Optimal Provision of Public Inputs in a Second Best Scenario*

Diego Martínez
Centro de Estudios Andaluces and Pablo Olavide University

A. Jesús Sánchez
Centro de Estudios Andaluces

September 23, 2005

Abstract

This paper studies the optimal provision of public inputs under two different tax settings: with lump-sum taxes and with taxes on labour. After obtaining the optimality rules for the provision of public inputs, a numerical simulation is carried out to compute the level of public spending in each scenario. We find that a second-best rule has to be followed when lump-sum taxes are not available, even though the labor taxes are set optimally. Another result we obtain is that the level of public input provided under the second best scenario is higher than that corresponding to the first best outcome. The effect of changes to some parameters on the level of public input is also studied.

JEL Classification: H21, H3, H41, H43

Keywords: public input, optimal taxation.

*Parts of this paper have been presented in seminars at centrA-Seville, Umea University, the ECB, the 60th Congress of IIPF, and the XII Encuentro de Economía Pública. We are grateful to T. Aronsson, A. Esteller, G. Fernandez de Cordoba, M.D. Morales and J. Rodero for their comments and suggestions. The usual disclaimer applies. D. Martinez thanks the hospitality of Umea University and of Fiscal Policies Division of the ECB, where parts of this paper were done. Both authors acknowledge the financial support by the Spanish Institute for Fiscal Studies. Corresponding author: D. Martinez. Centro de Estudios Andaluces. C/ Bailén, 50. 41001 Seville. Spain. Phone: 34 955 055 217. E-mail: diego.martinez@centrodeestudiosandaluces.es

1 Introduction

Conventional rule for the optimal provision of public goods claims that the sum of marginal rates of substitution between a public good and a private good over all individuals must be equal to the marginal rate of transformation (Samuelson, 1954). However, the pioneer contribution by Pigou (1947) questioned this rule when lump-sum taxation is not available. In such a case, Pigou stressed the deadweight loss caused by distortionary taxes, and argued that an over-supply of public goods may arise under Samuelson's environment.

However, Atkinson and Stern (1974) confirmed the finding of Stiglitz and Dasgupta (1971) that the marginal rate of transformation may well need to adjust downward when distortionary taxation is used. The basis for this claim is based on the feedback effect which may arise when the consumption of taxed goods is encouraged by public spending[1]. This entails that Samuelson's rule may lead to an under-supply of public goods, contrary to Pigou's approach. The controversy has persisted until the present day, with extensions for heterogeneous agents (King, 1986; Wilson, 1991), focusing on the complementarity nature of public spending and taxed goods (Wildasin, 1984; Chang, 2000), and with non-competitive markets (Aronsson and Sjogren, 2001).

Part of the current debate regarding public goods provision deals with the optimal *level* of public goods. Indeed, the controversy concerns more the quantity of public goods than the optimality rules derived from the first order conditions. Papers such as Wilson (1991), Chang (2000) and Gaube (2000) highlight this topic, in many cases using numerical examples (and counterexamples). The underlying idea of these papers is that using distortionary taxation leads to an optimal level of public goods below its first-best level; this idea is based on the argument that the optimal extent of public spending should be inversely related to the welfare cost derived from distorting taxation. However, Gaube (2000) shows that this statement is not as straightforward as it might seem. Particularly, the above argument holds as long as the public and private goods are normal and all groups of private commodities are gross substitutes.

All these issues have received very little attention in terms of public in-

[1] Ballard and Fullerton (1992) illustrate this issue with an example in which the provision by the state of a highway encourages the consumption of taxed goods such as petrol.

puts. However, we believe that the particular features of productive public spending deserve specific treatment, due to the economic growth implications and the social welfare consequences of the provision of public inputs. Early papers by Kaizuka (1965) and Sandmo (1972) study the optimality of public production factors in a first-best environment. Feehan and Matsumoto (2000) study the use of benefit taxation to provide productive public spending. Feehan and Matsumoto (2002) also show the differences between the first-best and the second-best rules in the provision of public inputs, and conclude that deviations from the first-best condition are inappropriate if taxes are set optimally.

In this paper, we use a simple general equilibrium model (adapted from Boadway and Keen, 1996) where public spending yields productive services to firms. Two different tax settings are available for government: a lump-sum tax and a tax on labour. After obtaining the optimality rules for the provision of public inputs under each scenario, a numerical simulation is carried out to compute the levels of public spending.

Our paper contains two main contributions. Firstly, in contrast to Feehan and Matsumoto (2002), we conclude that a second-best rule must be followed for the optimal public input provision when lump-sum taxes are not available. This is the case even where the taxes are set optimally and the production efficiency condition holds. This conclusion stems from the use of firm-augmenting public input; which generates profits that require the consideration of a second-best rule (Pestiau, 1976). However, the existence of a tax rate on rents equal to 100% guarantees the fullfilment of the production efficiency condition.

