\beta)\left[(1-\pi)\theta\psi - \pi\tau_2\right]}{(1+\beta)(1+\theta)} \right] \right)^{\frac{1}{1-\alpha}} \tag{43}$$

$$V2 = (2+n) \left[\begin{array}{c} \left[\pi \ln(1-\tau_2) + (1-\pi)\ln(\psi(1-\pi) + \pi\tau_2)\right] \\ + \left(\frac{\alpha}{1-\alpha}\right) \ln \left(\begin{array}{c} \left[\beta(1+\theta)\pi + (1+\beta)(1-\pi)\theta\psi\right] \\ -\left[\beta(1+\theta) + (1+\beta)\right]\pi\tau_2 \end{array} \right) \end{array} \right] + z_2^* \tag{44}$$

where z_2^{*}[7] represents a set of parameters independent of the policy term. The optimal tax is now represented by:

$$\frac{\partial V_2}{\partial \tau_2} = (2+n)\pi \left[\begin{array}{c} \left[\frac{-1}{(1-\tau_2)} + \frac{(1-\pi)}{(\psi(1-\pi)+\pi\tau_2)}\right] + \\ \left(\frac{\alpha}{1-\alpha}\right) * \left[\frac{-[\beta(2+\theta)+1]}{([\beta(1+\theta)\pi(1-\tau_2)+(1+\beta)[(1-\pi)\theta\psi-\pi\tau_2]])}\right] \end{array} \right] \tag{45}$$

Case 2 $\beta = \theta = \delta$

Setting $\frac{\partial V_2}{\partial \tau_2} = 0$ for the special case of $\beta = \theta = \delta$ yields:

$$\frac{\partial V_2}{\partial \tau_2} = \left(\begin{array}{c} \left[\frac{-1}{(1-\tau_2)} + \frac{(1-\pi)}{(\psi(1-\pi)+\pi\tau_2)}\right] \\ -\left[\frac{\alpha[\delta(2+\delta)+1]}{(1-\alpha)(1+\delta)*[\delta(\pi+(1-\pi)\psi)-(\delta+1)\pi\tau_2]}\right] \end{array} \right) = 0 \tag{46}$$

Although this does not allow for a set of results with significant degree of analytical tractability, it is possible to get an overview of how τ_2 whilst varying the proportions of rich [and poor] and the level of productivity of the poor as well as the discount rates for a given level of α.

[7] $z_2^* = z_2 + (2+n)\left[\ln(1-\alpha) + \left(\frac{\alpha}{1-\alpha}\right)\left[\ln(1-\alpha) - \ln(1+n) - \ln(1+\beta) - \ln(1+\theta)\right]\right]$

4.1.3 General Case: Intragenerational Transfer

We now consider the case where $R \neq (1+n)$ and proceed to find the optimal level of redistribution between the young rich and the poor. The steady state capital stock, $k_1 = \left(\frac{(1-\alpha)}{(1+n)}\left(\frac{\beta(1+\theta)\pi(1-\tau_1)+\theta(1+\beta)[(1-\pi)\psi+\pi\tau_1]}{(1+\beta)(1+\theta)}\right)\right)^{\frac{1}{(1-\alpha)}}$, is defined as before by Eqn (30). The welfare function under the current setting can be summed up as:

$$V1 = \left[\begin{array}{c} (2+n)\left[\pi \ln(1-\tau_1) + (1-\pi)\ln\left([(1-\pi)\psi + \pi\tau_1]\right)\right] \\ +\left[\alpha(3+n)-1\right]\ln(k) + \mathbf{z_1} \end{array} \right] \tag{47}$$

where $z_{\mathbf{1}} = z_{\mathbf{0}} + \ln\alpha$, is a set of parameters independent of the policy term. The optimal tax is derived as previously and this yields:

$$\frac{\partial V1}{\partial \tau_1} = \left[\begin{array}{c} (2+n)\pi\left[\frac{(1-\pi)}{[(1-\pi)\psi+\pi\tau_1]} - \frac{1}{(1-\tau_1)}\right] \\ +\left[\frac{[\alpha(3+n)-1]\boldsymbol{\pi}[\boldsymbol{\theta}-\boldsymbol{\beta}]}{(1-\alpha)\beta\pi(1+\theta)(1-\tau_1)+(1+\beta)\theta[(1-\pi)\psi+\pi\tau_1]}\right] \end{array} \right] \tag{48}$$

