Alethic Statements Are Not Intensional

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RESUMEN

De acuerdo con el punto de vista estándar, los enunciados aléticos (o modales) son *intensionales* en el sentido de que el Principio de Substitución no funciona en su caso. Así, por ejemplo, si en "Necesariamente, nueve es compuesto" se sustituye "nueve" por la expresión co-referencial "el número de los planetas" el enunciado en cuestión se convierte en falso. Se argumenta en este artículo que podemos evitar el adscribir intensionalidad a los enunciados aléticos distinguiendo entre usos *singulares* y usos *funcionales* de las descripciones definidas. De acuerdo con el uso singular, la descripción que se ha dado anteriormente designa en realidad a "el número *efectivo* de los planetas", que es substituible *salva veritate* por "nueve" en todos los enunciados aléticos. A su vez, de acuerdo con el uso funcional esta función es realmente una función de mundos posibles a números y así el Principio de Substitución no se viola tampoco en este caso, pues no puede mantenerse que tal función sea co-referencial con "nueve".

ABSTRACT

According to the standard view, alethic (or modal) statements are *intensional* in that the Principle of Substitution fails for them — e.g. substituting 'nine' in "Necessarily, nine is composite" with the co-referring 'the number of planets' turns this statement from true to false. It is argued in the paper that we could avoid ascribing intensionality to alethic statements altogether by separating between *singular* and *functional* uses of definite descriptions: on the singular use the description given above amounts to 'the *actual* number of planets', which is *salva veritate* substitutable to 'nine' in all alethic statements; on the functional use, in turn, that description is really a function from possible worlds to numbers, and thus the Principle of Substitution is not violated in this case either, since such a function cannot be held to be co-referential with 'nine'.

I. INTENSIONALITY

According to the following *Principle of Substitution* (PS), replacements between co-referential terms are *salva veritate*, or truth-preserving:

PS If in a true statement (or sentence) some expression *e* is replaced by (i.e. substituted with) an expression co-referential with it (i.e. with an expression sharing the referent with *e*), a true statement (sentence) results.

Following Quine and others,[1] let us say that a *position* of an expression in a statement (or sentence) is *intensional* (or *opaque*, *indirect*, *de dicto*, *notional*, *oblique*), if PS does not generally hold for expressions occurring in this position in the statement in question. Let us call a statement itself intensional, or opaque, etc. (with respect to a position), if it has an intensional position.

It is commonly held that *alethic statements* such as "Necessarily, S", "Possibly, S" are intensional. For example,

(1) Necessarily, 9 is composite

is evidently true. Since '9' appears to have the same referent as does 'the number of planets (in our solar system)',

(2) Necessarily, the number of planets is composite,

should be true, if PS were valid. However, (2) is certainly not true; thus, it is claimed, the position '9' occupies in (1) — and of course the one 'the number of planets' occupies in (2) — is not a position in which co-referring terms can be substituted *salva veritate*, i.e. it is an intensional position; consequently, alethic statements (1) and (2) are said to be intensional. In contrast, the occurrences of 'the number of planets' in "The number of planets is not composite" and "The number that is actually the number of planets is necessarily composite" are not intensional, or are extensional, *transparent*, *direct*, *de re*, *relational* (with respect to the position occupied by 'the number of planets'). I intend to show in this paper that despite appearance, alethic statements are not intensional, i.e. are extensional (i.e. PS holds unrestrictedly for them).

II. POSSIBLE WORLDS TREATMENT OF ALETHIC STATEMENTS

Possible worlds semanticists, e.g. Jaakko Hintikka,[2] account for the (alleged) intensionality of (1) and (2) in following manner: In "9 is the number of planets" we are concerned with identity only in one possible world, viz. the actual one, while in (1) and (2), we are concerned with all possible worlds (alternative to the actual one). Thus the expression 'the number of planets' in our identity statement refers to the number that happens to be the number of planets in the actual world, while there is *referential multiplicity* involved in (2), because with it we must take into account not only the actual number of planets but also the numbers of planets in other possible worlds. Accordingly, Hintikka restricts PS to obtain something like the following weaker principle:

PSR If in a true statement some expression *e* is substituted with an expression co-referential with it *in all relevant possible worlds*, a true statement results.[3]

I shall show, in section 5 below, that no restriction of this sort is really needed, but PS holds universally.

