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Redistribution and labour supply

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Abstract

Classical non-behavioural results on redistribution are not necessarily satisfied when labour supply reactions are taken into consideration. We postulate necessary and sufficient conditions to ensure redistribution in this wider setting. We also find that the functional specification of the labour supply may condition final results.

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1. Introduction

This note explores the effect of taxes on redistribution when labour supply behavioural reactions are taken into account. In this respect, redistribution is measured in the classical way, defined in terms of the transition from pre-tax to post-tax income distributions. We prove that non-behavioural (static) standard results on redistribution are not necessarily fulfilled in a more general behavioural (dynamic) framework.[2] To this extent, correct redistribution analysis requires incorporation of behavioural effects induced by taxes.

In this new setting, we have to distinguish among three different income distributions:

(i) the *initial pre-tax* income distribution, which corresponds to gross incomes in the absence of taxes,

(ii) the *initial post-tax* income distribution, which is the gross income distribution once changes in labour supply have been taken into consideration as a result of tax change,

(iii) the *final post-tax* income distribution, which reflects the distribution of *initial post-tax* incomes net of taxes.

[2] The seminal papers in static literature are Fellman (1976), Jakobsson (1976) and Kakwani (1977). Extensions to personal income taxes with positive thresholds can be found in Keen et al. (1996). As far as we know, Preston (1987, 1989) is the only reference using a dynamic approach.

In exploring redistribution, this conceptual distinction is theoretically relevant as it implies that net income and tax liability are endogenously determined. With this framework, the relevant transition to measure redistribution is the one from the *initial pre-tax* to the *final post-tax* income distribution, and not the one from the *initial post-tax* to the *final post-tax* income distribution, as the static setting assumes.

In this wider context, we postulate necessary and sufficient conditions to ensure redistribution. Moreover, standard Jakobsson-Fellman-Kakwani (JFK) results can be preserved under restricted conditions on the structure of the tax system and on the specification of the labour supply. As a result, we also conclude that labour supply specification matters and may influence the final redistributive results of a given tax reform. Therefore, testing the correct functional form for the labour supply would be a compulsory requirement in applied work.

The plan of the paper is as follows. Section 2 sets the model and quantifies progressivity in terms of wage elasticities. Section 3 offers some applications to alternative tax structures and labour supply functions. Finally, section 4 concludes.

2. The model

Assume a distribution of homogeneous individuals i=1,...,H with differences only in the exogenous gross wage rates $W=(w_1,...,w_H)$, with the same non-labour income m_i (assumed initially to be zero). Assume the *initial pre-tax* income vector $Y=(y_1,...,y_H)$ generated by

$y_i = w_i L_i$, where L_i is the pre-tax labour supply and the distribution Y is confined to positive pre-tax income levels $y_i \in \mathbf{R}_{++} \equiv (0,\infty)$, as usual.

Labour supply

The labour supply adopts this general form:

$$L_i = \begin{cases} L(w_i, m_i), & \dfrac{\partial L}{\partial w_i} \geq 0, \dfrac{\partial L}{\partial m_i} \leq 0 \\ \bar{L}, & w_i \geq \bar{w} \end{cases}$$

[OLE1]

where m_i is non-labour income, initially assumed to be zero, and $\bar{L}$ is the maximum labour supply, obtained for $\bar{w}$. This general form includes typical cases in the literature such as the CES (Stern, 1976 and Zabalza, 1983), linear (Hausman, 1980, 1981) and log-linear (Burtless and Hausman, 1978) specifications. These specific functional forms will be observed below.

Tax structure

The tax structure adopts the general form, T: $\mathbf{R}_+ \rightarrow \mathbf{R}$, such that T(u) is continuous, non-decreasing and differentiable on u and $\frac{\partial T}{\partial u} < 1, \quad \forall u$. These are the standard assumptions in the literature.