Case 3 $\beta = \theta = \delta$

It can be seen that for the special case of $\beta = \theta = \delta$ the second part of $\frac{\partial V1}{\partial \tau_1}$ boils down to zero and the equation reduces to$(1-\pi)(1-\tau_1) = [(1-\pi)\psi + \pi\tau_1]$ such that the optimal tax, as in the case of the Golden Rule, reduces to:

$$\tau_1^* = (1-\pi)(1-\psi) \tag{49}$$

Hence, for $\beta = \theta = \delta$, τ_1^* is the same for and outside the Golden Rule.

4.1.4 General Case: Intergenerational Transfer

When redistribution takes place via the pensions scheme outside the Golden Rule, the capital stock is defined as $k_2 = \left(\left[\frac{\alpha(1-\alpha)(1+\theta)[\beta\pi(1-\tau_2)+(1+\beta)(1-\pi)(\theta\psi)]}{(1+n)(1+\beta)[(1+\theta)\alpha+\pi\tau_2(1-\alpha)]}\right]\right)^{\frac{1}{(1-\alpha)}}$ (Eqn (31)). The welfare function now takes the following form:

$$V2 = \left[\begin{array}{c} (2+n)\left[\pi\ln(1-\tau_2) + (1-\pi)\ln\left(\psi(1-\pi)R_2 + \pi(1+n)\tau_2\right)\right] \\ +\left[(2+n)\left[(1+\pi)\alpha - \pi\right] + (1-\alpha)(1+n)\right]\ln k + z_2 \end{array} \right] \tag{50}$$

where $z_2 = z_0 + (2+n)\ln(1-\alpha) + [(2+n)\pi - (1+n)]\ln\alpha$, is a set of parameters independent of the policy term. The optimal level of tax now results in $\frac{\partial V2}{\partial \tau_2}$:

$$\left(\frac{-(2+n)\pi}{(1-\tau_2)} + \left[\begin{array}{c} \frac{(2+n)\pi(1-\pi)[(1-\alpha)]*[\psi(1-\pi)(1+\beta)+(1+\theta)\pi\beta(1-2\tau_2)]}{(\{[\psi(1-\pi)(1+\beta)]*[(1+\theta)\alpha+\pi(1-\alpha)\tau_2]\}+\{[(1-\alpha)(1+\theta)\pi]*[\beta\pi(1-\tau_2)+(1+\beta)(1-\pi)(\theta\psi)]\tau_2\})} \\ +\frac{\beta(2+n)\pi(1-\pi)}{([\beta\pi(1-\tau_2)+(1+\beta)(1-\pi)(\theta\psi)])} \end{array} \right] \right)$$
$$- \left(\frac{[(\beta\pi[\alpha(2+n)+(\alpha-1)[\pi-(1+n)(1-\pi)]]])*[\{[(1+\theta)\alpha+\pi(1-\alpha)\tau_2]\}+\{([\tau_2(1-\alpha)[\beta\pi(1-\tau_2)+(1+\beta)(1-\pi)(\theta\psi)]])\}]]}{((1-\alpha)*[\beta\pi(1-\tau_2)+(1+\beta)(1-\pi)(\theta\psi)])*[(1+\theta)\alpha+\pi\tau_2(1-\alpha)]} \right) \tag{51}$$

4.2 The Timing of Redistribution

Having considered the behaviour of the tax rate under some specific settings, the optimal level and timing of redistribution is now considered in a general equilibrium framework encompassing capital, output, wages, interest rates, consumption, the tax rate and welfare. The timing will be based the decision criteria of the planner (as highlighted previously). For a given set of parameters and tax rates, the welfare function with the highest value determines in which of the two periods it is optimal to effect the transfer. Alternatively, there might be a set of parameters and tax rates for which it is not possible to improve on the competitive equilibrium, in which case redistribution, in one form or another, is not optimal. We revert to simulation to show the general equilibrium effects of the taxes and how it influences on a host of macroeconomic variables.