Secondly, we offer an insight into the controversy regarding the level comparisons under different tax settings. We follow the approach suggested by Gronberg and Liu (2001), which is based on the sign of the marginal excess burden. Given the assumptions of our model, we cannot determine with certainty whether the first-best level will overcome the second-best level or not. In fact, the numerical simulation we have implemented indicates that the second best level exceeds the first-best level with the three utility functions used, being closely linked to the extent of profits. Other results concerning the impact of changes in the output elasticity with respect to public input and the number of households on the public input level are also provided.

The structure of the paper is as follows. Section 2 presents the main characteristics of the model. We obtain the conditions for the optimal provision of public inputs under each tax setting in the following section. Section

4 discusses the application of Gronberg and Liu's methodology to our case. Section 5 presents the main results and their interpretation. Finally, section 6 concludes.

2 The model

Let an economy be populated by n identical households whose utility function is expressed as:

$$u(x, l), \tag{1}$$

where x is a private good used as a numeraire and l is labour supplied. The properties of $u(x, l)$ are the standard ones to ensure a well-behaved function: strictly monotone, quasiconcave and twice differentiable. Weak separability in x and l is to be assumed[2]. The representative household faces the following budget constraint:

$$x = (\omega - \tau) l - \frac{T}{n}, \tag{2}$$

where ω is the wage rate, τ is the per unit tax on labour, and T is the lump-sum tax collected by the government. Household's optimization problem consists of maximizing (1) subject to (2) to yield the labour supply $l(\omega - \tau, T)$, and the indirect utility function $V(\omega - \tau, T)$. Defining the net wage rate as $\omega_N = \omega - \tau$, it is to be assumed that $l_{\omega_N} \geq 0$[3]. Output in the economy is produced using labour services and a public input g according to the following aggregate production function:

$$F(nl, g) \tag{3}$$

This function satisfies the usual assumptions: increasing in its arguments and strictly concave. Constant returns to scale are assumed in all factors, including the public input. Thus, we have a firm-augmenting-type public input which creates rents (Feehan, 1989). Output can be used costlessly as x or g. The labour market is perfectly competitive so that the wage rate is linked to the marginal productivity of labour:

[2] Non-separability would unnecessarily complicate the analysis.

[3] Hereafter, a subscript is used for partial derivatives.

$$\omega = F_L \left[nl \left(\omega - \tau, T \right), g \right], \tag{4}$$

where firms take g as given. It allows us to achieve the wage function $\omega(g, \tau, T, n)$, from which we can derive the following results for later use:

$$\omega_g = \frac{F_{Lg}}{1 - nF_{LL} l_{\omega_N}} \tag{5}$$

$$\omega_\tau = \frac{-nF_{LL} l_{\omega_N}}{1 - nF_{LL} l_{\omega_N}} \tag{6}$$

$$\omega_T = \frac{nF_{LL} l_T}{1 - nF_{LL} l_{\omega_N}} \tag{7}$$

The profits generated are equal to:

$$\pi(g, \tau, T, n) = F\left[nl \left(\omega(g, \tau, T, n) - \tau \right), g \right] - nl \left[\omega(g, \tau, T, n) - \tau \right] \omega(g, \tau, T, n) \tag{8}$$

We also obtain the following results for later use:

$$\pi_g = F_g - \left(nF_{LL} l_{\omega_N} \omega_g + F_{Lg} \right) nl \tag{9}$$

$$\pi_\tau = \left(1 - \omega_\tau \right) n^2 l F_{LL} l_{\omega_N} \tag{10}$$

$$\pi_T = -\frac{nlF_{LL} l_T}{1 - nF_{LL} l_{\omega_N}}, \tag{11}$$

The equilibrium condition in the labour market (4) and the expression for ω_T have been used to obtain the last equation. Note that the effect of public input on rents is ambiguous because g increases the output (and hence the profits) but also exerts a positive impact on the wage rate, reducing the rents.

The revenue raised by the government to finance public expenditure is:

$$g = n\tau l \left(\omega(g, \tau, T, n) - \tau \right) + \pi(g, \tau, T, n) + T \tag{12}$$

Note that all profits are fully taxed by the government because they are efficient resources for the public sector.