Redistribution

Our aim is to generalise the main standard results on redistribution of JFK. In doing so, we use the concept of a local residual progression. According to these authors, a necessary and sufficient condition for the existence of non-negative redistribution is that local residual progression is always less than or equal to one (and higher than or equal to zero), for every pre-tax income distribution. When comparing two tax systems, the necessary and sufficient condition for non-lower redistribution is that the residual progression should be reduced (and non-negative).

In order to allow for labour supply effects, we distinguish between the *initial post-tax income vector*[3] $Y' \in \mathbf{R}_{++}$, generated by $y'_i = w_i L'_i$, and the *final post-tax income vector* $X' \in \mathbf{R}_{++}$, generated by $x'_i = y_i' - T(y_i')$. L'_i is the post-tax labour supply:

$$L'_i(w_i, m_i) = \begin{cases} L(w'_i, m'_i), & \\ \overline{L}, & w'_i \geq \overline{w} \end{cases}$$

where w'_i is the marginal post-tax wage rate, $w_i(1\text{-}t)$, and m'_i is the virtual non-wage income. With this setting, redistribution focuses on the transition from the *initial pre-tax income distribution* (Y) to X'.

The redistribution effect is consistently defined with the Lorenz curve criterion of second-order relative inequality dominance as proposed by Atkinson (1970).

[3] Note that the initial post-tax income Y' corresponds to the actual tax base.

Definition

Given any initial pre-tax and final post-tax distributions, Y and X' $\in \mathbf{R}^{\mathbf{H}}_{++}$, a tax system is redistributive (progressive) if and only if X' $\geq_L$ Y; that is, if and only if X' weakly dominates Y:

$$X' \geq_L Y \quad \Leftrightarrow \quad \sum_{i=1}^{k} \frac{x'_{(i)}}{\mu(X')} \geq \sum_{i=1}^{k} \frac{y_{(i)}}{\mu(Y)} \quad , \forall k = 1, \ldots, H$$

where μ(X') and μ(Y) denote the mean of X' and Y, respectively. The terms $x'_{(i)}$ and $y_{(i)}$ are the i^{th} smallest elements of the corresponding distributions.

Making use of this definition we can state the following proposition, which is a natural extension of JFK:

Proposition

Given any initial pre-tax and final post-tax distributions Y and X' $\in \mathbf{R}^{\mathbf{H}}_{++}$, a necessary and sufficient condition for a tax system to be non-negative redistributive (progressive) is

$$0 \leq \eta_{x',y} = \eta_{y',y} \eta_{x',y'} \leq 1$$

That is, the local elasticity of x' with respect to y (local residual progression) is always not greater than one (and not lower than zero). As can be noticed, JFK's condition ($0 \leq \eta_{x',y'} \leq 1$) is a particular case when labour supply behaviour is discarded, so that $\eta_{y',y}=1$, which implies $\eta_{x',y}=\eta_{x',y'}$. It is worth noting that $\eta_{y',y}$ is the key concept here, which captures labour supply behaviour changes, and we shall call it the dynamic component.

This key concept $\eta_{y',y}$ can be expressed in terms of wage-income elasticities as:

$$\eta_{y',y} = \frac{\eta_{y',w}}{\eta_{y,w}}$$

Since $\eta_{y',w} = \eta_{L',w} + 1$, it can be also written as a function of wage-labour supply elasticities:

$$\eta_{y',y} = \frac{\eta_{L',w} + 1}{\eta_{L,w} + 1}$$

Note that these elasticities are expressed in terms of the exogenous pre-tax wage rate as it identifies individuals.