4.2.1 Choice of parameters:

Our parameter choices for the population growth rates and the discount rates are in line with the range of values used for macro simulations (see de la Croix and Michel, 2002; Krueger and Kubler, 2006). In line with the demographic transitions affecting different regions of the world, there is a wide range of parameters that could have been used ranging from 0 for Europe to 2.4 percent for Africa with 1.3 percent for the world[8] (*United Nations,1999*). For the purpose of the simulations we assume that population grows by 1

[8]Data pertain to 1995-2000.

	α	β	θ	π	*Consumption*				ψ	n
$\beta = \theta$	0.3	**0.96**	**0.96**	0.4	*Rich*		*Poor*		0.75	0.01
	y	**k**	**w**	**R**	Y	O	Y	O	$\boldsymbol{\tau}$	**Welfare**
V_0	0.587	0.1694	0.411	1.04	0.2097	0.2093	0.1572	0.1569	0	-3.4890
V_1	0.587	0.1694	0.411	1.04	*0.1782*	*0.1779*	*0.1782*	*0.1779*	0.1500	-3.4688
V_2	0.568	0.1521	0.398	1.121	0.1869	0.2011	0.1619	0.1743	0.0794	-3.4708
	α	β	θ	π					ψ	n
$\beta > \theta$	0.3	**0.96**	**0.9**	0.4					0.75	0.01
	y	**k**	**w**	**R**	Y	O	Y	O	$\boldsymbol{\tau}$	**Welfare**
V_0	0.583	0.1652	0.408	1.058	0.2081	0.2114	0.1610	0.1533	0	-3.4878
V_1	0.582	0.1646	0.407	1.061	0.1765	0.1797	*0.1824*	0.1741	0.1509	-3.4672
V_2	0.562	0.1469	0.394	1.149	0.1838	0.2028	0.1657	0.1714	0.0849	-3.4703

Table 1: Baseline Simulations

percent from one period to the next. The share of capital in production is set at 0.3. The discount rate of the rich (β) is 0.96, whilst that of the poor (θ) is initially 0.96 and it is then lowered to 0.9 for comparative purposes . In the baseline, we assume that the poor earn 75 percent of what the rich earn and we assume that the rich, π, make up 40 percent of the economy, with the poor accounting for the remaining 60 percent. $(n = 0.01; \alpha = 0.3, \beta = 0.96, \theta = 0.96/0.9, \psi = 0.75, \pi = 0.4)$

4.2.2 Simulation Results

For the baseline, two situations for the discount rates are considered: in the first setting the discount rate of the poor (θ) and rich (β) is set to be the same at 0.96 whilst in the second case $\beta\,(0.96) > \theta\,(0.9)$ is considered. The baseline results are shown in Table *1*.

The first case of $\beta = \theta$ is analysed for a general overview of the results. V_0 represents the welfare function without redistribution whilst V_1 and V_2 are defined as previously, that is, with transfers in the first and second periods respectively. Without redistribution, we can see that the agents consume what they earn over their lifetime, and in fact, ratio of the consumption of the poor relative to that of the rich is equal to ψ, the difference in the level of productivity between the two types of agents. When redistribution takes place in the fist period, for $\beta = \theta$, it can be seen that the capital stock is not (adversely) affected relative to the competitive equilibium and as a result, the output, wage and interest rate are the same. With redistribution from the young rich to the young poor, the tax rate is 15% - and this confirms the earlier result that for $\beta = \theta$, the optimal level of transfer in the first period is defined by $\tau = (1 - \pi)(1 - \psi)$. There is an increase in aggregate welfare relative to the competitive equilibrium and in this setting, we find that the consumption levels of the rich and the poor equalise with that of the rich falling and that of the poor increasing. When intergenerational redistribution takes place, the adverse impact of the redistribution on the capital stock results in a lower output and wage and higher interest rate. However, despite the distortion introduced in terms of capital formation, intergenerational redistribution still represents an improvement over the competitive equilibrium. The same mechanism operates as regard consumption viz the rich consume less than the competitive level whilst the poor consume more. It can be seen the welfare is higher for V_1 compared to V_2, suggesting that for the given set of parameters, intergenerational transfer is optimal. Each young agent would then fund for his own retirement and there is no need for pensions as a redistributive instrument in this set-up.