3 First order conditions with lump-sum and distorting taxes

In this section we obtain the first order conditions for the optimal provision of public inputs in two different cases: with a lump-sum tax ($\tau = 0$) and with a distorting tax on labour ($T = 0$). The optimization problem of the government for these two cases is as follows:

$$\begin{aligned} & Max \quad V(\omega - \tau, T) \\ s.t. \quad : \quad & g = n\tau l(\omega(g, \tau, T, n) - \tau) + \pi(g, \tau, T, n) + T, \end{aligned} \tag{13}$$

that is, the government chooses the values of g and T or τ to maximize the representative household's utility subject to the budget constraint[4].

The first order conditions in the scenario with only lump-sum taxes are as follows:

$$V_{\omega_N}\omega_g - \mu + \mu\pi_g = 0 \tag{14}$$

$$V_{\omega_N}\omega_T + V_T + \mu\pi_T + \mu = 0, \tag{15}$$

where μ is the Lagrange multiplier. Since the tax is lump-sum, the social and the private marginal utility of income coincide ($V_T = -\mu = -\lambda$, where λ is the private marginal utility of income). Using Roy's identity and the equations (7) and (11) as well, the optimalily rule for the provision of public inputs can be expressed as follows:

$$\frac{nV_{\omega_N}\omega_g}{\lambda} = 1 - \pi_g \tag{16}$$

The LHS of (16) is the benefit from one aditional unit of public input, while the RHS is the marginal cost of providing (MCP) the public input. This term can be interpreted as the marginal rate of transformation between the public input and a numeraire (MRT) corrected by the feedback effect stemming from the public input provision. This second term (MRTC, hereafter) measures the reduction in the MRT as a result of the indirect benefit caused by spending public funds on tax revenue-increasing public inputs. This feedback effect is a

[4] Wildasin (1986) demonstrates that it is relevant to distinguish between maximizing the per capita utility and maximizing the total utility.

key feature in the provision of public inputs in the case where a consumption public good is considered (Feehan and Matsumoto, 2002).

On the basis of (16), if (5), (9) and Roy's identity are taken into consideration, an important condition containing efficiency implications can be derived:

$$F_g = 1 \tag{17}$$

This is the production efficiency condition in the provision of public inputs. It means that the production effects of the public input (its marginal productivity) are equal to its MRT.

Next we focus on the optimal provision of public inputs when distorting taxes are used. In such a case, the first order conditions for g and τ derived from (13) are:

$$V_{\omega_N}\omega_g - \eta + \eta\tau n l_{\omega_N}\omega_g + \eta\pi_g = 0 \tag{18}$$

$$(\omega_\tau - 1)\, V_{\omega_N} + \eta n l + \eta\,(\omega_\tau - 1)\,\tau n l_{\omega_N} + \eta\pi_\tau = 0, \tag{19}$$

where η is the Lagrange multiplier. After some manipulation with equations (18) and (19), using again Roy's identity, and (6) and (10), the second-best condition for the optimal provision of g can be written as follows:

$$\frac{nV_{\omega_N}\omega_g}{\lambda} = \frac{1}{1 - \frac{\tau l_{\omega_N}}{l}}\left(1 - \tau n l_{\omega_N}\omega_g - \pi_g\right) \tag{20}$$

In essence, the interpretation of equation (20) is the following. As before, the LHS are the marginal benefits received by the household when an additional unit of public input is provided. On the other hand, two terms can be distinguished in the RHS (Schob, 1994). The first of those, the marginal cost of public funds ($MCPF$), measures the social cost of each unit collected through distortionary taxes. In this respect, we follow the recent paper by Liu (2004) in which the role of the MCPF is justified based on a cost-benefit analysis. The second term is the MRTC.

When expressions (5) and (9) are inserted into (20), algebraic manipulation also results in the production efficiency condition (17). This result was first found by Diamond and Mirrless (1971) and later confirmed by Feehan and Matsumoto (2002) and Dahlby and Wilson (2003) in their studies of fiscal federalism issues.

However, in contrast to Feehan and Matsumoto (2002), we have to follow a second-best rule when taxes different to lump-sum are used. As was

pointed out in the Introduction, this is a consequence of defining a firm-augmenting public input creating rents. As long as these profits accrue to the government the production efficiency condition holds but the optimality rule for the public input provision requires the consideration of a second-best scenario[5].

Since there are no differences in terms of production efficiency between lump-sum and distorting taxes, condition (17) cannot be used to see what happens with the quantities of public input provided under each tax setting. Hence, in the section 5 we use a numerical approach to solve the equation systems defined by the first order conditions (14)-(15) and (18)-(19) plus the government budget constraint.

4 The second-best level and the marginal excess burden approach

On the basis of previous optimality rules, Gaube (2000) and Chang (2000) have suggested several criteria for level comparisons between the first and second-best environments. The support for their approaches is related to the complementarity or substitutability relationships among private goods, and between these and public goods as well. Unfortunately, this procedure has a limitation in our case: the public input does not enter the utility function as an argument, and consequently cannot be directly defined as a substitute or complement to the (taxed) private goods.