III. FUNCTIONS *À LA* FREGE

As a preliminary, a reminder of Frege's application of functions in logic and philosophy of language may be in order. For Frege, there is a fundamental, undefinable difference between *Gegenstände*, or *objects*, and *functions*. In his paper "Function und Begriff" (1891) [Frege (1967), pp. 126f. / (1984), pp. 138f.], and elsewhere, Frege explains this difference by means of an arithmetical example as follows: A function such as $2x^3+x$, where '*x*' indicates an empty place, or is a place-holder, is *incomplete*, for it does not designate an object — only after it is properly *supplemented*, we get an object, e.g. the number 132, when we supplement this function by the number four (i.e. when we apply this function to 4 as an argument).

One of Frege's greatest ideas is to apply this familiar notion of a mathematical function more generally.[4] For example, $x^2=4$ may be regarded as a function as well, viz. a function that gives as a result the *truth value the True* for the arguments 2 and −2 and the truth value *the False* for all other arguments. Functions that return a truth value on application Frege calls *Begriffe*, or *concepts*. Those objects that give the True as a result when a concept is applied to it, are said to *fall under* that concept.

Concepts (and other functions) can also be used outside mathematical discourse; for example, *x is mortal* is a function that returns the True when applied to mortals and the False for the rest, i.e. all and only mortals fall under *being a mortal*. Besides one-place or unary concepts (and other functions) there are of course also many-place concepts, or *relations*, such as $x>y$ and *x gives y to z*. I shall call relations concepts as well. Frege often calls an expression of a concept, i.e. an expression that has a concept as a referent (*Bedeutung*), a *Begriffswort* — I shall use the word *predicate* for this purpose.

Concepts and other functions are non-objects — however, *extensions* or, in general, what Frege calls *Wertverläufe*, or *courses of values*, are objects that correspond to concepts or, in general, to functions. The extension of the concept *being mortal*, for example, is the set of mortal things; the extension of $x^2=4$ is the set {−2, 2}. The notion of a course of values of a function may be seen as a generalization of that of extension of a concept: the course of values of a concept is its extension, while the course of values of a function

that is not a concept is a logical object that is the same for any two functions which always return the same value for the same argument — thus for instance x^2-1 and *(x+1)(x−1)* have the same course of values [see here especially Frege (1893), § 22]. For my present purposes I shall use the word *extension* in an extended sense to cover also functions that are not concepts: I say that two such functions have the same extension if their courses of values coincide. Further, I say that two functions (including concepts) are the same, or identical, if they have the same extension [see e.g. Frege (1967), p. 184 / (1984), p. 200; Frege (1969), pp. 131-3, 197-8 / (1979), pp. 120-2, 182]. This is based on the following consideration [see esp. Frege (1969), pp. 128, 197-8 / (1979), pp. 118, 182]: For singular terms '*a*' and '*b*' it is clear that they have the same referent if they are *salva veritate* substitutable with each other in all fully extensional positions in all statements. The analogous criterion for the sameness of function turns on the sameness of extension, for two predicates (or expressions of functions in general) are *salva veritate* substitutable in all extensional positions if, and only if, they share the extension. Thus, it is natural to hold that if we say that there is between concepts (and other functions) a relation corresponding to the relation of identity between objects, this holds whenever these concepts have the same extension.

Concepts considered so far take objects as arguments, or, since it is perhaps clearer to speak in terms of expressions, so far I have only mentioned predicates that become statements when supplemented by singular terms (terms that refer to objects). These statements are about objects named in them. However, they are about the concepts named in them just as well. Thus, "The object *a* is an *X*", where '*X*' indicates an empty place for a concept, may be regarded as a *second-level* concept *being a concept the object a falls under* [see esp. Frege (1879), §10; (1884), §53; (1893), §22]. The most notable higher-level concepts relate to *quantifiers*. Quantification is about concepts, and thus indeed of second level: "There are horses", for example, says that the concept *being a horse* falls within the second-level concept *being exemplified*, and "All human beings are mortal" that there is a second-level relation of *subordination* between the concepts *being a human being* and being mortal.