For $\beta > \theta$, the capital stock is now lower, compared to$\beta = \theta$, resulting in lower output and wages whilst the interest rate goes up. Welfare is higher than that of the competitive equilibrium and any redistribution from the rich to the poor, in any period, results to an even lower capital stock and as a result a lower output and wage rate and a higher interest rate. There is an increase in the level of tax rates compared to the case of $\beta = \theta$. . As for the case of $\beta = \theta$ welfare is unambiguously higher with redistribution, with welfare higher for intragenerational redistribution compared to intergenerational redistribution. For $\beta > \theta$ the optimal redistribution is thus still from young to young. One of the interesting aspects of the results is the fact that although $\tau_1$is almost twice τ_2, the welfare is still higher under intragenerational transfer implying that the welfare costs in terms of capital formation are fairly high with intergenerational transfer. The main results of the two cases for the given set of parameters are summarised in Table *2*.

(i)	(ii)	(i) vs (ii)
$\beta = \theta$	$\beta > \theta$	
$k_0^\delta = k_1^\delta > k_2^\delta$	$k_0 > k_1 > k_2$	$k^\delta > k$
$y_0^\delta = y_1^\delta > y_2^\delta$	$y_0 > y_1 > y_2$	$y^\delta > y$
$w_0^\delta = w_1^\delta > w_2^\delta$	$w_0 > w_1 > w_2$	$w^\delta > w$
$R_0^\delta = R_1^\delta < R_2^\delta$	$R_0 < R_1 < R_2$	$R^\delta < R$
$\tau_1^\delta > \tau_2^\delta$	$\tau_1 > \tau_2$	$\tau^\delta < \tau$
$V_0^\delta < V_2^\delta < V_1^\delta$	$V_0 < V_2 < V_1$	$V_0^\delta < V_0 < \mathbf{V}_2^\delta < \mathbf{V}_2 < V_1^\delta < V_1$

Table 2: Summary of Results for Selected Parameters

	k	**y**	**w**	**R**	cyr	cor	cyp	cop			**Tax**		**Welfare**		
								(0)	(1)	(2)	(1)	(2)	(0)	(1)	(2)
α	$-$	$-$	$-$	$+$	$-$	$+$	$-$	$+$	$+$	$-$	0	$-$	$-$		
β	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$			$+$		$-$		
θ	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$+$			$-$		-	-	0
π	$+$	$+$	$+$	$-$	$+$	- + -	$+$	-	$+$	0	$-$		$+$		
ψ	$+$	$+$	$+$	$-$	$+$	- + -	$+$	-++			$-$		$+$		
$n?$	$-$	$-$	$-$	$+$	-	$+$	-	$+$			0	$-?$	$-$		

Table 3: Impact of a 1 percent change

The results in Table *2* are in line with the previous findings that redistribution leads to a lower capital stock when the discount rate of the rich, β is greater than θ, the discount rate of the poor. As a result, $k^\delta > k$ which in turn yields $y^\delta > y; w^\delta > w$ and $R^\delta < R$. Though not shown in Table *2*, with redistribution the consumption of the rich is always lower than that for competitive equilibrium, whilst that of the poor is always higher.

Having considered the results we now proceed to show the impact of a 1% change in the exogenous variables on the endogenous variables compared to the baseline. The results for $\beta > \theta$ are summarised in Table *3*. The case without redistribution is not considered in the table, although it is considered when the findings are discussed.

Changes in the exogenous parameters for $\beta = \theta$ and $\beta > \theta$ are unambiguous in so far as the impact on capital and consequently output, wages and interest rate are concerned. However, the main difference lies at the level of taxes and welfare. For $\beta = \theta$, the tax rate changes for young to young only when the proportions of rich and the level of productivity of the poor change since $\tau = (1 - \pi)(1 - \psi)$. Any change in the other parameters does not affect the optimal level of tax from young to young.

Changes in α and n have an inverse impact on capital. Since output and wages are positively related to the capital level and the interest rate is negatively related, output and wages are negatively related to changes in α and n whilst interest rates are positively related. Whilst this leaves unchanged the optimal level of transfer from young to young, the tax rate is inversely related if the transfer is from young to old. Changes in α and n have an unambiguous inverse impact on welfare for both modes of transfer. However, it has to be noted that the impact of n is infinitesimal compared to a change in α.