An alternative manner to gain an insight into whether the second-best level may exceed the first-best level, we shall follow the approach suggested by Gronberg and Liu (2001), which is better suited to our environment. The crucial point is the concept of marginal excess burden (MEB). Previously, the total excess burden (TEB) needs to be defined:

Definition 1 *The total excess burden (TEB) of a tax system is the difference between the equivalent variation measure (absolute value) of the loss in utility due to taxation and the revenue collected.*

Formally,

$$TEB = -E(\omega(g,n), V(\omega(g,\tau,n)-\tau)) + E(\omega(g,n), V(\omega(g,n))) + \pi(g,n) - R, \tag{21}$$

[5] We later offer a possible interpretation of this point linked to the concept of externality.

where the difference between both expenditure functions $E(.)$ is the equivalent variation when distorting taxes are included, and $R = \tau l\left(\omega\left(g,\tau,n\right)-\tau\right)+\pi\left(g,\tau,n\right)$ is the per capita government revenue[6]. $E(\omega\left(g,n\right),V\left(\omega\left(g,n\right)\right))$ is the value of total time endowment available $(\bar{L})$ in the first-best scenario, i. e., $\omega\bar{L}$. Since in a lump-sum environment an exogenous income deriving from economic rents is also availaible, the term $\pi\left(g,n\right)$ has to be considered to compute the TEB.

Hence, the MEB can be defined as $MEB = \frac{d(TEB)}{dR}$. Gronberg and Liu (2001) claim that if the utility function is strictly quasi-concave and the $MEB > 0$ for all R, then the second-best public good level lies below the first-best level (Sufficient condition).

Applying Shephard's lemma and the implicit function theorem and after some algebraic manipulation, it renders:

$$\begin{aligned}\frac{d\left(TEB\right)}{dR} &= \left(H+l+l_c\right)\frac{\partial\omega}{\partial g}\frac{dg}{dR}\\ &\quad -\frac{\partial E}{\partial V}V_{\omega_N}\left[\frac{\partial\omega}{\partial g}\frac{dg}{dR}+\frac{\partial\omega}{\partial\tau}\left(\frac{1-\tau l_{\omega_N}\frac{\partial\omega}{\partial g}\frac{dg}{dR}-\frac{\partial\pi}{\partial g}\frac{dg}{dR}}{l-\tau l_{\omega_N}\left(1-\frac{\partial\omega}{\partial\tau}\right)+\frac{\partial\pi}{\partial\tau}}\right)-1\right]-1+\frac{\partial\pi}{\partial g}\frac{dg}{dR},\end{aligned} \tag{22}$$

where l_c is the compensated labour supply function. Given the assumptions of the model, it is not possible to elucidate the sign of this derivative. Note that not only is there no clear-cut relationship between the public input and profits (recall expression (9)), but also the multiple interactions among involved variables lead to an indeterminate sign for expression (22). For example, the feedback effect caused by the public input provision on labour tax revenue (the second term in the numerator between brackets) ussually implies a tendency towards a reduction in the MEB (Gronberg and Liu, 2001). But things are different in our case: a decrease in the government revenue requirements as a result of the feedback effect may increase the MEB. This occurs because of the positive relationship between the tax rate on labour and the wage rate, so that a smaller tax rate negatively affects the households' utility by reducing wage rates[7].

[6] This measure has been studied by Kay (1980) and Triest (1990). See, for example, the survey by Hakonsen (1998).

[7] Obviously, on the other hand, taxation damages the households' utility as individuals have to work more.

Another point which is worth nothing of note is the sensitivity of the MEB with respect to the number of households[8]. In particular, the higher the number of households, the greater the likelihood of a negative MEB. This can be seen within our framework by assuming a Cobb-Douglas specification for the utility function (which involves $\frac{\partial \omega}{\partial \tau} = l_{\omega_N} = 0$). Then the MEB becomes:

$$MEB = F_{Lg}\frac{dg}{dR}\left((H + l + l_c) - \frac{\partial E}{\partial V}V_{\omega_N} - nl\right) + \frac{\partial E}{\partial V}V_{\omega_N} - 1 + F_g, \quad (23)$$

which is decreasing in n.