IV. Singular vs. functional use of definite descriptions

Let us consider the sentence,

(3) The secretary-general of the UN is wise.

This does not amount to a definite statement unless it is somehow made clear or understood, *which* secretary-general is referred to. That is, a more fully articulated version of (3) is, for instance,

The secretary-general of the UN *in 2005* is wise.

In other words, a definite description such as 'The secretary-general of the UN' is not a referring expression (singular term) in itself, but must somehow be supplemented by a mention of a moment of time. (Of course, such a supplementation is not always given explicitly.)

There is another interpretation of (3): Somebody who thinks that nobody gets elected to the high office mentioned in (3) unless he or she is wise, might use (3) as a way of expressing something like the following:

> (4) All secretary-generals of the UN are (or have been, are and will be) wise.

(Cf. "The horse is herbivorous" as a way of saying, "All horses are herbivorous"). In this case, 'The secretary-general of the UN' in (3) is not a singular referring expression at all but is, to utilize the Fregean approach devised above, functional (as well as quantificational) in character: Just as "All horses are herbivorous" states that the relation of subordination holds between the functions *being a horse* and *being herbivorous*, (3), under the interpretation (4), states that this relation holds between the functions *being a secretary-general of the UN* and *being wise*.

It is straightforward to extend this distinction between *singular* and *functional* uses of definite descriptions to alethic statements such as (2), "Necessarily, the number of planets is composite". This amounts to expressing the distinction between the intensional and extensional interpretation of statements like (2) in terms of singular and functional uses. Accordingly, I suggest that the definite description 'the number of planets' is used in (2) singularly, strictly speaking, only when (2) is interpreted, with respect to 'the number of planets', extensionally (or *de re*) — in which case (2) might be rendered by something like, "The *actual* number of planets (viz. 9) is necessarily composite". On the other hand, under intensional (*de dicto*) interpretation[5] of (2) 'the number of planets' is used functionally: It names or designates (refers to) a function from possible worlds to individuals — the value of this function for a possible world w is the number of planets in w. Then (2) says, strictly speaking: For every possible world w, the number of planets in w is composite.

V. Farewell to alethic intensionality

The account just given avoids the failure of PS altogether (i.e. avoids intensionality). All we need is to be clear about how we are using the key expressions, singularly or functionally. Let us confirm this by considering again

(2) as an example. Under the extensional (*de re*) reading of (2), the definite description 'the number of planets' amounts to, when more fully articulated, 'the number of planets in the actual world'. This "actualized" definite description stands for 9 irrespective of any considerations relating to possible worlds, and thus is unrestrictedly co-referential with '9'. In this case, there is not even an apparent failure of substitutivity, since "The number that actually is the number of planets, viz. 9, is necessarily composite" — which follows, by PS, from the co-reference of '9' and 'the actual number of planets', and (1), interpreted extensionally — is unproblematically true.

Secondly, on the functional interpretation of 'the number of planets' in (2), identity cannot, in any sense, hold between the referent of '9', which is an object (number), and the referent of 'the number of planets in a world x', which is a function, since there is a mismatch of entities: Although it may be said that a relation corresponding to identity holds between two functions when they are coextensive, there is certainly no such relation between an object and a function. Thus, in this case the co-reference condition is not fulfilled and thus PS is not applicable to begin with.

However, this is not the end of the matter because instead of an identity statement involving a proper name and a "proper" definite description, as in "9 is the actual number of planets", we might originally just as well have considered a case without proper names. Thus, let us consider the question of substitution more generally, e.g. the substitution of a definite description with a co-referential expression in the statement,

(5) Necessarily, the shortest ichthyologist is an ichthyologist.