For the baseline case for $\beta > \theta$, the capital stock is unambiguously and positively related to any change in β, θ, π and ψ - and as a result, for any increase in these parameters, output and wages increase whilst the interest rates fall. For $\beta \neq \theta$, an increase in β can be interpreted as an increase in the relative myopia of the poor relative to the rich - as a result, this leads to an increase in the level of transfers from the rich to the poor in both settings. Similarly, an increase in θ, the discount rate of the poor can be perceived as a reduction in the relative level of myopia and this reduces the tax rate from the rich to the poor. Whilst without transfers there is a positive relationship between the discount rates and welfare, with transfers the relationship is negative. The relationships between capital (hence, output and wages) and π and ψ is positive, whilst it is negative with respect to the interest rates. An increase in π has an inverse impact on the interest rate whilst having a positive impact on welfare is consistent with the view that a homogeneous economy, in terms of having more rich people, requires a lower level of transfer from the rich to the poor. In the same vein, an increase in the number of rich people in the economy leads to an increase in welfare. The same reasoning applies to ψ - an increase in the level of productivity of the poor relative to the rich leads to a reduction in the gap between the rich and the poor and as a result a lower level of transfer is required whilst the economy has a higher level of welfare overall.

Sensitivity Analysis: Changes in the discount rate In so far, consistent with Pigou's "faulty telescopic faculty", it has been assumed that agents suffer from partial myopia in that they discount the future. Starting with the case where agents do not discount the future, that is they have perfect foresight, a combination of cases where agents discount the future at high rates are considered. Except for β and θ, all the other parameters are taken to be the same as in the previous experiments.

Case 4 $\beta = \theta = 1$

When none of the agents discount the future, this can be summarised as $\beta = \theta = 1$. When the transfer is from young to young, the general result $\tau = (1 - \pi)(1 - \psi)$ still applies whilst if there is an intergenerational transfer, the optimal level of tax is lower. This also results in a higher level of capital stock (output and wages) and a lower level of interest rate. Compared to the initial baseline, the aggregate level of welfare is lower in all three cases.

Case 5 $\beta = 1; \theta \approx 0$

In the situation where the rich do not discount the future whilst the poor have a very high discount rate, welfare is unambiguously lower at all levels. There is a significant decline in capital (output and wages) whilst the interest rate goes up markedly as well. The optimal transfer from the rich to the poor reaches nearly 24%, the highest in the set of simulations considered for these set of parameters. It is also found that the rich consume more in the second period of their lifetime whilst the poor have a very low level of consumption when old. The other main result is that whilst transfer in the first period still results in a higher level of welfare relative to the competitive equilibrium, intergenerational transfer yields a welfare level lower than the competitive level.

Case 6 $\beta = 1; \theta \to 0$

Assuming that the rich do not discount the future, we initially begin with a similar discount rate for the poor. We then proceed by allowing the poor to discount the future at a higher rate. We find that as the wedge increases marginally, this leads to an increase in the level of welfare under all settings. However, shortly afterwards, welfare begins to decline. Intragenerational transfers raise welfare by the biggest margin throughout and though transfers