To summarize this section, according to Gronberg and Liu's (2001) criterium, a higher level of public input may be provided in a second-best environment than that under lump-sum tax. Moreover, issues concerning the impact of public inputs on rents and the number of households need to be taken into account. Next we provide the results of a numerical simulation implemented to determine whether may the first-best level be below the second-best level in the provision of public inputs

5 Numerical simulation and results

We use Newton-Raphson's well-known method for the numerical exercise, due to its high convergence speed and its simple structure. The Newton-Raphson method is rather simple. Let f be the function to be optimized:

$$\begin{array}{rcl} f : X \subseteq \mathbb{R}^n & \longrightarrow & \mathbb{R}^m \\ x & \longrightarrow & f(x) \end{array}$$

where f is differentiable. Given $x_0 \in X \subseteq \mathbb{R}^n / \exists \nabla f(x_0)^{-1}$, the process has the following steps for each iteration $i > 0$. Firstly, we evaluate $\nabla f(x_i)$[9]. Secondly, if $\exists \nabla f(x_i)^{-1}$, we calculate the point we use in the next iteration: $x_{i+1} = x_i - f(x_i) * \nabla f(x_i)^{-1}$. Finally, the stop criterion is defined as follows: given $\epsilon > 0$, if $\|x_{i+1} - x_i\| < \epsilon$, then x_{i+1} is the root of the function f[10]; otherwise, the procedure continues up to the condition to be fulfilled.

[8] Gronberg and Liu (2001) point out the relevance of this issue in their footnote 18.

[9] The gradient of this function can be obtained analytically or numerically.

[10] There are other possible ways of setting the stopping criterion. For example, ϵ could be defined as the per cent pre-specified relative tolerance error. Under these conditions, we would have the following criterion: $\|\frac{x_{i+1} - x_i}{x_{i+1}}\| < \epsilon$.

With regard to this simulation, we consider the set of first order conditions of problem (13) under each scenario as the objective function. On this basis, we obtain the optimum values for the variables of the problem. Three different utility functions have been used in an attempt to achieve results which are as general as possible. We have considered a quasi-linear utility function (Gronberg and Liu, 2001); a Cobb-Douglas utility functions (Atkinson and Stern, 1974; Wilson, 1991); and a CES utility function (Wilson, 1991b; Gaube, 2000). Specifically,

$$U(x, H) = x + 2H^{\frac{1}{2}} \tag{24}$$

$$U(x, H) = a \log x + (1 - a) \log H \tag{25}$$

$$U(x, H) = (x^{\rho} + H^{\rho})^{\frac{1}{\rho}} , \tag{26}$$

where $H = \bar{L} - l$ is leisure, $a \in (0, 1)$, and $\rho = 0.5$. The production function is given by $F(nl, g) = (nl)^{\alpha} g^{1-\alpha}$, where $\alpha \in (0, 1)$, and the total time endowment $(\bar{L})$ is fixed and equal to 24. The exercise we carry out sets as the following parameters benchmark : $a = 0.5$, $\alpha = 0.7$, and $n = 100$. Next, the results are obtained by solving the lump-sum problem -equations (14) and (15)- and the distortionary taxation problem -equations (18) and (19). The government budget constraint is also used in both non-linear equation systems. Routines used in the simulations are available upon request.

Tables 1-3 show the main results. Public input provided under the second-best tax setting is always higher than that corresponding to the first-best environment. In principle, this is contrary to the bulk of existing literature, which generally states that the first-best level exceeds the second-best level. Indeed, our approach based on the MEB highlights the difference between the traditional public good and the public input considered here. In fact, the sign of the MEB is ambiguous in our framework under a number of non-restrictive assumptions.

When the results are explored more extensively one realizes that the key variable determining the level of public input is profits. Whereas the values of tax rates under both environments are very low, almost the whole public input is financed on the basis of economic profits fully taxed by the government. This is in line with the characterization of the public input as firm-augmenting. Recall that the public input is said to be firm-augmenting

when the production function exhibits constant returns to scale in all factors, including the public input. In this case, profits arise and this can be interpreted as an externality. As long as the government becomes the owner of these profits, the positive effect of public inputs is taken into account, thus the externality is internalized. In other words, since all profits accrue to government, the optimal quantity of public input is directly related to the extent of these profits.

Our simulation allows additional results to be obtained regarding the sensitivity to changes in parameters of the utility and in production function and in the number of households. Firstly, in the case of the Cobb-Douglas utility function (Table 2), it can be seen how the public input level increases when the preference for the private good goes up (parameter a). The higher the preference for the private good, the smaller the preference for leisure, and consequently more time is devoted to work. Given the assumptions of the model, this means enhanced production, and hence more profits[11].

Secondly, the output elasticity to public input $(1-\alpha)$ has a positive relationship with the public input level. Certainly, this is a very logical as result. In a sense, this elasticity measures the extent of the externality deriving from public input provision. The size of this effect, which is completely internalized by the government when all rents are taxed, determines the level of public input, and a direct relationship between $1-\alpha$ and g is to be expected.