This can be extended to characterise redistribution for two alternative tax structures. Given any initial pre-tax income distribution Y, assume two alternative taxes T' and T''. These tax structures generate, respectively, two initial post-tax distributions, Y' and Y'', and two final post-tax distributions, X' and X''. Then, T' is more redistributive than T'' if and only if local residual progression from Y to X' is always not greater than the one from Y to X'':

$$0 \leq \eta_{x',y} \leq \eta_{x'',y}$$

which is equivalent to:

$$0 \leq \eta_{y',y}\eta_{x',y'} \leq \eta_{y'',y}\eta_{x'',y''}$$

3. Applications to alternative taxes and labour supply specifications

In the following, let us analyse the labour supply effects and the particular value of $\eta_{y',y}$ for different taxes under different labour supply specifications. In general, we prove that the condition mentioned above for positive redistribution is difficult to satisfy, even in the homogeneous case. As an illustration of this, firstly we analyse the case of the proportional tax, for which standard zero-redistribution is only guaranteed under restricted conditions.

Secondly, we study linear progressive taxation applied to alternative labour supply specifications in order to search for conditions that guarantee positive redistribution.

3.1 Proportional Tax Case

We see that under very restricted conditions a proportional tax achieves non-negative redistribution with this tax behavioural framework.

Initial pre-tax income y_i is:

$$y_i = \begin{cases} w_i L(w_i, 0), & 0 \le w_i \le \overline{w} \\ w_i \overline{L}, & w_i \ge \overline{w} \end{cases}$$

Post-tax labour supply L_i' is:

$$L_i' = \begin{cases} L(w_i(1-t),\ 0), & 0 \le w_i(1-t) \le \overline{w} \\ \overline{L}, & w_i(1-t) \ge \overline{w} \end{cases}$$

Initial post-tax income y_i' is:

$$y'_i = \begin{cases} w_i L(w_i(1-t),\ 0), & 0 \le w_i(1-t) \le \overline{w} \\ w_i \overline{L}, & w_i(1-t) \ge \overline{w} \end{cases}$$

Final post-tax income x_i' is:

$$x_i' = \begin{cases} w_i(1-t)L(w_i(1-t),\ 0), & 0 \le w_i(1-t) \le \overline{w} \\ w_i(1-t)\overline{L} & w_i(1-t) \ge \overline{w} \end{cases}$$

As $h_{x',y'} = 1$, the necessary and sufficient condition to guarantee redistribution neutrality between $(\mathbf{0}, \overline{\boldsymbol{w}})$ is $h_{x',y} = h_{y',y} = 1$. Hence, $h_{L',w} = h_{L,w}$. This stringent condition is satisfied in cases such as the log-linear labour supply specification and, when m_1=0, in the cases of the Cobb-Douglas and the linear labour supply functions.

3.2 The linear tax under different labour supply specifications

Now we study the affine tax system as analysed by Atkinson and Stiglitz (1980), Atkinson (1995) and Hall and Rabushka (1995):

$T(y) = -Z + ty,\ Z \geq 0 \ and\ 1 > t \geq 0$

This tax scheme will be examined below under three alternative labour supply specifications: the CES, linear and log-linear.

3.2.1 *CES*

The CES utility function is defined as follows:

$U(y, L) = y^{r} + a(\overline{L} - L)^{r} \quad a > 0,\ r < 1$

from which the pre-tax labour supply L can be recovered as:

$$L(w_i, 0) = \frac{\overline{L}\ (w_i / \boldsymbol{a})^{s}}{w_i + (w_i / \boldsymbol{a})^{s}}, \quad w_i \geq 0$$

where $\overline{L}$ represents the maximum labour and σ = 1/(1-ρ) is the elasticity of substitution.

Post-tax labour supply L' is:

$$L'(w_i, 0) = \frac{\overline{L}\,(w_i(1-t)/a)^s - Z}{w_i(1-t) + (w_i(1-t)/a)^s}, \quad 0 \leq w_r \leq w_i$$

where w_r is the reservation wage.