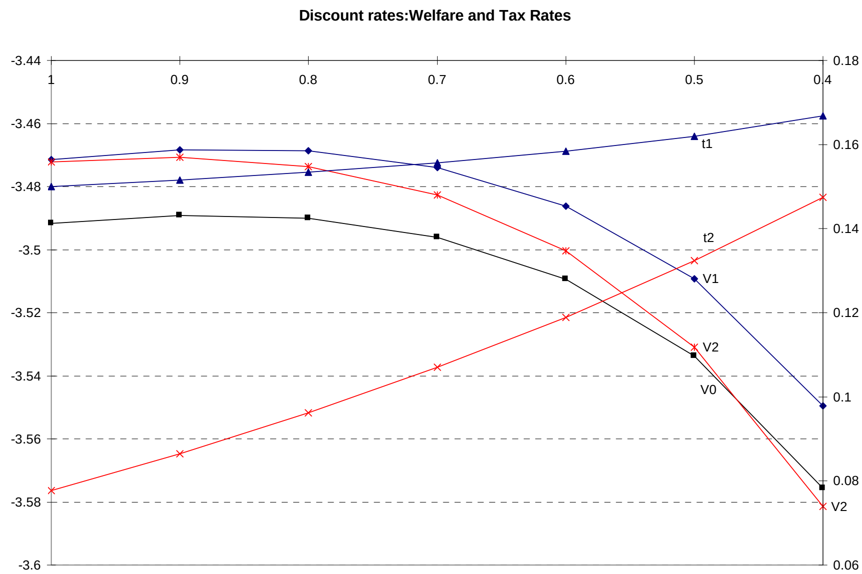

Figure 1: Wedge in Discount Rates and Welfare

through pensions initially yield higher welfare than the competitive equilibrium, this is eventually dwarfed. The fact that welfare increases initially as the poor discount the future at a higher level can be attributed to the fact that the initial increase in consumption boosts welfare by a higher level than the associated costs. However, as the wedge grows larger, the associated costs in terms of higher interest rates and taxes far exceeds the accompanying welfare gains. The impact of the wedge on welfare in summarised in Figure *1*. It can be seen that initially, the tax rate for intragenerational transfers is about twice the rate for intergenerational transfers. However, as the poor discount the future by a higher amount, though the tax rate increases in both settings, the increase is more pronounced in the case of τ_2 and we can find that the wedge narrows. In the extreme case, τ_2 is marginally higher than τ_1. The increase in τ_2 compounds the costs associated with the crowding out of capital and thus leads to a lowering of welfare.

Changes in proportions of rich and productivity of poor We have seen that the proportions of rich and the productivity of the poor will play an important role in determining welfare in this set-up. An economy with a low proportion of rich has a lower level of utility (relative to baseline) and similiarly, as the gap between the rich and the poor widens, this has detrimental effects on welfare. Any changes in π, will affect the capital accumulation process and hence welfare. The same applies to ψ. Keeping all the other parameters unchanged relative to the baseline, π and ψ are allowed to vary and the resulting impact on the endogenous variables considered. The general equilibrium results are summarised in Figure *2*.

The first set of diagrams represent the evolution of π, the rich individuals in the economy. We find that when π is set to 1% it unambiguously leads to a lower capital stock (relative to the baseline) and hence the interest rate is higher, nearing 20% compared to 4% for the baseline (output and wages are lower).. We also find that there is a wedge between the capital stock under various modes of distribution the capital stock is lower for intergenerational transfer than intragenerational transfers whilst the latter is marginally lower than the competitive level. We also find that for low levels of π, the tax rate is higher. As π increases, we find that the capital stock increases and the interest rates fall and dynamic inefficiency, where the population growth rate exceeds the interest rate, creeps in after π has increased beyond a certain level. Dynamic inefficiency creeps in faster in the case of intragenerational transfers. The tax rate falls gradually as well and welfare increase unambiguously.

The same mechanism applies to the level of productivity of the poor, ψ. When the productivity of the poor is low, the impact on capital accumulation is relatively severe and it leads to a lower capital stock. Welfare is the lowest among all the experiments considered and the tax rate is highest as well. We however find that as ψ starts to increase, the capital stock increases consistently, with the wedge between k_2 still apparent, and the interest rates start to converge and fall (with dynamic inefficiency beyond a certain level). The tax rate falls as well and they are eventually equal before τ_2 exceeds τ_1. There is a convergence and increase in welfare as ψ increases with the convergence faster at lower levels of ψ.

In so far as n is concerned, its impact in the current set up is relatively subdued in the sense that there has to be significant changes in the population growth rate for there to be any significant impact on the variables under