Thirdly, there is a direct link between the number of households and the level of public inputs. The proportion by which this occurs is linear, that is, if population increases by 10 times, the level of public input goes up by the same proportion. Again, profits define the optimal provision of public spending. On the basis of expression (8), and since the production function exhibits constant return to scale (i. e., it is homogeneous of degree 1), it can be claimed that the function $\pi(.)$ is homogeneous of degree 1. Accordingly, increments in the number of households are followed by increases in profits at the same rate, and consequently by identical increases in the public input.

6 Concluding remarks

This paper has dealt with an issue to which the existing literature has not paid much attention: the optimal *level* of public inputs under different tax

[11] It can be proved that the derivative of expression (8) with respect to a is positive. See Appendix 1.

settings. Previous contributions have focused on the case of consumption public goods or have discussed the optimal *rules* of productive public spending. However, both the social welfare implications of taxation and the characterization of public inputs as growth-enhacing public instruments make this issue highly relevant for policy-makers.

We have built a simple general equilibrium model where public inputs provide productive services to firms. Two different tax settings have been considered: one with lump-sum taxation and another using per unit tax on labour. Firstly, we have obtained the optimal conditions for the provision of public inputs under each scenario, concluding that we have found that a second-best rule must be considered when lump-sum taxes are not available, even though the production efficiency condition holds and taxes are optimally set.

Secondly, we have shed some light on the controversy concerning level comparisons between both tax settings. Contrary to the existing literature on public goods, which generally holds that the first-best level is higher than the second-best level, the relationship is the reverse for the case of public inputs. This result is a consequence of the extent of profits, which are used in our framework to internalize the externality of the public input. Moreover, we have detected a positive relationship between the level of public input and its output elasticity in the production function, and with the number of households in the economy.

This paper raises various policy implications. Firstly, optimality rules and levels in the public input provision require a different treatment to those corresponding to public goods. This is due to the more intense feedback effect derived from public inputs and certain characteristics of the externalities involved in their provision. The second policy implication is precisely related to th nature of this externality. As long as the public input provision may generate external effects, benefit-based taxation becomes an efficient source of resources for the government and a means of improving efficiency.

Further research on this issue is warranted. Gronberg and Liu's criterion for level comparisons could be studied in a more detailed way for the case of public inputs. Similarly, It would be interesting to carry out a further research into the behavior of the MEB with public inputs, and according to certain parameters of the model, namely the number of households, output elasticity to the public input, and so on.

References

[1] Atkinson, A. B. and Stern, N. H. (1974), Pigou, taxation and public goods, Review of Economic Studies 41, 119-128.

[2] Aronsson, T. and Sjogren T., (2001), Income taxation, commodity taxation and the provision of public goods under labour market distortions, FinanzArchiv, 59, 347-370.

[3] Ballard, C. L. and Fullerton, D. (1992), Distortionary taxes and the provision of public goods, *Journal of Economic Perspectives*, 6: 117-131.

[4] Boadway, R. and Keen, M. (1996), Efficiency and the optimal direction of federal-state transfer, International Tax and Public Finance 3, 137-155.

[5] Chang, M. (2000), Rules and levels in the provision of public goods: the role of complementarities between public good and taxed commodities, International Tax and Public Finance 7, 83-91.

[6] Dahlby, B. and Wilson, L. S. (2003), Vertical fiscal externalities in a federation, *Journal of Public Economics* 87, 917-930.

[7] Diamond, P. A. and Mirrlees, J. A. (1971), Optimal taxation and public production I-II, American Economic Review 61, 8-27 and 261-278.

[8] Feehan, J. P. (1989), Pareto-efficiency with three varieties of public input, Public Finance/Finances Publiques 44, 237-248.

[9] Feehan, J. P. and M. Matsumoto (2000), Productivity-enhancing public investment and benefit taxation: the case of factor-augmenting public inputs, Canadian Journal of Economics 33 (1), 114-121.

[10] Feehan, J. P. and Matsumoto, M. (2002), Distortionary taxation and optimal public spending on productive activities, Economic Inquiry, 40 (1), 60-68.

[11] Gaube, T. (2000), When do distortionary taxes reduce the optimal supply of public goods?, Journal of Public Economics 76, 151-180.

[12] Gronberg, T. and Liu, L. (2001), The second-best level of a public good: an approach based on the marginal excess burden, Journal of Public Economics Theory 3 (4), 431-451.

[13] Hakonsen, L. (1998), An investigation into alternative representations of the marginal cost of public funds, International Tax and Public Finance 5, 329-343.