Again, it may initially seem that the substitutivity principle PS is bound to be violated, because on the assumption that the shortest ichthyologist is the same as the richest spy (as it is typically sloppily put), the following need not share its truth value with (5):

Necessarily, the richest spy is an ichthyologist.

However, this apparent violation is again seen to be only apparent. For we must ask, What, exactly, is the expression we are substituting for in (5)? If it is, first, 'the shortest ichthyologist in the actual world', then (5) amounts to,

(6) The shortest ichthyologist of the actual world is necessarily an ichthyologist.

Now, it is clear that the only possibilities for an expression, 'a', to be co-referential with 'the shortest ichthyologist of the actual world' are (i) proper names, e.g. 'Joe Jones', (ii) *fully articulated* definite descriptions, e.g. 'the

richest spy of the actual world', and (iii) *fully articulated* indexicals and de-monstratives, e.g. 'you'-as-used-in-a-context and 'that'-as-used-in-a-context-with-a-demonstration. It should be clear that whatever truth value (6) has (it certainly appears to be false), this truth value is shared by

The person who is *a* is necessarily an ichthyologist.

As the second interpretation of (5), corresponding to the *de dicto* or functional reading of 'the number of planets', the only relation that gives rise to co-reference is the following "identity" between functions:

(7) The shortest ichthyologist in $x \approx$ the richest spy in x.

Here '$\approx$' stands for that identity-like relation between functions, which, as in-dicated, turns on the coincidence of extension or "course of values",[6] or, in other words, (7) is true just in case for every possible world w, the shortest ichthyologist in w is the same as the richest spy in w. However, there is no problem with this case either, because (7) is certainly false, and, in general it is immediately seen that if in (5), taken *de dicto*, we replace 'the shortest ich-thyologist' with an "incomplete" definite description that is co-referential with it (i.e. the referent of which is $\approx$-related to *the shortest ichthyologist in x*), we have an alethic statement that has the same truth value as (5), taken *de dicto*, has: For example, since 'the shortest fish-scientist in x' *is* co-referential with 'the shortest ichthyologist in x',

Necessarily, the shortest fish-scientist is an ichthyologist,

is, by substitutivity, true (assuming that (5) is true), which is as it should be. In short, a replacement with a co-referring functional expression, co-referring in the sense that referents are $\approx$-related, is *salva veritate*, and 'the shortest fish-scientist in x' names, while 'the richest spy in x' does not name, the same function as does 'the shortest ichthyologist in x'.

In the similar manner, we see that substitutions between predicates in alethic statements are always *salva veritate* as well, i.e. that such statements are *not* intensional with respect to predicates either. All in all, alethic state-ments are *not*, *pace* the received view, intensional.

VI. EXTENSIONALITY

Especially Quine [see, for instance, Quine (1956); (1960), Ch. 6] has held dear the idea of extensionality, holding that we should consider only

statements that are thoroughly extensional, and should also dispense with "intensional objects" (such as Fregean senses). The treatment of alethic statements I have given above may be seen important in that it makes such statements extensional and thus legitimate objects of "serious philosophy", and also in that no "creatures of darkness" are involved.[7]

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NOTES

[1] Quine (1953) and elsewhere. See also, e.g., Whitehead & Russell (1927), p. 187 and App. C; Carnap (1928), §§ 43-5 and Carnap (1947), e.g. § 11.

[2] For an early statement, see Hintikka (1957).

[3] See, e.g., Hintikka (1969). For instance, according to PSR we can get from (1) to (2) only if there are nine planets in all alternative worlds.

[4] This is a great idea especially with respect to the development of logic, for it leads directly to the introduction of quantifiers.

[5] There is an obvious terminological difficulty in my calling these interpretations extensional and intensional, for intensionality is defined as failure of substitutivity and I hold that there are no such failures (in connection with alethic statements). Perhaps I should say that I use the term intensional in the sense: PS apparently fails.

[6] Whenever such an identity-like relation holds, I think we may just as well say that 'the F in x' is co-referential with 'the G in x'.

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