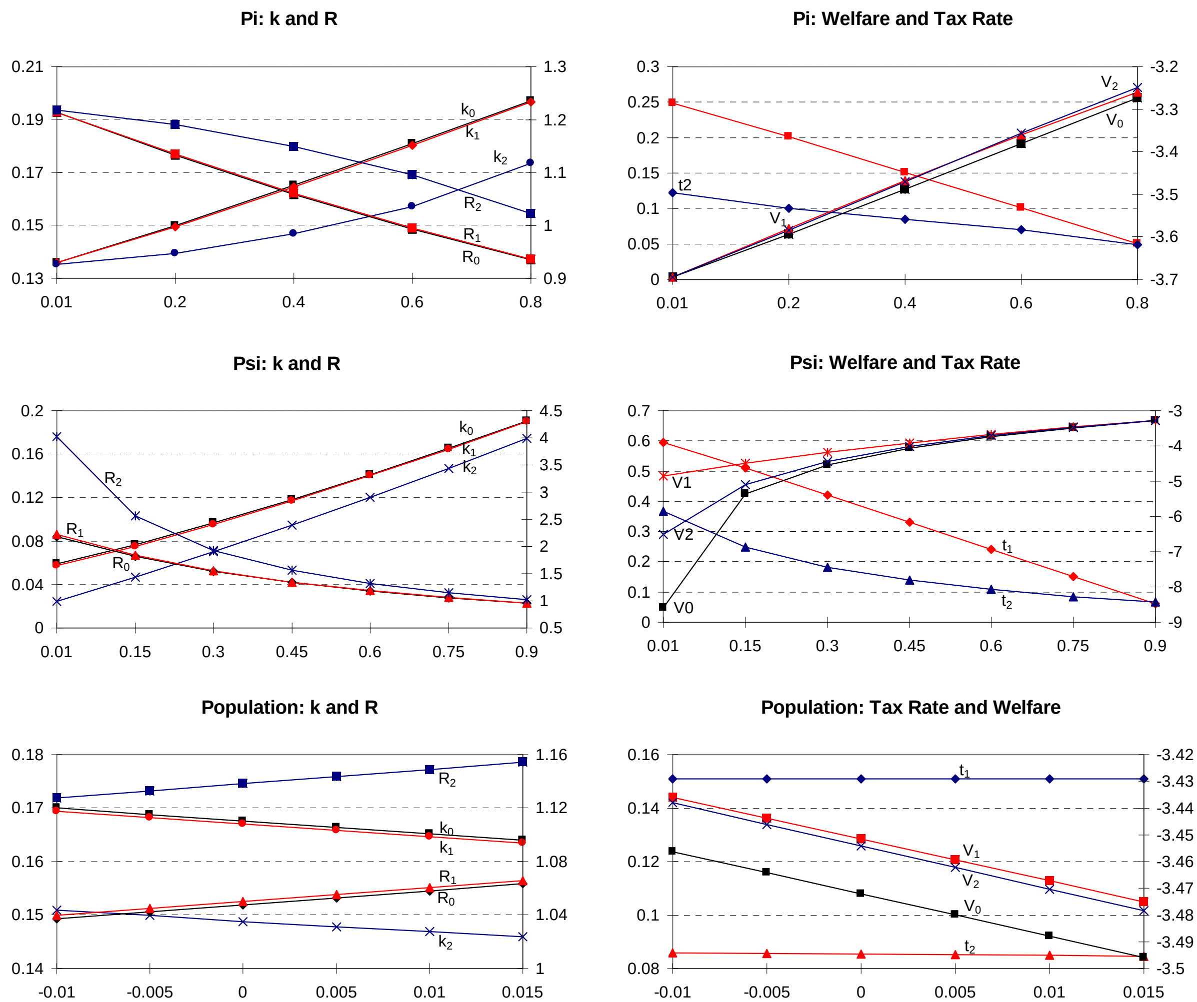

Figure 2: GE Effects of Changes in Exogenous Parameters

consideration. In Figure *2*, we consider changes ranging from a 1% fall to a 1.5% increase in population from one period to the next. The findings are in line with expectations in that as the population growth rate increases, capital available per worker falls and the interest rate rises. The wedge in capital stock is still apparent and this is reflected in the interest rate as well. The impact on the tax rate is infinitesimal though it does tend to fall as the population growth rate increases. In the case of the population growth rate increasing from -1.5% to 1%, for intragenerational transfer, the fall is only 1 basis point whilst in the case of intergenerational transfer, it is 13 basis points. Welfare falls as the population growth rate increases.

4.3 When is pensions an optimal tool for redistribution?

In so far, based on the results for the given set of parameters in the baseline, a transfer from the rich to the poor through taxes (young-to-young) is almost always welfare improving. The same applies to transfers through pensions with some restrictions in the sense that there are certain instances where pensions can yield a lower welfare than the competitive equilibrium (as depicted when considering the discount rate changes). We now consider under what conditions pensions can be an optimal instrument for redistribution. For ease of exposition, the parameter values are set as before and we only consider how the decision is affected by changes in the proportions of rich and the level of productivity of the poor; the two parameters that seem to have the highest incidence on the optimal timing of redistribution. For pensions to be the optimal redistribution instrument, we require, $V_2 - V_1 > 0$.