[14] Kaizuka, K. (1965), Public goods and decentralization of production, Review of Economics and Statistics 47, 118-120.

[15] Kay, J. A. (1980), The deadweight loss from a tax system, Journal of Public Economics 13, 111- 119.

[16] King, M. A. (1986), A pigouvian rule for the optimum provision of public goods, Journal of Public Economics 30, 273-291.

[17] Konishi, H. (1993), A note on public good provision and commodity taxes, The Economic Studies Quarterly 44 (2), 178-184.

[18] Liu, L (2004), The marginal cost of funds and the shadow prices of public sector inputs and outputs, International Tax and Public Finance 11, 17-29.

[19] Pestieau, P. (1976), Public intermediate goods and distortionary taxation, European Economic Review 7, 351-357.

[20] Pigou, A. C. (1947), A study in Public Finance, Third edition, Macmillan: London.

[21] Samuelson, P. A. (1954), The pure theory of public expenditure, Review of Economics and Statistics 36, 387-389.

[22] Sandmo, A. (1972), Optimality rules for the provision of collective factors of production, Journal of Public Economics, 1, 149-157.

[23] Schob, R. (1994), On the marginal cost and marginal benefit of public funds, *Public Finance* 49 (1), 87-106.

[24] Stiglitz, J. E. and Dasgupta, P. S. (1971), Differential taxation, public goods and economic efficiency, Review of Economics Studies 38, 151-174.

[25] Triest, R. K. (1990), The relationship between the marginal cost of public funds and the marginal excess burden, American Economic Review 80, 557-566.

[26] Wildasin, D. E. (1984), On public good provision with distortionary taxation, Economic Inquiry 22, 227-243.

[27] Wildasin, D. E. (1986), Urban Public Finance, Harwood Academic. Chur, Switzerland.

[28] Wilson, J. D. (1991), Optimal public good provision with limited lump-sum taxation, American Economic Review 81 (1), 153-166.

Appendix 1

Proposition 1 *Let* $\pi = [nl(\omega(g,\tau,T,n)-\tau)]^{\alpha} g^{1-\alpha} - nl(\omega(g,\tau,T,n)-\tau)\omega(g,\tau,T,n)$ *be the profits. As we use the Cobb-Douglas utility function, then* $l(\omega_N) = \bar{L}a + \dfrac{(1-a)T}{n\omega_N}$ *is the labour supply depending on the net wage* ($\omega_N = \omega(g,\tau,T,n) - \tau$) *. Under these conditions,* $\dfrac{\partial \pi}{\partial a} > 0$.

Proof. *From the above expression of* π*, we have:*

$$\begin{aligned}\frac{\partial \pi}{\partial a} &= \alpha[nl(\omega(g,\tau,T,n)-\tau)]^{\alpha-1} g^{1-\alpha} l'(\omega(g,\tau,T,n)-\tau)\frac{\partial \omega}{\partial a} \\ &\quad -nl(\omega(g,\tau,T,n)-\tau)\frac{\partial \omega}{\partial a} - nl'(\omega(g,\tau,T,n)-\tau)\omega(g,\tau,T,n)\frac{\partial \omega}{\partial a}\end{aligned}$$

Using the equation (4) we simplify:
$\dfrac{\partial \pi}{\partial a} = -nl(\omega(g,\tau,T,n)-\tau)\dfrac{\partial \omega}{\partial a} \Longrightarrow sign(\dfrac{\partial \pi}{\partial a}) \neq sign(\dfrac{\partial \omega}{\partial a})$.

As a result of this, if $\dfrac{\partial \omega}{\partial a} < 0$ *then* $\dfrac{\partial \pi}{\partial a} > 0$*.So, we are interested in demonstrating that* $\dfrac{\partial \omega}{\partial a} < 0$

Let us combine (4) and $l(\omega - \tau) = \bar{L}a + \dfrac{(1-a)T}{(\omega-\tau)n}$*, so that it renders*

$$\omega = \alpha \left(\frac{g}{n}\right)^{1-\alpha} \left(\bar{L}a + \frac{(1-a)T}{(\omega-\tau)n}\right)^{\alpha-1}$$

Using the Implicit Function Theorem, it yields

$$\frac{\partial \omega}{\partial a} = \alpha(1-\alpha)\left(\frac{g}{n}\right)^{1-\alpha}\left(\bar{L}a + \frac{(1-a)T}{(\omega-\tau)n}\right)^{\alpha} \frac{-\left(\bar{L} + \frac{T}{n}\frac{\partial}{\partial a}\left(\frac{1-a}{\omega-\tau}\right)\right)}{\left(\bar{L}a + \frac{(1-a)T}{(\omega-\tau)n}\right)^2}$$

where

$$\frac{\partial}{\partial a}\left(\frac{1-a}{\omega-\tau}\right) = \frac{-(\omega-\tau) + (1-a)\dfrac{\partial \omega}{\partial a}}{(\omega-\tau)^2}$$