It can be proved that for any $\alpha>0$, $\sigma>0$ (or $\rho<1$), $Z \geq 0$ and $1 > t > 0$, and $w_i \geq w_r$,

$$\eta_{L',w} > \eta_{L,w}$$

So

$$\eta_{y',w} > \eta_{y,w}$$

Hence

$$\eta_{y',y} > 1$$

Positive redistribution is not guaranteed, as $\eta_{x',y'}<1$. Note also that under the proportional tax case, Z=0 and t>0 case, negative redistribution arises, unless σ converges to zero, which is the Cobb-Douglas case. There is a regressivity behavioural bias.

3.2.2 *The linear labour supply* (Hausman, 1980, 1981)

Pre-tax labour supply L is:

$$L_i(w_i, 0) = \begin{cases} aw_i, & 0 \leq w_i \leq \overline{w} \\ \overline{L}, & w_i \geq \overline{w} \end{cases}$$

where a>0

Post-tax labour supply L' is:

$$L_i'(w_i, 0) = \begin{cases} aw_i(1-t) - bZ, & 0 \leq w_r \leq w_i \leq \overline{w}' \\ \overline{L}, & w_i \geq \overline{w}' \end{cases},$$

where b>0 and $\overline{w} < \overline{w}'$.

It can be proved that for any a, b>0, Z > 0 and 1 > t > 0, and $\overline{w} \geq w_i \geq w_r$,

$$h_{L',w} > h_{L,w} = 1$$

So

$$h_{y',y} > 1$$

Nevertheless, positive redistribution is produced -between w_r and the lowest of $\overline{w}$ and w^*=2/b(1-t)- as:

$$h_{x',y} = h_{x',y'} h_{y',y} < 1$$

Note also that under the proportional tax case (Z=0 and t>0), the polar case of zero redistribution arises. In addition, in this case Z<0 implies negative redistribution (between w_r and the lowest of $\overline{w}$ and w^*). In general, for m_i=m, the zero redistribution case is reached at Z=-m. There is also a regressivity dynamic bias.

3.2.3 *The iso-elastic labour supply specification* (Burtless and Hausman 1978)

Pre-tax labour supply L is:

$$L_i(w_i, m_i) = \begin{cases} A w_i^a m_i^b, & 0 \leq w_i \leq \overline{w} \\ \overline{L}, & w_i \geq \overline{w} \end{cases}$$

where A, α>0 and β≤0 and m_i>0.

Post-tax labour supply L' is:

$$L_i'(w_i, m_i) = \begin{cases} A(w_i(1-t))^a (m_i + Z)^b, & 0 \leq w_i \leq \overline{w}' \\ \overline{L}, & w_i \geq \overline{w}' \end{cases},$$

where $\overline{w} < \overline{w}'$.

It can be proved that for any A,α>0, β≤0, Z >0 and 1 > t > 0, and $\bar{w} \geq w_i \geq 0$,

$$\eta_{L',w} = \eta_{L,w} = \alpha$$

So

$$\eta_{y',w} = \eta_{y,w}$$

Hence

$$\eta_{y',y} = 1$$

Positive redistribution is produced for Z>0 (between 0 and $\bar{w}$). Note also that zero redistribution arises under the proportional tax case Z=0 and that negative redistribution is induced under Z<0. Moreover, for any tax system (with non-negative virtual incomes), $\eta_{y',y} = 1$ so $\eta_{x',y} = \eta_{x',y'}$ (between 0 and $\bar{w}$) which is the JFK result under no-tax behaviour. There is behaviour neutrality.

4. Concluding remarks

Making use of the concept of local residual progression, this paper decomposes redistribution into two components:

(i) the contribution to progression due to the impact on labour supply behaviour induced by the tax change, captured by the transition from the initial pre-tax to the final post-tax income distribution,

(ii) the contribution to progression as a consequence of the actual tax liability, quantified by the move from the initial to the final post-tax income distribution.

This decomposition allows a generalisation of the standard JFK conditions on redistribution when labour supply reactions to taxes are taken into account. In this richer framework, we also found that the labour supply specification is relevant in evaluating redistribution of taxation. Further research may explore the extension of the concept of redistribution to incorporate the notion of equality of opportunities.

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