Figure *3* depicts how the optimal instrument varies as the proportions of rich(LHS) and productivity of the poor (RHS) change, keeping other things constant. In the baseline π had been set at 40% and for the given set of parameters, it was optimal to transfer from the rich to the poor in the first period. Allowing π to increase results in a bridging of the gap between the two modes of transfers. Intragenerational transfer is optimal until the proportion reaches 50.1%, when the two modes of transfers yield the same welfare. If π exceeds 50.1%, then an intergenerational transfer scheme is optimal. The same mechanism applies to the level of productivity ψ which had been set at 75% in the baseline and for the given set of parameters, it was optimal to transfer in the first period. By allowing ψ to increase,

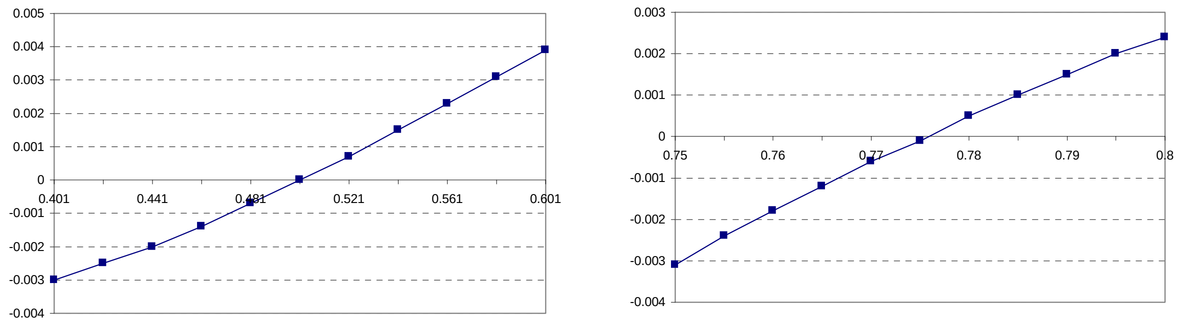

Figure 3: Optimal Timing of Redistribution

intragenerational transfer remains optimal until ψ increases to 77.5% when welfare is the same irrespective of the mode of redistribution in place. For ψ exceeding 77.5%, it is optimal to transfer through pensions.

The mode of transfer will also determine the type of pensions scheme in place. With intergenerational transfer, there is a hybrid system with the rich saving through a FF scheme viz the provision of their own pension whilst the poor although they might save also receive an additional transfer when old in the form of a PAYG scheme. On the other hand, with intragenerational transfer, all redistribution takes place in the first period and every individual is responsible for the provision of their own pension in retirement. The system is FF in that case.

The above results suggest that it is optimal to redistribute in the first period if an economy is characterised by pervasive inequality and the productivity of the poor relative to the rich is low. Under those circumstances, redistributing in the first period yields a higher welfare than redistributing through pensions. However, in an economy where inequality is relatively low, transferring through the pensions scheme is optimal.

In so far, we have assumed that either π or ψ vary but they do not vary simultaneously. For instance, if π falls to 1%, ψ needs to increase to 80.55% to ensure that welfare is the same under both settings. If ψ is higher (lower), then it is optimal to transfer from young-to-young (old-to-old). On the other hand if π increases to 90%, ψ can be as low as 10.95% and yet welfare will be the same under both settings.

5 Conclusion

In this paper, we have considered the impact of redistribution in an overlapping generations economy and considered some of the implications in a general equilibrium framework. We adopt a relatively underutilised approach to consider the pensions scheme as a redistribution instrument as opposed to the more conventional income support. Our results confirm some of the earlier findings on redistribution that it can be costly and there are distortions that arise in the economic decision making of the recipients. In our framework, this takes the form of a crowding out of capital (Feldstein (1974)) resulting in an adverse impact on output and wages whilst causing interest rate to rise. We also find that the level and timing of redistribution matters as well. Our key finding suggests that in cases of high inequality, that is, when there is a very small proportion of rich and the poor have a low productivity, intragenerational transfer is the optimal instrument. On the other hand, when inequality is fairly low, then it is optimal to redistribute though intragenerational transfers. We also find that whilst intragenerational transfers almost always represent an improvement over competitive equilibrium, intergenerational transfers can, in situations of high myopia on behalf of the poor, yield a lower welfare than the competitive level. In sum, our findings suggest that with heterogeneity, there can be a role for pensions as a redistributive instrument that can potentially enhance welfare above the competitive level.

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