Then,

$$\frac{\partial\omega}{\partial a} = \alpha(1-\alpha)\left(\frac{g}{n}\right)^{1-\alpha}\left(\bar{L}a + \frac{(1-a)T}{(\omega-\tau)n}\right)^{\alpha}\frac{\dfrac{-\bar{L}n(\omega-\tau)^2 - T\,(\omega-\tau) + T(1-a)\dfrac{\partial\omega}{\partial a}}{n(\omega-\tau)^2}}{\dfrac{\left(\bar{L}a(\omega-\tau)n + (1-a)T\right)^2}{(\omega-\tau)^2 n^2}} =$$

$$= \alpha(1-\alpha)\left(\frac{g}{n}\right)^{1-\alpha}\left(\bar{L}a + \frac{(1-a)T}{(\omega-\tau)n}\right)^{\alpha}\left(\frac{-\bar{L}n^2(\omega-\tau)^2 - nT\,(\omega-\tau) + nT(1-a)\dfrac{\partial\omega}{\partial a}}{\left(\bar{L}a(\omega-\tau)n + (1-a)T\right)^2}\right)$$

Finally, we obtain:

$$\frac{\partial\omega}{\partial a} = \frac{\alpha(1-\alpha)\left(\frac{g}{n}\right)^{1-\alpha}\left(\bar{L}a + \frac{(1-a)T}{(\omega-\tau)n}\right)^{\alpha}\dfrac{-n^2\bar{L}(\omega-\tau)^2 - Tn(\omega-\tau)}{\left(\bar{L}a(\omega-\tau)n + (1-a)T\right)^2}}{1 - \alpha(1-\alpha)\left(\frac{g}{n}\right)^{1-\alpha}\left(\bar{L}a + \frac{(1-a)T}{(\omega-\tau)n}\right)^{\alpha}\dfrac{T(1-a)n}{\left(\bar{L}a(\omega-\tau)n + (1-a)T\right)^2}}$$

As $Num(\frac{\partial\omega}{\partial a}) < 0$ *and* $Den(\frac{\partial\omega}{\partial a}) > 0$, *we conclude* $\frac{\partial\omega}{\partial a} < 0$. ■

Tables

Table 1: Quasi-linear utility.

	Benchmark	$n = 100$		$\alpha = 0.7$	
		$\alpha = 0.6$	$\alpha = 0.8$	$n = 1$	$n = 1000$
Lump-sum					
Public Input	327,2234	316,5251	274,2790	3,2722	3272,2340
Profits	327,2134	316,5151	274,2690	3,2721	3272,1340
Distorsionary					
Public Input	327,4674	316,7426	274,5220	3,2746	3274,6740
Profits	327,2847	316,5968	274,3169	3,2728	3272,8470

Source: Benchmark ($n = 100$, $\alpha = 0.7$)

Table 2: Cobb-Douglas utility.

	Benchmark	$n = 100, \alpha = 0.7$		$n = 100, a = 0.5$		$a = 0.5, \alpha = 0.7$	
		$a = 0.1$	$a = 0.9$	$\alpha = 0.6$	$\alpha = 0.8$	$n = 1$	$n = 1000$
Lump-sum							
Public Input	214,9042	42,9957	386,8126	260,6041	160,5114	2,1490	2149,0420
Profits	214,8942	42,9857	386,8026	260,5941	160,5014	2,1489	2148,9420
Distorsionary							
Public Input	215,0591	43,0118	387,1064	260,7841	160,6477	2,1505	2150,5910
Profits	214,9391	42,9878	386,8904	260,6641	160,5277	2,1493	2149,3910

Source: Benchmark ($n = 100$, $\alpha = 0.7$, $a = 0.5$)

Table 3: CES utility ($\rho = 0.5$).

	Benchmark	$n = 100$		$\alpha = 0.7$	
		$\alpha = 0.6$	$\alpha = 0.8$	$n = 1$	$n = 1000$
Lump-sum					
Public Input	126,6753	128,0770	111,8926	1,2667	1266,7530
Profits	126,6653	128,0670	111,8826	1,2666	1266,6530
Distorsionary					
Public Input	126,7560	128,1486	111,9813	1,2675	1267,5600
Profits	126,6853	128,0896	111,8977	1,2668	1266,8530

Source: Benchmark ($n = 100$, $\alpha = 0.7$)